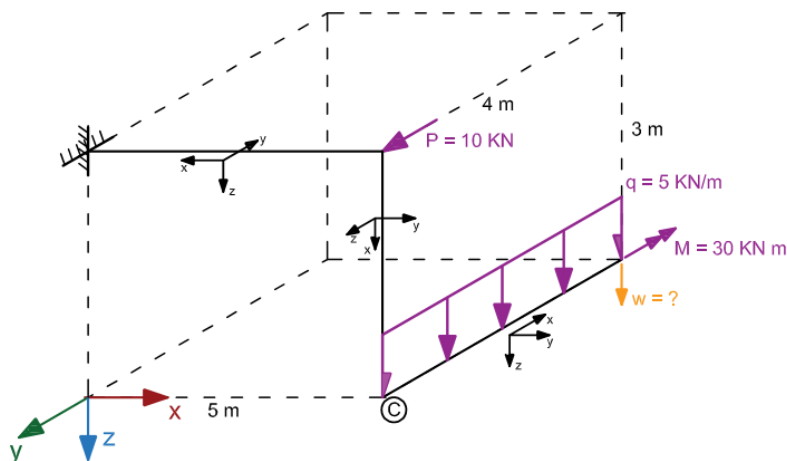
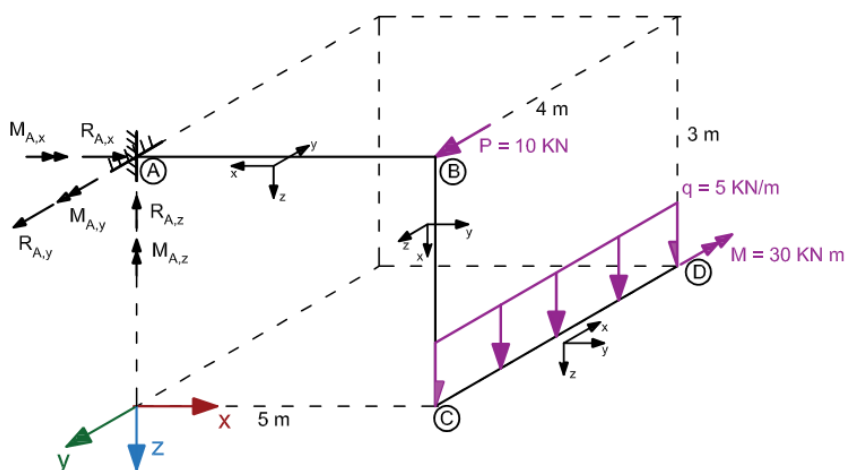


Dany jest hiperstatyczny układ przestrzenny o schemacie statycznym i obciążeniu jak na rysunku. Zakładamy, że $GI_s = 0,5 EI_y$ oraz $EI_y = EI_z$. Obliczyć wskazane przemieszczenie w:



1. Obliczenie reakcji od obciążenia danego



$$\sum X = 0 \rightarrow R_{A,x} = 0,$$

$$\sum Y = 0 \rightarrow R_{A,y} + P = 0 \rightarrow R_{A,y} = -P \rightarrow R_{A,y} = -10 \text{ kN},$$

$$\sum Z = 0 \rightarrow R_{A,z} - q \cdot 4 \text{ m} \rightarrow R_{A,z} = q \cdot 4 \text{ m} \rightarrow R_{A,z} = 5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} \rightarrow R_{A,z} = 20 \text{ kN},$$

$$\sum M_{xA} = 0 \rightarrow M_{A,x} - q \cdot 4 \text{ m} \cdot 2 \text{ m} = 0 \rightarrow M_{A,x} = q \cdot 4 \text{ m} \cdot 2 \text{ m} = 5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} \cdot 2 \text{ m} \rightarrow M_{A,x} = 40 \text{ kN m},$$

$$\sum M_{yA} = 0 \rightarrow M_{A,y} - q \cdot 4 \text{ m} \cdot 5 \text{ m} - M = 0 \rightarrow M_{A,y} = q \cdot 4 \text{ m} \cdot 5 \text{ m} + M = 5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} \cdot 5 \text{ m} + 30 \text{ kN m} \rightarrow M_{A,y} = 130 \text{ kN m},$$

