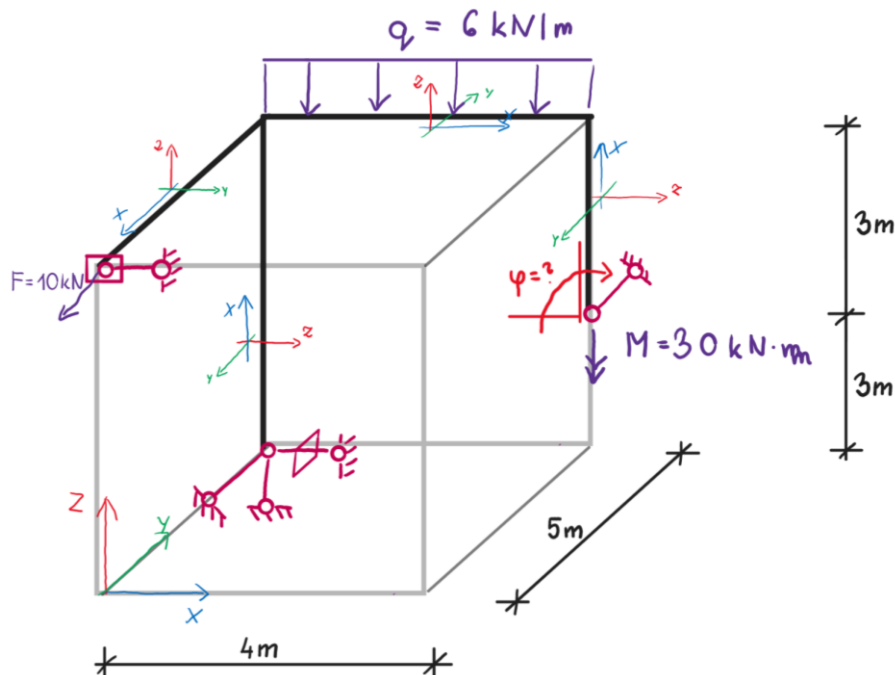
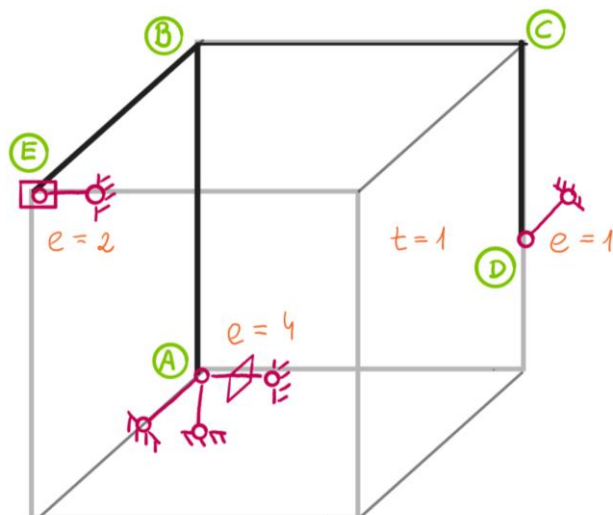


1. Rozwiązanie układu przestrzennego metodą sił i obliczenie przemieszczenia



Dane: $GI_x = 0,769 EI_y$; $EI_y = EI_z = \text{const.}$

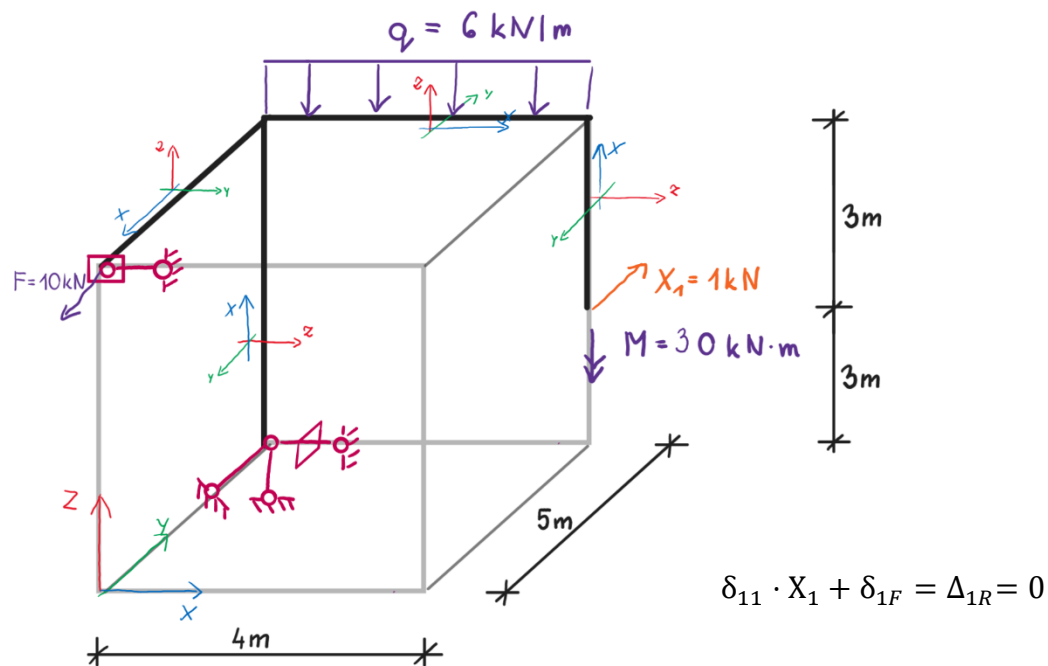
2. Obliczenie stopnia statycznej niewyznaczalności i sprawdzenie geometrycznej niezmienności układu



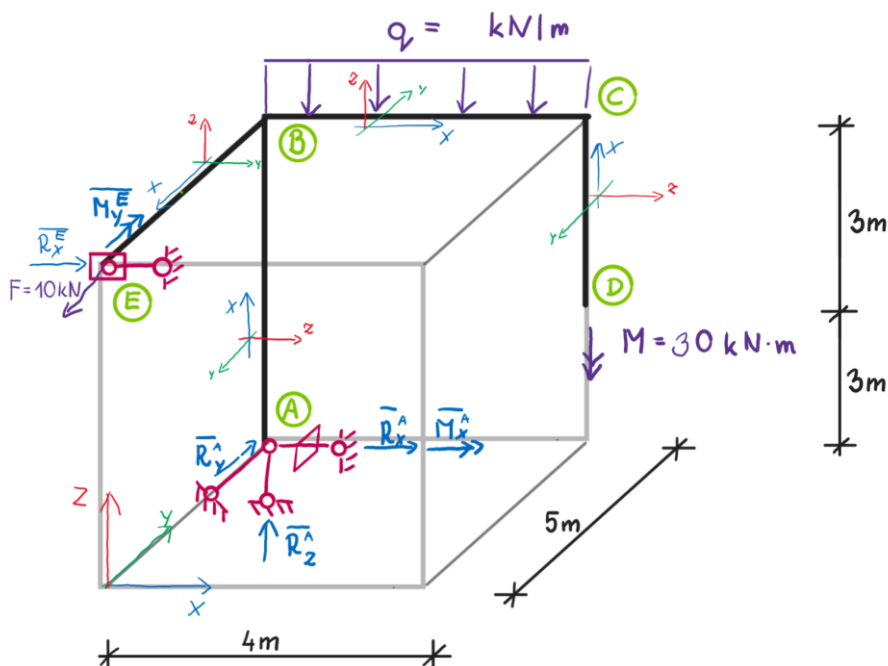
$$n_h = e - 6 \cdot t = 7 - 6 \cdot 1 = 1$$

Bryła sztywna jest podparta za pomocą więzi elementarnych blokujących przesuwu na wszystkich kierunkach X Y i Z i obrót względem osi X w węźle A. Za potencjalną oś obrotu uznano oś na kierunku osi Y w węźle A. Kierunki pozostałych więzi nie przecinają kierunku osi obrotu, co wyklucza jakiegokolwiek ruch bryły.

