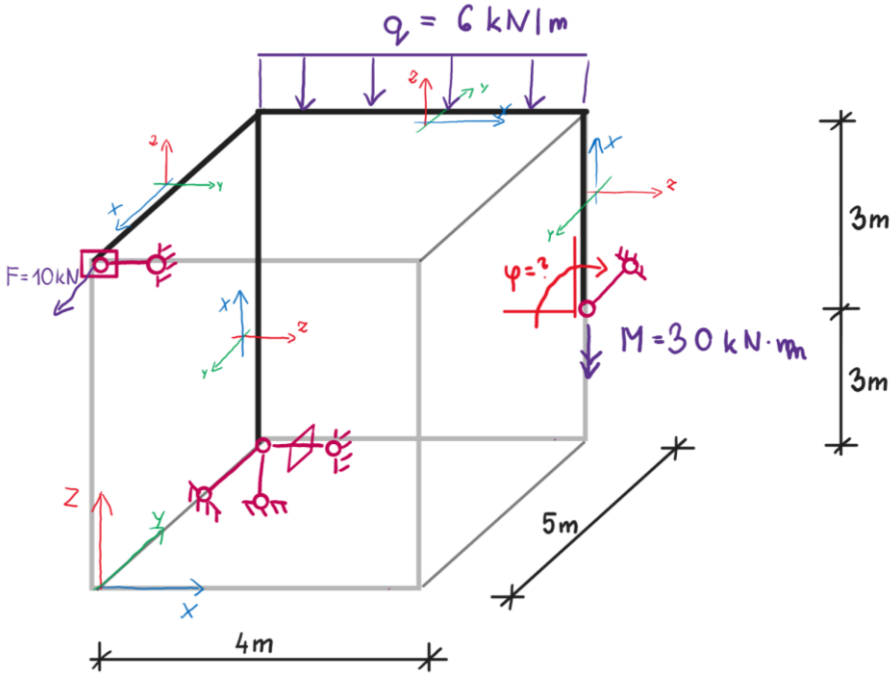
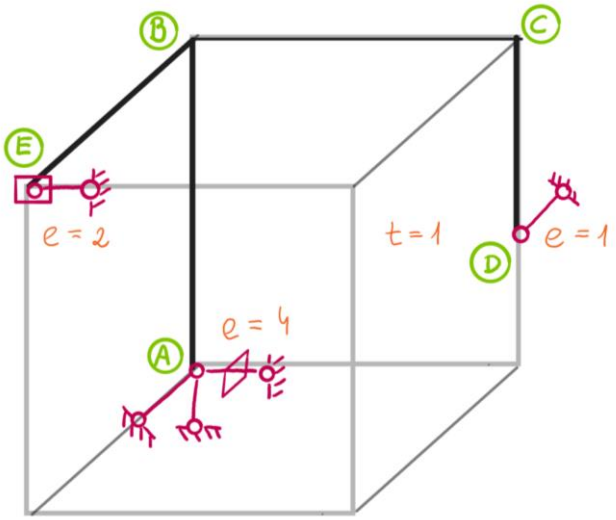
Opracowała:
dr inż. Olga Szyłko-Bigus

1. **Rozwiązanie układu przestrzennego metodą sił i obliczenie przemieszczenia**



Dane: $\mathrm{GI}_x = 0{,}769\ \mathrm{EI}_y;\ \mathrm{EI}_y = \mathrm{EI}_z = const.$

2. **Obliczenie stopnia statycznej niewyznaczalności i sprawdzenie geometrycznej niezmienności układu**