$$\sum M_{z,A} = 0 \rightarrow M_{A,z} = 0.$$

1. Obliczenie sił wewnętrznych i sporządzenie wykresów

Siły osiowe:

$$N_{x,DC} = N_{x,CD} = 0,$$

$$N_{x,CB} = N_{x,BC} = q \cdot 4 \text{ m} = 5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} = 20 \text{ kN m},$$

$$N_{x,BA} = N_{x,AB} = 0.$$

Siły tnące:

$$V_{y,DC} = V_{y,CD} = 0,$$

$$V_{y,CB} = V_{y,BC} = 0,$$

$$V_{z,BA} = V_{z,AB} = P = 10 \text{ kN},$$

$$V_{z,DC} = 0,$$

$$V_{z,CD} = q \cdot 4 \text{ m} = 5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} = 20 \text{ kN m},$$

$$V_{z,CB} = V_{z,BC} = 0,$$

$$V_{z,BA} = V_{z,AB} = -q \cdot 4 \text{ m} = -5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} = -20 \text{ kN m}.$$

Momenty skręcające:

$$M_{x,DC} = M_{x,CD} = M = 30 \text{ kN m},$$

$$M_{x,CB} = M_{x,BC} = 0,$$

$$M_{x,BA} = M_{x,AB} = -q \cdot 4 \text{ m} \cdot 2 \text{ m} = -5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} \cdot 2 \text{ m} = -40 \text{ kN m}.$$

Momenty zginające:

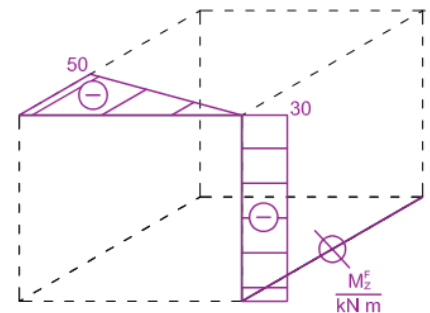
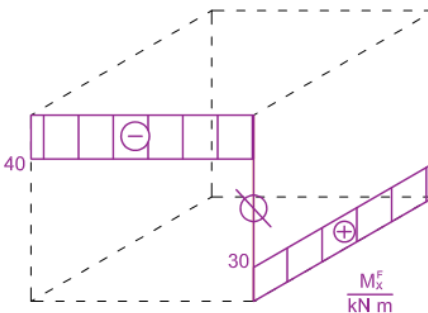
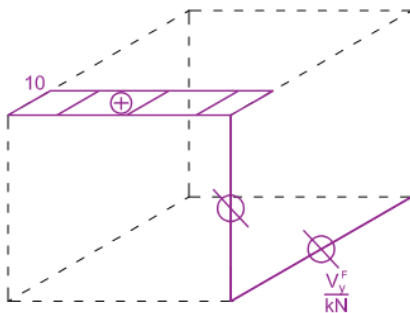
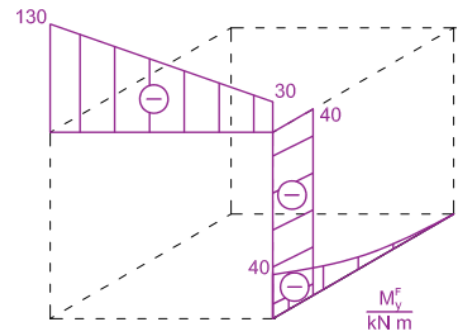
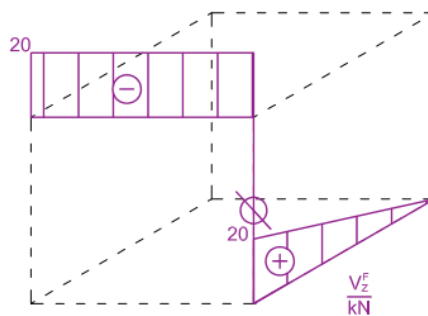
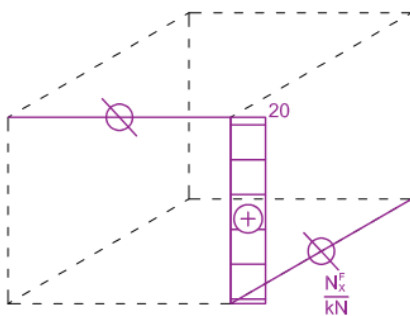
$$M_{y,DC} = 0,$$

$$M_{y,CD} = -q \cdot 4 \text{ m} \cdot 2 \text{ m} = -5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} \cdot 2 \text{ m} = -40 \text{ kN m},$$