3. Dobranie układu podstawowego metody sił i postać ogólna układu równań metody sił



4. Rozwiązanie układu podstawowego metody sił od wpływów mechanicznych



Obliczenie reakcji:

$$\sum Y = 0 \Rightarrow \bar{R}_{yA}^F - F = 0 \Rightarrow \bar{R}_{yA}^F = 10 \text{ kN}$$

$$\sum Z = 0 \Rightarrow \bar{R}_{zA}^F - q \cdot 4 \text{ m} = 0 \Rightarrow \bar{R}_{zA}^F = 24 \text{ kN}$$

$$\sum M_x^A = 0 \Rightarrow \bar{M}_{xA}^F + F \cdot 6 \text{ m} = 0 \Rightarrow \bar{M}_{xA}^F = -60 \text{ kNm}$$

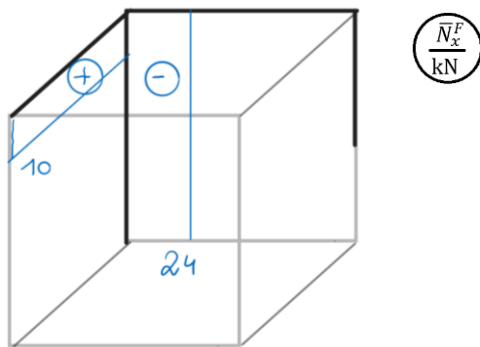
$$\sum M_z^A = 0 \Rightarrow \bar{R}_{xE}^F \cdot 5 \text{ m} - M = 0 \Rightarrow \bar{R}_{xE}^F = 6 \text{ kN}$$

$$\sum M_y^A = 0 \Rightarrow \bar{R}_{xE}^F \cdot 6 \text{ m} + \bar{M}_{yE}^F + q \cdot 4 \text{ m} \cdot 2 \text{ m} = 0 \Rightarrow \bar{M}_{yE}^F = -84 \text{ kNm}$$

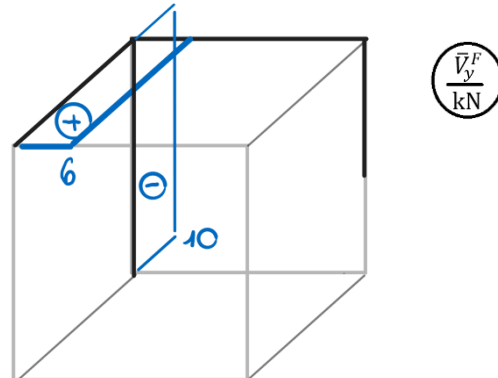
$$\sum X = 0 \Rightarrow \bar{R}_{xA}^F + \bar{R}_{xE}^F = 0 \Rightarrow \bar{R}_{xA}^F = -6 \text{ kN}$$

Obliczenie sił wewnętrznych

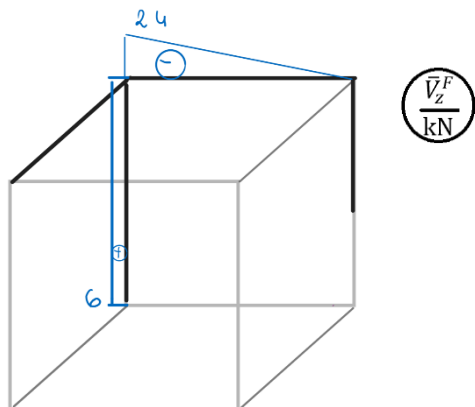
$$\begin{aligned}\bar{N}_{xEB}^F &= \bar{N}_{xBE}^F = F = 10 \text{ kN} \\ \bar{N}_{xAB}^F &= \bar{N}_{xBA}^F = -\bar{R}_{zA}^F = -24 \text{ kN} \\ \bar{N}_{xDC}^F &= \bar{N}_{xCD}^F = \bar{N}_{xCB}^F = \bar{N}_{xBC}^F =\end{aligned}$$



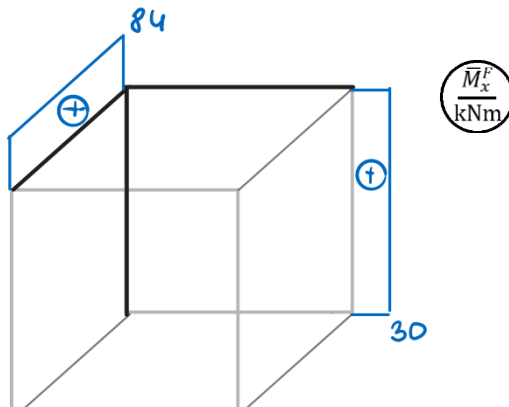
$$\begin{aligned}\bar{V}_{yEB}^F &= \bar{V}_{yBE}^F = \bar{R}_{xE}^F = 6 \text{ kN} \\ \bar{V}_{yDC}^F &= \bar{V}_{yCD}^F = \bar{V}_{yCB}^F = \bar{V}_{yBC}^F = 0 \\ \bar{V}_{yAB}^F &= \bar{V}_{yBA}^F = \bar{R}_{yA}^F = 10 \text{ kN}\end{aligned}$$



$$\begin{aligned}\bar{V}_{zEB}^F &= \bar{V}_{zBE}^F = \bar{V}_{zDC}^F = \bar{V}_{zCD}^F = 0 \\ \bar{V}_{zAB}^F &= \bar{V}_{zBA}^F = -\bar{R}_{xA}^F = 6 \text{ kN} \\ \bar{V}_{zCB}^F &= 0 \\ \bar{V}_{zBC}^F &= -q \cdot 4 \text{ m} = -24 \text{ kN}\end{aligned}$$