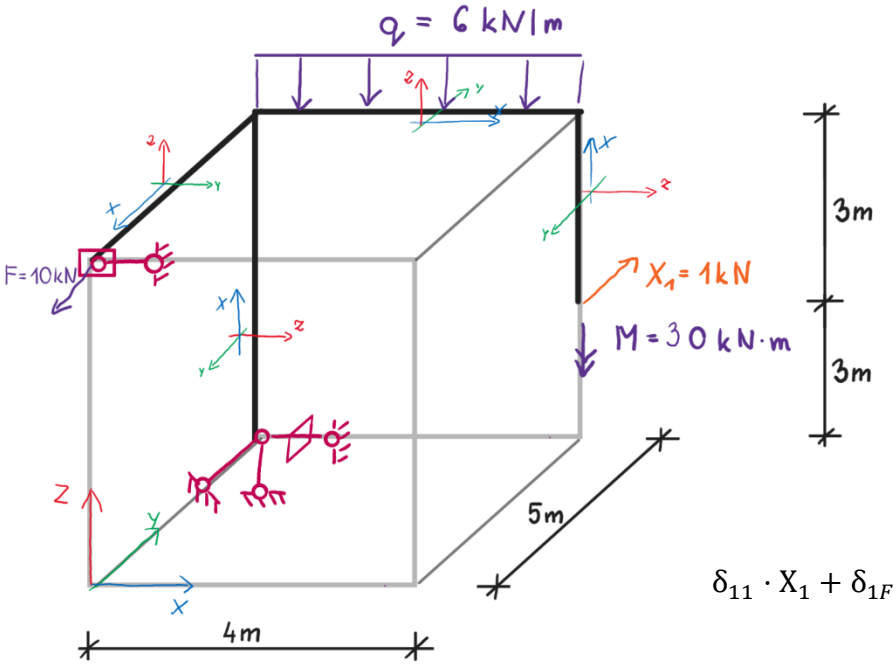


$$\mathrm{n}_h = e - 6 \cdot t = 7 - 6 \cdot 1 = 1$$

Bryła sztywna jest podparta za pomocą więzi elementarnych blokujących przesuwy na wszystkich kierunkach X Y i Z i obrót względem osi X w węźle A. Za potencjalną oś obrotu uznano oś na kierunku osi Y w węźle A. Kierunki pozostałych więzi nie przecinają kierunku osi obrotu , co wyklucza jakikolwiek ruch bryły.

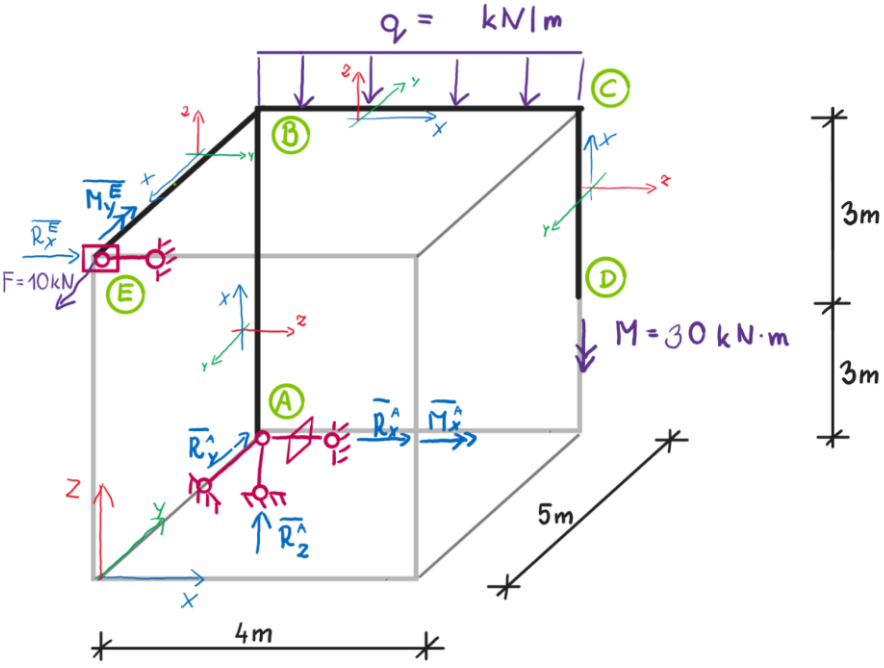
Opracowała:
dr inż. Olga Szyłko-Bigus

3. Dobranie układu podstawowego metody sił i postać ogólna układu równań metody sił



$$\delta_{11} \cdot X_1 + \delta_{1F} = \Delta_{1R} = 0$$

4. Rozwiązanie układu podstawowego metody sił od wpływów mechanicznych



Obliczenie reakcji:

$$\sum Y = 0 \Rightarrow \bar{R}^F_{yA} - F = 0 \Rightarrow \bar{R}^F_{yA} = 10\,\text{kN}$$

$$\sum Z = 0 \Rightarrow \bar{R}^F_{zA} - q \cdot 4\,\text{m} = 0 \Rightarrow \bar{R}^F_{zA} = 24\,\text{kN}$$

$$\sum M^A_x = 0 \Rightarrow \bar{M}^F_{xA} + F \cdot 6\,\text{m} = 0 \Rightarrow \bar{M}^F_{xA} = -60\,\text{kNm}$$

$$\sum M^A_z = 0 \Rightarrow \bar{R}^F_{xE} \cdot 5\,\text{m} - M = 0 \Rightarrow \bar{R}^F_{xE} = 6\,\text{kN}$$