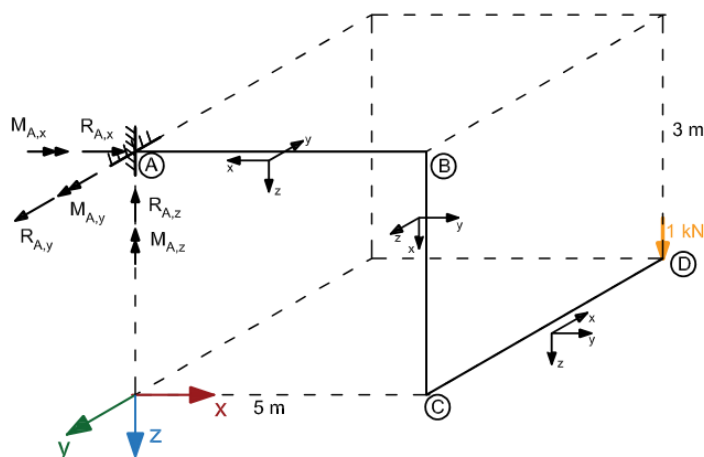
$$M_{y,CB} = M_{y,BC} = -q \cdot 4 \text{ m} \cdot 2 \text{ m} = -5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} \cdot 2 \text{ m} = -40 \text{ kN m},$$

$$M_{y,BA} = -M = -30 \text{ kN m},$$

$$M_{x,AB} = -M - q \cdot 4 \text{ m} \cdot 5 \text{ m} = -30 \text{ kN m} - 5 \frac{\text{kN}}{\text{m}} \cdot 4 \text{ m} \cdot 5 \text{ m} = -130 \text{ kN m}.$$



2. Obliczenie reakcji od obciążenia jednostkowego



$$\sum X = 0 \rightarrow R_{A,x} = 0,$$

$$\sum Y = 0 \rightarrow R_{A,y} = 0,$$

$$\sum Z = 0 \rightarrow R_{A,z} = 1 \text{ kN},$$

$$\sum M_{xA} = 0 \rightarrow M_{A,x} - 1 \text{ kN} \cdot 4 \text{ m} = 0 \rightarrow M_{A,x} = 4 \text{ kN m},$$

$$\sum M_{yA} = 0 \rightarrow M_{A,y} - 1 \text{ kN} \cdot 5 \text{ m} = 0 \rightarrow M_{A,y} = 5 \text{ kN m},$$

$$\sum M_{z,A} = 0 \rightarrow M_{A,z} = 0.$$

Siły osiowe:

$$N_{x,DC} = N_{x,CD} = 0,$$

$$N_{x,CB} = N_{x,BC} = 1 \text{ kN},$$

$$N_{x,BA} = N_{x,AB} = 0.$$

Siły tnące:

$$V_{y,DC} = V_{y,CD} = 0,$$

$$V_{y,CB} = V_{y,BC} = 0,$$

$$V_{z,BA} = V_{z,AB} = 0,$$

$$V_{z,DC} = V_{z,CD} = 1 \text{ kN},$$

$$V_{z,CB} = V_{z,BC} = 0,$$

$$V_{z,BA} = V_{z,AB} = -1 \text{ kN}.$$

Momenty skręcające:

$$M_{x,DC} = M_{x,CD} = 0,$$

$$M_{x,CB} = M_{x,BC} = 0,$$

$$M_{x,BA} = M_{x,AB} = -1 \text{ kN} \cdot 4 \text{ m} = -4 \text{ kN m}.$$

Momenty zginające:

$$M_{y,DC} = 0,$$

$$M_{y,CD} = -1 \text{ kN} \cdot 4 \text{ m} = -4 \text{ kN m},$$

$$M_{y,CB} = M_{y,BC} = -1 \text{ kN} \cdot 4 \text{ m} = -4 \text{ kN m},$$

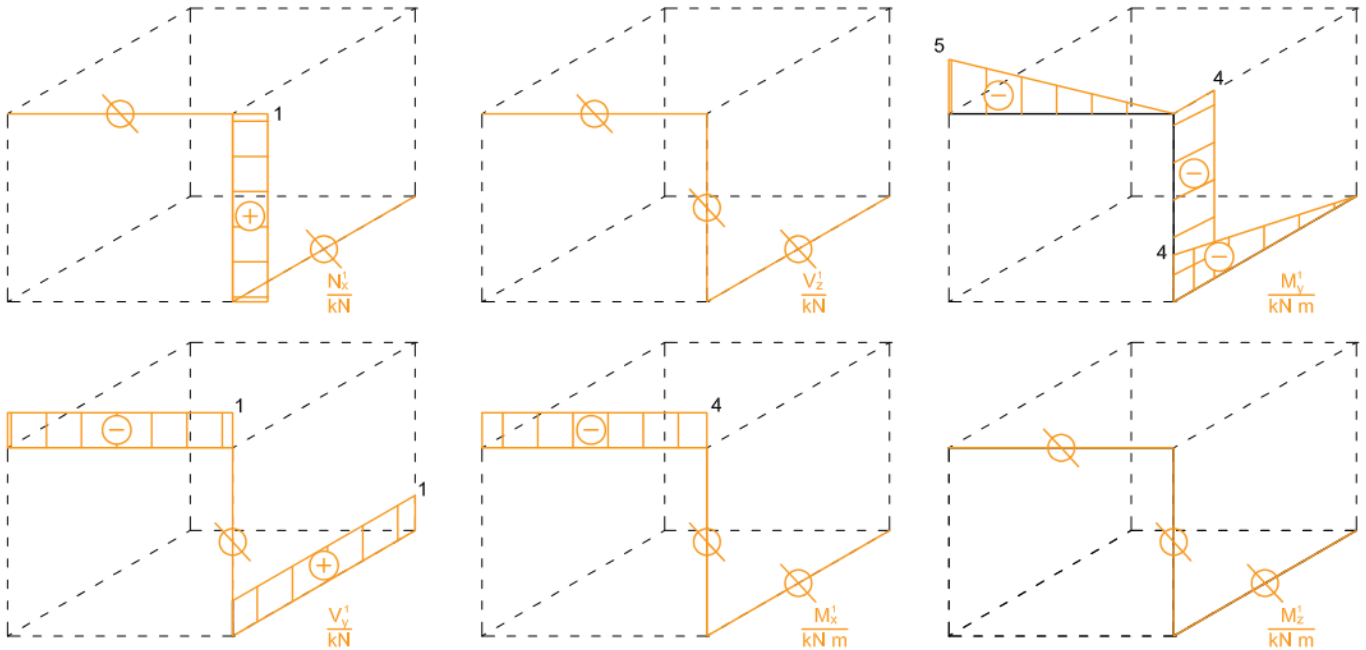
$$M_{y,BA} = 0,$$

$$M_{y,AB} = -1 \text{ kN} \cdot 5 \text{ m} = -5 \text{ kN m}.$$

$$M_{z,DC} = M_{z,CD} = 0,$$

$$M_{z,CB} = M_{z,BC} = 0,$$

$$M_{z,BA} = M_{z,AB} = 0.$$



3. Obliczenie szukanego przemieszczenia

$$\begin{aligned}
 1kN \cdot w &= \int \frac{M_y^1 M_y^F}{EI_y} dx + \int \frac{M_z^1 M_z^F}{EI_z} dx + \int \frac{M_x^1 M_x^F}{GI_x} dx = \frac{1}{GI_x} \cdot 5m \cdot (-40kN \cdot m) \cdot (-4kN \cdot m) \\
 &+ \frac{4m}{6EI_y} [0 \cdot 0 + 4 \cdot (-10kN \cdot m) \cdot (-2kN \cdot m) + (-40kN \cdot m) \cdot (-4kN \cdot m)] \\
 &+ \frac{1}{EI_y} 3m \cdot (-40kN \cdot m) \cdot (-4kN \cdot m) \\
 &+ \frac{5m}{6EI_y} [0 \cdot (-30kN \cdot m) + 4 \cdot (-80kN \cdot m) \cdot (-2,5kN \cdot m) + (-150kN \cdot m) \cdot (-5kN \cdot m)], \\
 w &= 3448,3 \frac{kN \cdot m^3}{EI_y}
 \end{aligned}$$