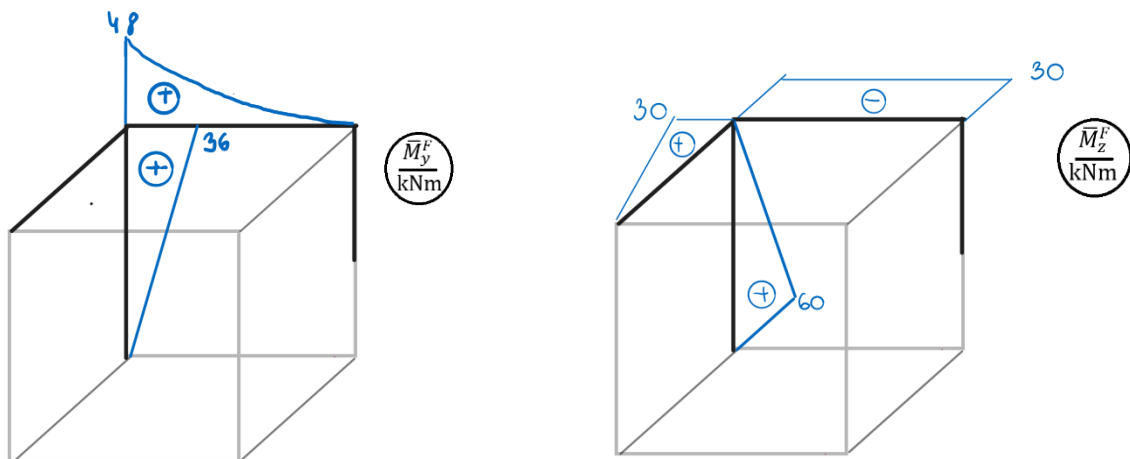


$$\begin{aligned}\bar{M}_{xEB}^F &= \bar{M}_{xBE}^F = -\bar{M}_{yE}^F = 84 \text{ kNm} \\ \bar{M}_{xDC}^F &= \bar{M}_{xCD}^F = M = 30 \text{ kNm} \\ \bar{M}_{xAB}^F &= \bar{M}_{xBA}^F = 0 \\ \bar{M}_{xCB}^F &= \bar{M}_{xCB}^F = 0\end{aligned}$$

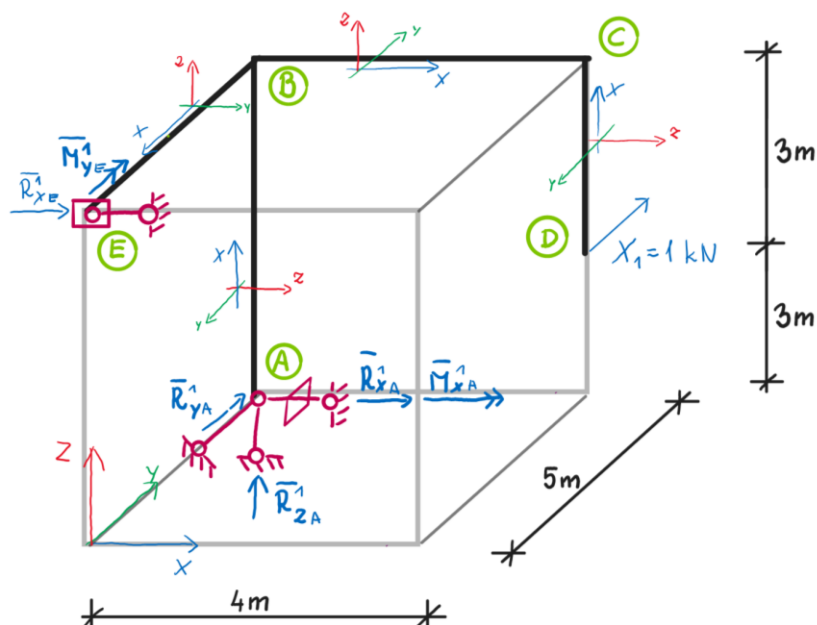


$$\begin{aligned}\bar{M}_{yEB}^F &= \bar{M}_{yBE}^F = 0 \\ \bar{M}_{yDC}^F &= \bar{M}_{yCD}^F = 0 \\ \bar{M}_{yAB}^F &= 0 \\ \bar{M}_{yBA}^F &= -\bar{R}_{xA}^F \cdot 6 \text{ m} = 36 \text{ kNm} \\ \bar{M}_{yCB}^F &= 0 \\ \bar{M}_{yBC}^F &= q \cdot 4 \text{ m} \cdot 2 \text{ m} = 48 \text{ kNm}\end{aligned}$$

$$\begin{aligned}\bar{M}_{zEB}^F &= 0 \\ \bar{M}_{zBE}^F &= \bar{R}_{xE}^F \cdot 3 \text{ m} = 30 \text{ kNm} \\ \bar{M}_{zDC}^F &= \bar{M}_{zCD}^F = 0 \\ \bar{M}_{zAB}^F &= -\bar{M}_{xA}^F = 60 \text{ kNm} \\ \bar{M}_{zBA}^F &= -\bar{M}_{xA}^F - \bar{R}_{yA}^F \cdot 6 \text{ m} = 0 \\ \bar{M}_{yCB}^F &= \bar{M}_{zBC}^F = -M = -30 \text{ kNm}\end{aligned}$$



5. Rozwiązanie układu podstawowego metody sił od $X_1=1\text{kN}$



Obliczenie reakcji:

$$\sum Y = 0 \Rightarrow \bar{R}_{yA}^1 + X_1 = 0 \Rightarrow \bar{R}_{yA}^1 = -1 \text{ kN}$$

$$\sum Z = 0 \Rightarrow \bar{R}_{zA}^1 = 0$$

$$\sum M_x^A = 0 \Rightarrow \bar{M}_{xA}^1 - X_1 \cdot 3 \text{ m} = 0 \Rightarrow \bar{M}_{xA}^1 = 3 \text{ kNm}$$

$$\sum M_z^A = 0 \Rightarrow \bar{R}_{xE}^1 \cdot 5 \text{ m} + X_1 \cdot 4 \text{ m} = 0 \Rightarrow \bar{R}_{xE}^{F1} = -0,8 \text{ kN}$$

$$\sum M_y^A = 0 \Rightarrow \bar{R}_{xE}^1 \cdot 6 \text{ m} + \bar{M}_{yE}^1 = 0 \Rightarrow \bar{M}_{yE}^1 = 4,8 \text{ kNm}$$

$$\sum X = 0 \Rightarrow \bar{R}_{xA}^1 + \bar{R}_{xE}^1 = 0 \Rightarrow \bar{R}_{xA}^1 = 0,8 \text{ kN}$$

Obliczenie sił wewnętrznych

$$\bar{N}_{xEB}^1 = \bar{N}_{xBE}^1 = 0$$

$$\bar{N}_{xAB}^1 = \bar{N}_{xBA}^1 = -\bar{R}_{zA}^1 = 0$$

$$\bar{N}_{xDC}^1 = \bar{N}_{xCD}^1 = 0$$

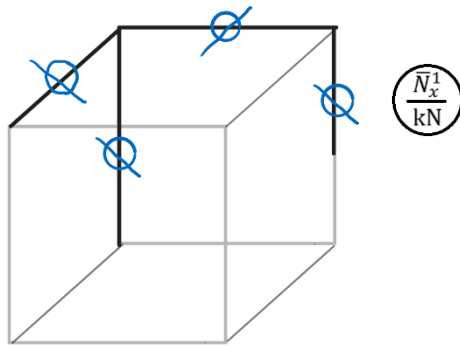
$$\bar{N}_{xCB}^1 = \bar{N}_{xBC}^1 = 0$$