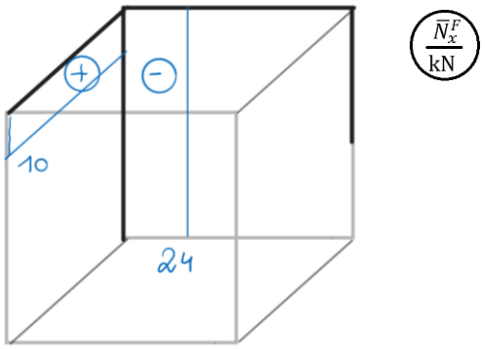
Opracowała:
dr inż. Olga Szyłko-Bigus

$$\sum M_y^A = 0 \Rightarrow \bar{R}_{xE}^F \cdot 6\,\text{m} + \bar{M}_{yE}^F + q \cdot 4\,\text{m} \cdot 2\,\text{m} = 0 \Rightarrow \bar{M}_{yE}^F = -84\,\text{kNm}$$

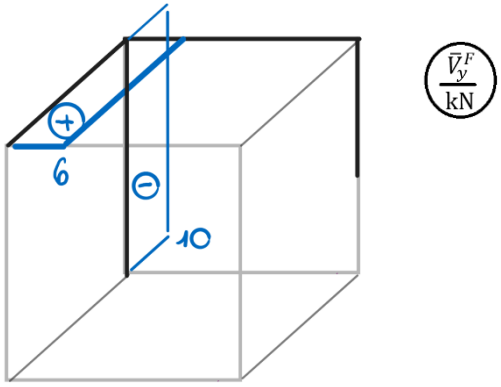
$$\sum \text{X} = 0 \Rightarrow \bar{R}_{xA}^F + \bar{R}_{xE}^F = 0 \Rightarrow \bar{R}_{xA}^F = -6\,\text{kN}$$

Obliczenie sił wewnętrznych

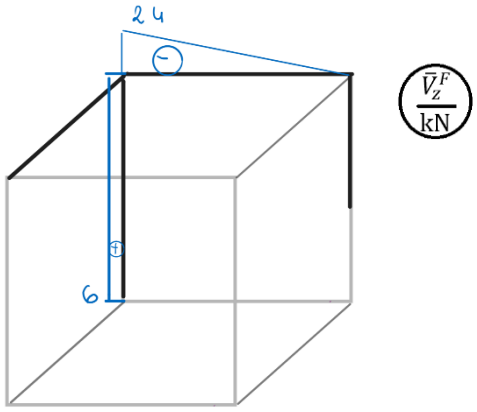
$\bar{N}_{xEB}^F = \bar{N}_{xBE}^F = F = 10\,\text{kN}$
$\bar{N}_{xAB}^F = \bar{N}_{xBA}^F = -\bar{R}_{zA}^F = -24\,\text{kN}$
$\bar{N}_{xDC}^F = \bar{N}_{xCD}^F = \bar{N}_{xCB}^F = \bar{N}_{xBC}^F =$



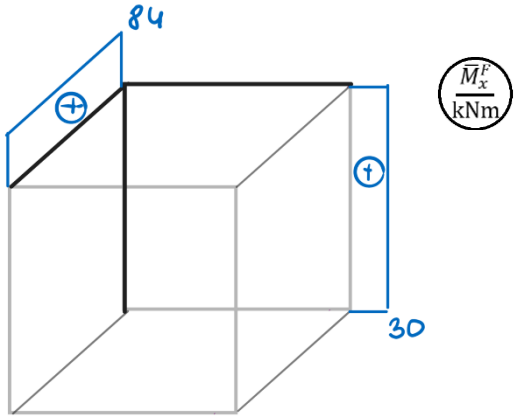
$\bar{V}_{yEB}^F = \bar{V}_{yBE}^F = \bar{R}_{xE}^F = 6\,\text{kN}$
$\bar{V}_{yDC}^F = \bar{V}_{yCD}^F = \bar{V}_{yCB}^F = \bar{V}_{yBC}^F = 0$
$\bar{V}_{yAB}^F = \bar{V}_{yBA}^F = \bar{R}_{yA}^F = 10\,\text{kN}$