$$\bar{V}_{yEB}^1 = \bar{V}_{yBE}^1 = \bar{R}_{xE}^F = -0,8 \text{ kN}$$

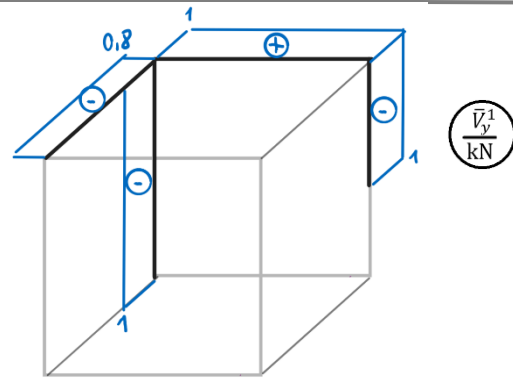
$$\bar{V}_{yDC}^F = \bar{V}_{yCD}^F = X_1 = 1 \text{ kN}$$

$$\bar{V}_{yCB}^1 = \bar{V}_{yBC}^1 = X_1 = 1 \text{ kN}$$

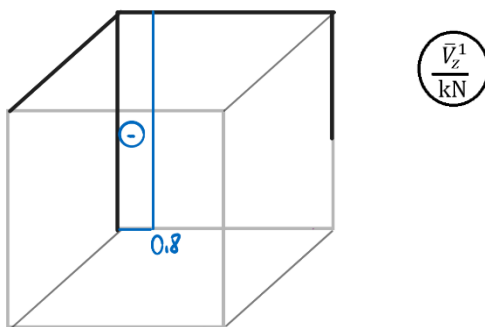
$$\bar{V}_{yAB}^1 = \bar{V}_{yBA}^1 = \bar{R}_{yA}^1 = -1 \text{ kN}$$



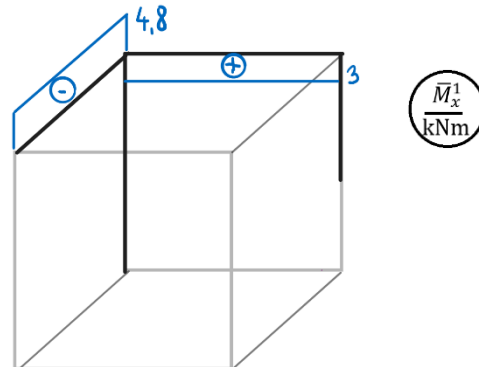
$$\begin{aligned}\bar{V}_{zEB}^1 &= \bar{V}_{zBE}^1 = \bar{V}_{zDC}^1 = \bar{V}_{zCD}^1 = 0 \\ \bar{V}_{zAB}^1 &= \bar{V}_{zBA}^1 = -\bar{R}_{xA}^1 = -0,8 \text{ kN} \\ \bar{V}_{zCB}^1 &= \bar{V}_{zBC}^1 = 0\end{aligned}$$



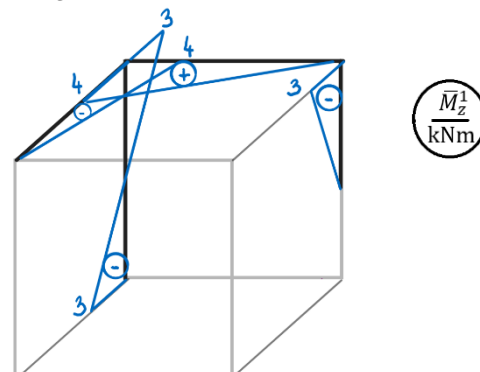
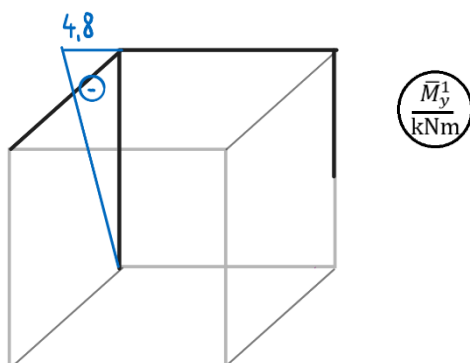
$$\begin{aligned}\bar{M}_{xEB}^1 &= \bar{M}_{xBE}^1 = \bar{M}_{yE}^F = -4,8 \text{ kNm} \\ \bar{M}_{xDC}^1 &= \bar{M}_{xCD}^1 = \\ \bar{M}_{xAB}^1 &= \bar{M}_{xBA}^1 = 0 \\ \bar{M}_{xBC}^1 &= \bar{M}_{xCB}^1 = X_1 \cdot 3 \text{ m} = 3 \text{ kNm}\end{aligned}$$



$$\begin{aligned}\bar{M}_{yEB}^1 &= \bar{M}_{yBE}^1 = 0 \\ \bar{M}_{yDC}^1 &= \bar{M}_{xCD}^1 = 0 \\ \bar{M}_{yAB}^1 &= 0 \\ \bar{M}_{yBA}^1 &= -\bar{R}_{xA}^1 \cdot 6 \text{ m} = \\ &= -0,8 \text{ kN} \cdot 6 \text{ m} = -4,8 \text{ kNm} \\ \bar{M}_{yCB}^1 &= \bar{M}_{yBC}^1 = 0\end{aligned}$$



$$\begin{aligned}\bar{M}_{zEB}^1 &= 0 \\ \bar{M}_{zBE}^1 &= \bar{R}_{xE}^F \cdot 5 \text{ m} = \\ &= -0,8 \text{ kN} \cdot 5 \text{ m} = -4 \text{ kNm} \\ \bar{M}_{zDC}^1 &= 0 \\ \bar{M}_{zCD}^1 &= -X_1 \cdot 3 \text{ m} = -3 \text{ kNm} \\ \bar{M}_{zAB}^1 &= -\bar{M}_{xA}^1 = -3 \text{ kNm} \\ \bar{M}_{zBA}^1 &= -\bar{M}_{xA}^1 - \bar{R}_{yA}^1 \cdot 6 \text{ m} = \\ &= -3 \text{ kNm} - (-1) \text{ kN} \cdot 6 \text{ m} = 3 \text{ kNm} \\ \bar{M}_{yCB}^1 &= 0 \\ \bar{M}_{zBC}^1 &= X_1 \cdot 4 \text{ m} = 4 \text{ kNm}\end{aligned}$$



6. Obliczenie współczynników i wyrazów wolnych układu równań metody sił

$$\delta_{11} \cdot X_1 + \delta_{1F} = \Delta_{1R} = 0$$

$$\begin{aligned} 1 \text{ kN} \cdot \delta_{11} &= \sum_p \int \frac{\bar{M}_x^1 \bar{M}_x^1}{GI_x} dx + \sum_p \int \frac{\bar{M}_y^1 \bar{M}_y^1}{EI_y} dx + \sum_p \int \frac{\bar{M}_z^1 \bar{M}_z^1}{EI_z} dx + \sum_s \frac{\bar{S}_\delta^1 \bar{S}_\delta^1}{k_\delta} \\ 1 \text{ kN} \cdot \delta_{1F} &= \sum_p \int \frac{\bar{M}_x^1 \bar{M}_x^F}{GI_x} dx + \sum_p \int \frac{\bar{M}_y^1 \bar{M}_y^F}{EI_y} dx + \sum_p \int \frac{\bar{M}_z^1 \bar{M}_z^F}{EI_z} dx + \sum_s \frac{\bar{S}_\delta^1 \bar{S}_\delta^F}{k_\delta} \end{aligned}$$

Ustalenie jednostek współczynników:

$$[\delta_{11}] = [\delta_{1F}] = \frac{1 \text{ kNm} \cdot \text{kNm} \cdot \text{m}}{\text{kN} EI_y} = \frac{1 \text{ kNm} \cdot \text{kNm} \cdot \text{m}}{\text{kN} \frac{\text{kN}}{\text{m}^2} \cdot \text{m}^4} = \text{m}$$

Obliczenie współczynników i wyrazów wolnych układu równań metody sił:

$$\begin{aligned} \delta_{11} &= \frac{1}{GI_x} [5 \cdot (-4,8) \cdot (-4,8) + 4 \cdot 3 \cdot 3] \text{kNm}^3 + \frac{1}{EI_y} \left[\frac{1}{2} \cdot 6 \cdot (-4,8) \cdot \frac{2}{3} \cdot (-4,8) \right] \text{kNm}^3 \\ &\quad + \frac{1}{EI_z} \left[\frac{1}{2} \cdot 5 \cdot (-4) \cdot \frac{2}{3} \cdot (-4) + \frac{1}{2} \cdot 3 \cdot (-3) \cdot \frac{2}{3} \cdot (-3) + \frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 + \frac{1}{2} \cdot 3 \cdot (-3) \cdot \frac{2}{3} \cdot (-3) + \frac{1}{2} \cdot 3 \cdot 3 \cdot \frac{2}{3} \cdot 3 \right] \text{kNm}^3 \\ &= \frac{1}{0,769 EI_y} [115,2 + 36] \text{kNm}^3 + \frac{1}{EI_y} [46,08] \text{kNm}^3 \\ &\quad + \frac{1}{EI_y} [26, (6) + 9 + 21, (3) + 9 + 9] \text{kNm}^3 = 317,6990 \frac{\text{kNm}^3}{EI_y} \\ \delta_{1F} &= \frac{1}{GI_x} [5 \cdot (-4,8) \cdot 84] \text{kNm}^3 + \frac{1}{EI_y} \left[\frac{1}{2} \cdot 6 \cdot (-4,8) \cdot \frac{2}{3} \cdot 36 \right] \text{kNm}^3 \\ &\quad + \frac{1}{EI_z} \left[\frac{1}{2} \cdot 5 \cdot (-4) \cdot \frac{2}{3} \cdot 30 + \frac{1}{2} \cdot 4 \cdot 4 \cdot (-30) + \frac{1}{2} \cdot 6 \cdot 60 \cdot \frac{1}{3} \cdot (-3) \right] \text{kNm}^3 \\ &= \frac{1}{0,769 EI_y} [-2016] \text{kNm}^3 + \frac{1}{EI_y} [-345,6] \text{kNm}^3 \\ &\quad + \frac{1}{EI_z} [-200 - 240 - 180] \text{kNm}^3 = -3587,1865 \frac{\text{kNm}^3}{EI_y} \end{aligned}$$

7. Postać szczegółowa układu równań metody sił i jego rozwiązanie

$$\begin{aligned} 317,6990 \frac{\text{kNm}^3}{EI_y} \cdot X_1 - 3587,1865 \frac{\text{kNm}^3}{EI_y} &= 0 \\ X_1 &= 11,2911 \end{aligned}$$

8. Obliczenie sił wewnętrznych i sporządzenie wykresów

$$R_{xij}^F = \bar{R}_{xij}^1 \cdot X_1 + R_{xij}^F$$

$$N_{xij}^F = \bar{N}_{xij}^1 \cdot X_1 + \bar{N}_{xij}^F$$

$$V_{yij}^F = \bar{V}_{yij}^1 \cdot X_1 + \bar{V}_{yij}^F$$

$$V_{zij}^F = \bar{V}_{zij}^1 \cdot X_1 + \bar{V}_{zij}^F$$

$$M_{yij}^F = \bar{M}_{yij}^1 \cdot X_1 + \bar{M}_{yij}^F$$

$$V_{zij}^F = \bar{V}_{zij}^1 \cdot X_1 + \bar{V}_{zij}^F$$

$$M_{zij}^F = \bar{M}_{zij}^1 \cdot X_1 + \bar{M}_{zij}^F$$

Obliczenie reakcji:

$$R_{xA}^F = \bar{R}_{xA}^1 \cdot X_1 + R_{xA}^F = 0,8\text{kN} \cdot 11,2911 + (-6)\text{kN} = 3,0329\text{kN}$$

$$R_{yA}^F = \bar{R}_{yA}^1 \cdot X_1 + R_{yA}^F = -1\text{kN} \cdot 11,2911 + 10\text{kN} = -1,2911\text{kN}$$

$$R_{zA}^F = \bar{R}_{zA}^1 \cdot X_1 + R_{zA}^F = 0 \cdot 11,2911 + 24\text{kN} = 24\text{kN}$$

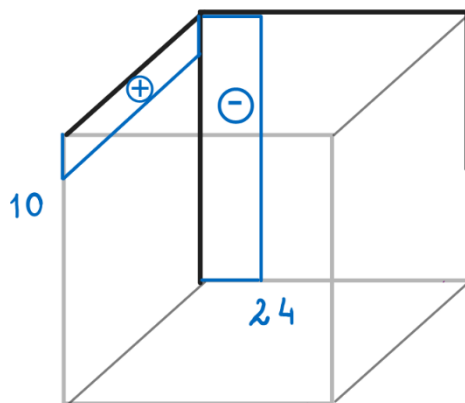
$$M_{xA}^F = \bar{M}_{xA}^1 \cdot X_1 + M_{xA}^F = 3\text{kNm} \cdot 11,2911 + (-60)\text{kNm} = -26,1266\text{kNm}$$

$$R_{xE}^F = \bar{R}_{xE}^1 \cdot X_1 + R_{xE}^F = -0,8\text{kN} \cdot 11,2911 + 6\text{kN} = -3,0329\text{kN}$$

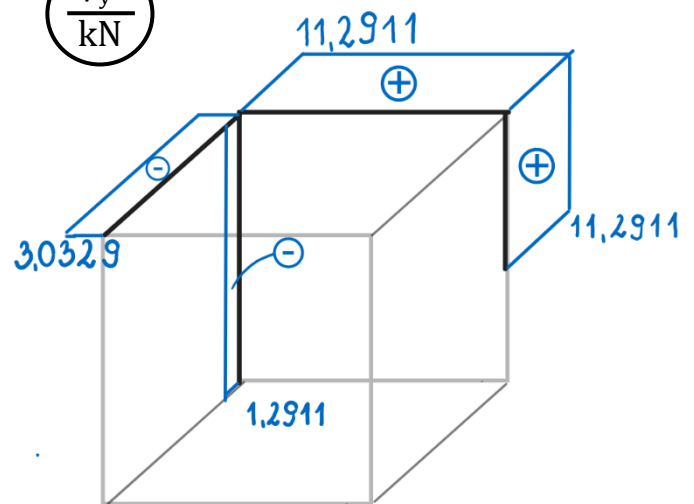
$$M_{yE}^F = \bar{M}_{yE}^1 \cdot X_1 + M_{yE}^F = 4,8\text{kNm} \cdot 11,2911 - 84\text{kNm} = -29,8025\text{kNm}$$

Sporządzenie wykresów:

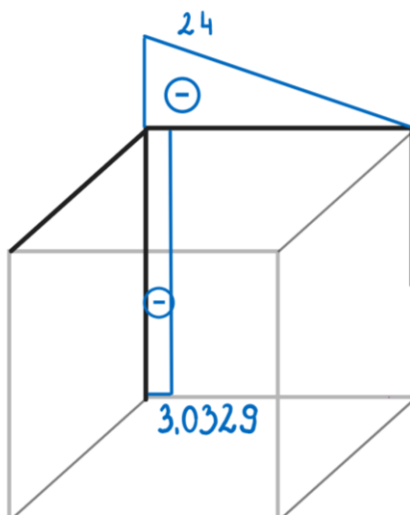
$$\frac{N_x}{\text{kN}}$$



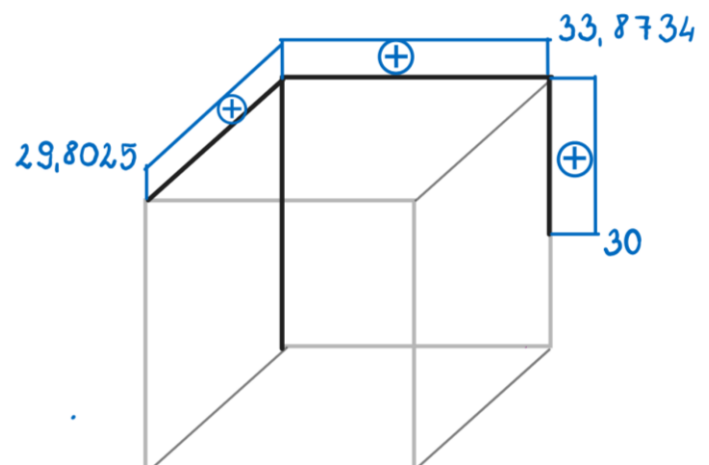
$$\frac{V_y}{\text{kN}}$$



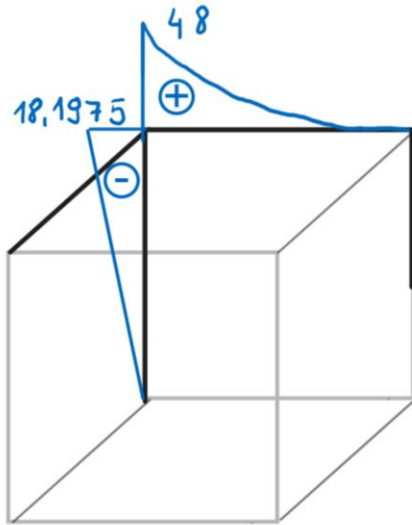
$$\frac{V_z}{\text{kN}}$$



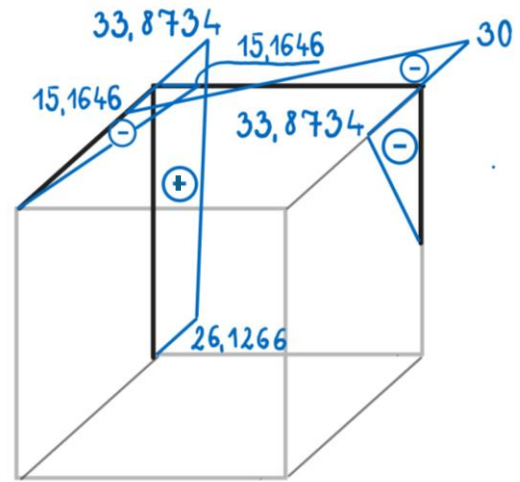
$$\frac{M_x}{\text{kNm}}$$



$\frac{M_y}{\text{kNm}}$

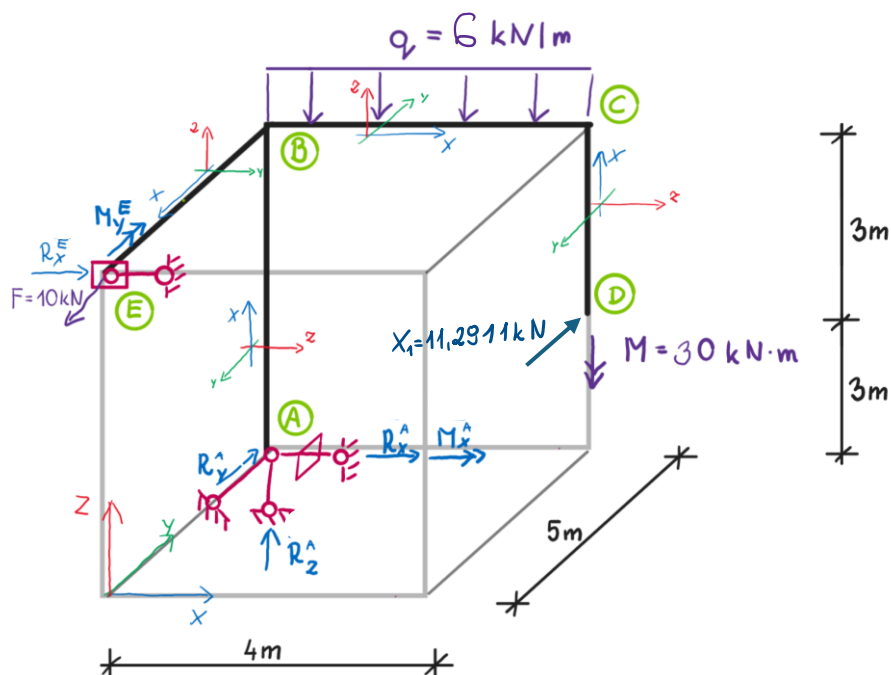


$\frac{M_z}{\text{kNm}}$



9. Kontrola statyczna rozwiązania

Globalna równowaga sił



$$\sum X = 0 \Rightarrow R_{xA}^1 + R_{xE}^1 = 3,0329 \text{ kN} + (-3,0329) \text{ kN} = 0$$

$$\sum Y = 0 \Rightarrow R_{yA} + X_1 - F = -1,2911 \text{ kN} + 11,2911 \text{ kN} - 10 \text{ kN} = 0$$

$$\sum Z = 0 \Rightarrow R_{zA} - q \cdot 4 \text{ m} = 24 \text{ kN} - 6 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} = 0$$

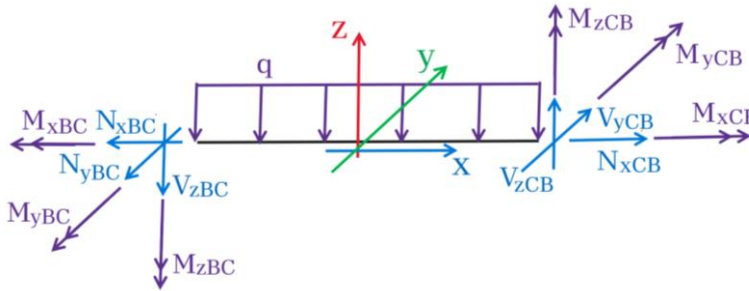
$$\sum M_x^A = 0 \Rightarrow M_{xA} - X_1 \cdot 3 \text{ m} + F \cdot 6 \text{ m} = -26,1266 \text{ kNm} - 11,2911 \text{ kN} \cdot 3 \text{ m} + 10 \text{ kN} \cdot 6 \text{ m} = 0$$

$$\sum M_y^A = 0 \Rightarrow R_{xE} \cdot 6 \text{ m} + M_{yE} - q \cdot 4 \text{ m} \cdot 2 \text{ m} =$$

$$-3,0329 \text{ kN} \cdot 6 \text{ m} - 1,2911 \text{ kN} + 6 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} \cdot 2 \text{ m} = 0$$

$$\sum M_z^A = 0 \Rightarrow R_{xE} \cdot 5 \text{ m} + X_1 \cdot 4 \text{ m} - M = -3,0329 \text{ kN} \cdot 5 \text{ m} + 11,2911 \cdot 4 \text{ m} - 30 \text{ kNm} = 0$$

Równowaga sił w przęcie BC (liczona względem lokalnego układu współrzędnych):



$$\sum X = 0 \Rightarrow N_{xCB} - N_{zBC} = 0 + 0 = 0$$

$$\sum Y = 0 \Rightarrow V_{yCB} - V_{yBC} = 11,2911 \text{ kN} - 11,2911 \text{ kN} = 0$$

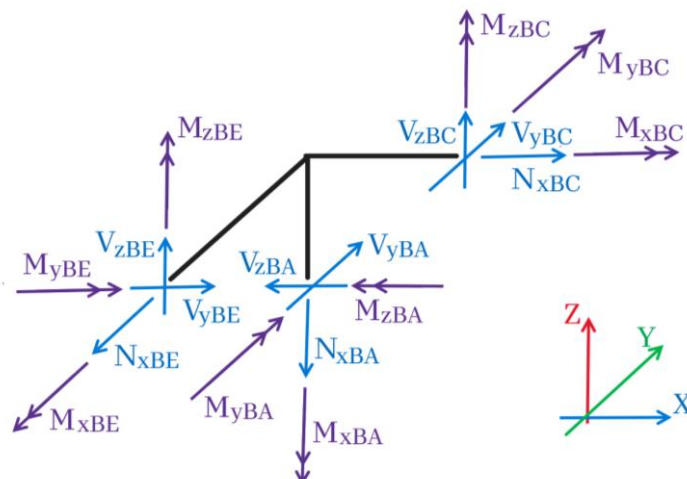
$$\sum Z = 0 \Rightarrow V_{zCB} - V_{zBC} - q \cdot 4 \text{ m} = 0 - (-24) \text{ kN} - 6 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} = 0$$

$$\sum M_{x,B} = 0 \Rightarrow M_{xCB} - M_{xBC} = 33,8734 \text{ kNm} - 33,8734 \text{ kNm} = 0$$

$$\begin{aligned} \sum M_{y,B} = 0 \Rightarrow M_{yCB} - M_{zBC} - V_{zCB} \cdot 4 \text{ m} + q \cdot 4 \text{ m} \cdot 2 \text{ m} \\ = 0 - 48 \text{ kNm} - 0 \cdot 4 \text{ m} + 6 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} \cdot 2 \text{ m} = 0 \end{aligned}$$

$$\begin{aligned} \sum M_{z,B} = 0 \Rightarrow M_{zCB} - M_{zBC} + V_{yCB} \cdot 4 \text{ m} \\ = -30 \text{ kNm} - 15,1646 \text{ kNm} + 11,2911 \text{ kNm} \cdot 4 \text{ m} = 0 \end{aligned}$$

Równowaga sił w węźle B (liczona względem globalnego układu współrzędnych):



$$\sum X = 0 \Rightarrow N_{xBC} - V_{zBA} + V_{yBE} = 0 - (-3,0329)\text{kN} + (-3,0329)\text{kN} = 0$$

$$\sum Y = 0 \Rightarrow V_{yBC} + V_{yBA} - N_{xBE} = 11,2911\text{kN} + (-1,2911)\text{kN} - 10\text{kN} = 0$$

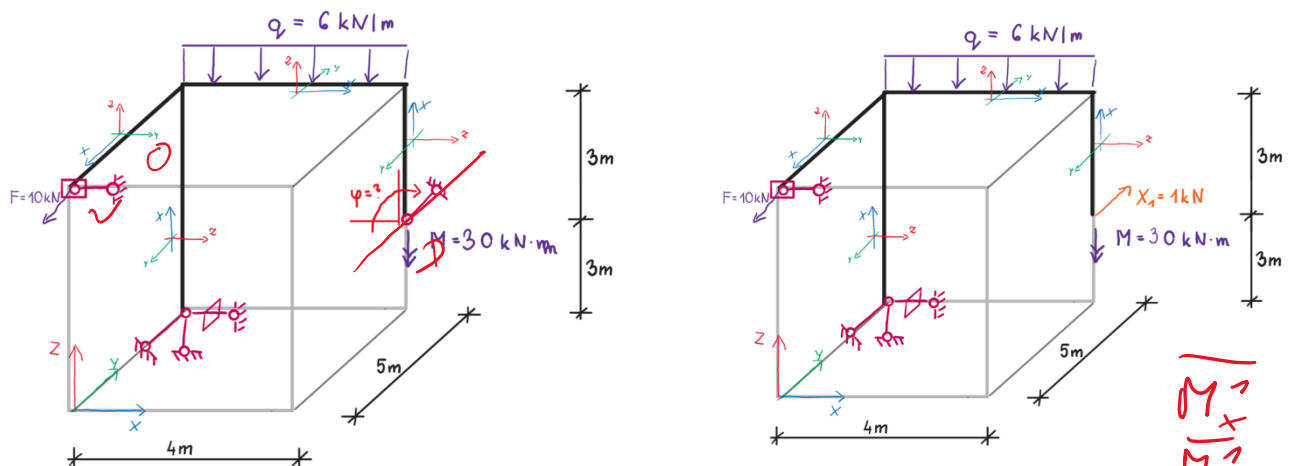
$$\sum Z = 0 \Rightarrow V_{zBC} - N_{xBA} + V_{zBE} = -24\text{kN} - (24)\text{kN} + 0 = 0$$

$$\sum M_x = 0 \Rightarrow M_{xBC} - M_{zBA} + M_{yBE} = 33,8734\text{kNm} - (33,8734)\text{kNm} + 0 = 0$$

$$\sum M_y = 0 \Rightarrow M_{yBC} + M_{yBA} - M_{xBE} = 48\text{kNm} + (-18,1975)\text{kNm} - 29,8025\text{kNm} = 0$$

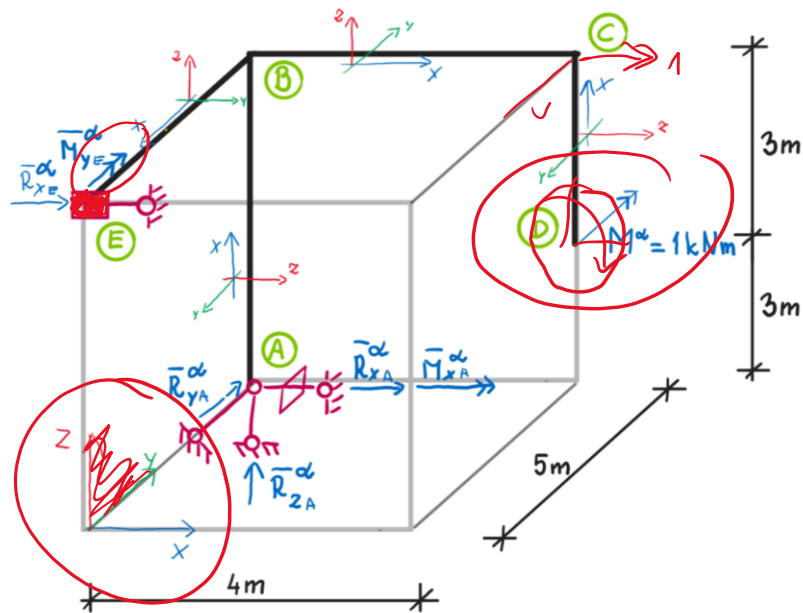
$$\sum M_z = 0 \Rightarrow M_{zBC} - M_{xBA} + M_{zBE} = 15,1646\text{kNm} - 0 + (-15,1646)\text{kNm} = 0$$

10. Kontrola kinematyczna rozwiązania



$$\begin{aligned} 1\text{kN} \cdot \delta_{1F} &= \sum_p \int \frac{\bar{M}_x^1 M_x^F}{GI_x} dx + \sum_p \int \frac{\bar{M}_y^1 M_y^F}{EI_y} dx + \sum_p \int \frac{\bar{M}_z^1 M_z^F}{EI_z} dx + \sum_s \frac{\bar{S}_\delta^1 S_\delta^F}{k_\delta} \\ &= \frac{1}{GI_x} [5 \cdot (-4,8) \cdot 29,8025 + 4 \cdot 3 \cdot 33,8734] \text{kNm}^3 \\ &\quad + \frac{1}{EI_y} \left[\frac{1}{2} \cdot 6 \cdot (-4,8) \cdot \frac{2}{3} \cdot (-18,1975) \right] \text{kNm}^3 \\ &\quad + \frac{1}{EI_z} \left\{ \frac{1}{2} \cdot 5 \cdot (-4) \cdot \frac{2}{3} \cdot (-15,1646) + \frac{1}{2} \cdot 3 \cdot (-3) \cdot \frac{2}{3} \cdot (-33,8734) + \frac{4}{6} \right. \\ &\quad \cdot \left[15,1646 \cdot 4 + 4 \cdot \frac{15,1646 - 30}{2} \cdot 2 + (-30) \cdot 0 \right] + \frac{6}{6} \\ &\quad \cdot \left[(-26,1266) \cdot 3 + 4 \cdot \frac{-15,1646 - 26,1266}{2} \cdot 0 + (-33,8734) \cdot (-3) \right] \left. \right\} \text{kNm}^3 \\ &= \frac{1}{0,769EI_y} [-715,26 + 406,4807] \text{kNm}^3 + \frac{1}{EI_y} [174,696] \text{kNm}^3 \\ &\quad + \frac{1}{EI_y} [101,0973 + 101,6202 + 0,8763 + 23,2404] \text{kNm}^3 = -0,0034 \frac{\text{kNm}^3}{EI_y} \end{aligned}$$

11. Obliczenie szukanego przemieszczenia



Obliczenie reakcji:

$$\sum Y = 0 \Rightarrow \bar{R}_{yA}^{\alpha} = 0$$

$$\sum Z = 0 \Rightarrow \bar{R}_{zA}^{\alpha} = 0$$

$$\sum M_x^A = 0 \Rightarrow \bar{M}_{xA}^{\alpha} = 0$$

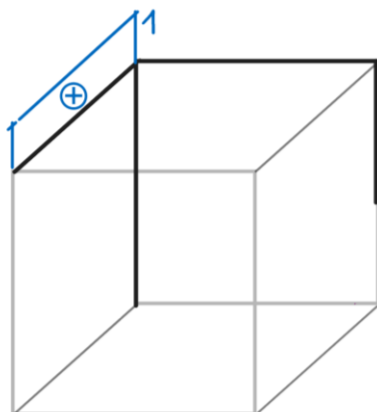
$$\sum M_z^A = 0 \Rightarrow \bar{R}_{xE}^{\alpha} = 0$$

$$\sum M_y^A = 0 \Rightarrow \bar{R}_{xE}^{\alpha} \cdot 6 \text{ m} + \bar{M}_{yE}^{\alpha} + M^{\alpha} = 0 \Rightarrow \bar{M}_{yE}^{\alpha} = -1 \text{ kNm}$$

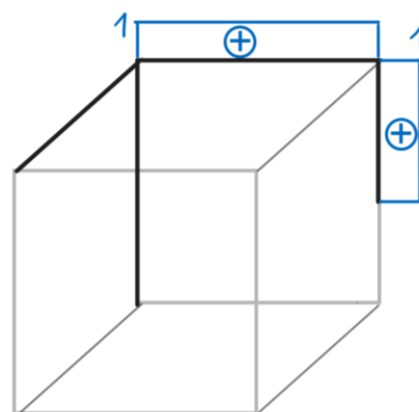
$$\sum X = 0 \Rightarrow \bar{R}_{xA}^{\alpha} + \bar{R}_{xE}^{\alpha} = 0 \Rightarrow \bar{R}_{xA}^{\alpha} = 0$$

Sporządzenie wykresów:

$\frac{\bar{M}_x}{\text{kNm}}$



$\frac{\bar{M}_y}{\text{kNm}}$



Obliczenie szukanego przemieszczenia:

$$\begin{aligned} 1 \text{ kN} \cdot \delta_{\alpha F} &= \sum_p \int \frac{\bar{M}_x^\alpha M_x^F}{GI_x} dx + \sum_p \int \frac{\bar{M}_y^\alpha M_y^F}{EI_y} dx + \sum_p \int \frac{\bar{M}_z^\alpha M_z^F}{EI_z} dx + \sum_s \frac{\bar{S}_\delta^\alpha S_\delta^F}{k_\delta} \\ &= \frac{1}{GI_x} [5 \cdot 1 \cdot 29,8025] \text{ kNm}^3 + \frac{1}{EI_y} \left[\frac{4}{6} \cdot [1 \cdot 48 + 4 \cdot 1 \cdot 12 + 1 \cdot 0] \right] \text{ kNm}^3 \\ &= \frac{1}{0,769 EI_y} 149,0125 \text{ kNm}^3 + \frac{1}{EI_y} 64 \text{ kNm}^3 = 257,7744 \frac{\text{kNm}^3}{EI_y} \end{aligned}$$