$\bar{V}_{zEB}^F = \bar{V}_{zBE}^F = \bar{V}_{zDC}^F = \bar{V}_{zCD}^F = 0$
$\bar{V}_{zAB}^F = \bar{V}_{zBA}^F = -\bar{R}_{xA}^F = 6\,\text{kN}$
$V_{zCB}^F = 0$
$\bar{V}_{zBC}^F = -q \cdot 4\,\text{m} = -24\,\text{kN}$

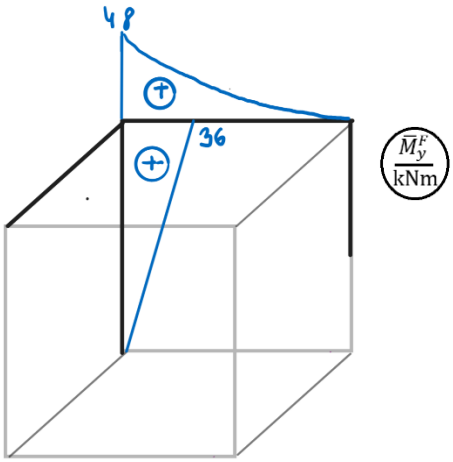


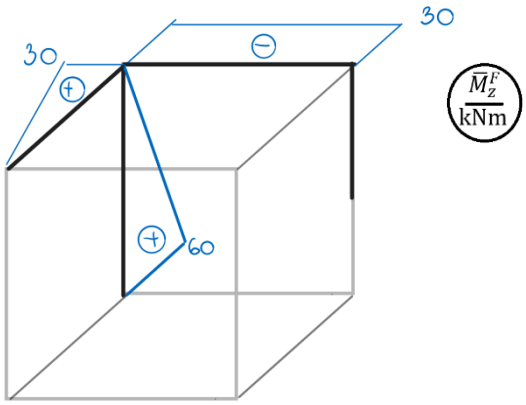
$\bar{M}_{xEB}^F = \bar{M}_{xBE}^F = -\bar{M}_{yE}^F = 84\,\text{kNm}$
$\bar{M}_{xDC}^F = \bar{M}_{xCD}^F = M = 30\,\text{kNm}$
$\bar{M}_{xAB}^F = \bar{M}_{xBA}^F = 0$
$\bar{M}_{xBC}^F = \bar{M}_{xCB}^F = 0$



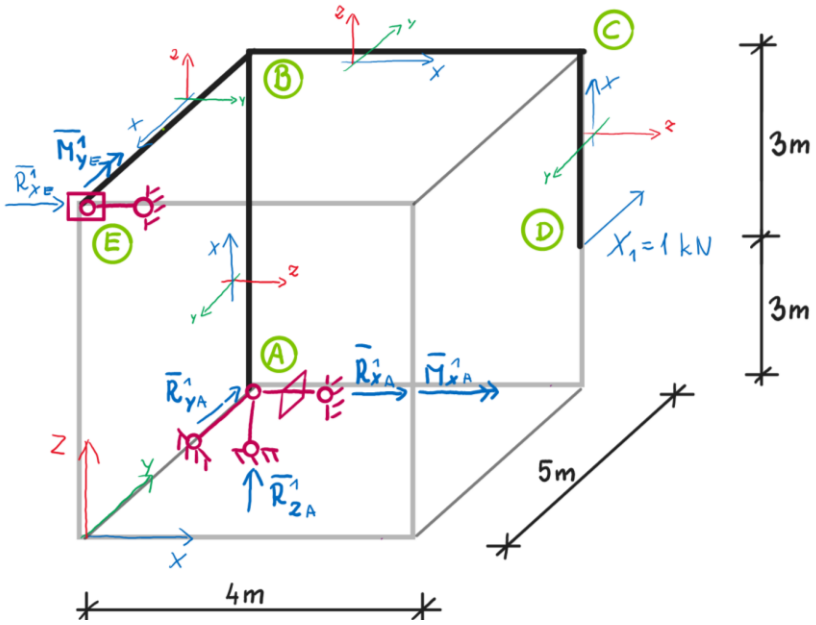
$\bar{M}_{yEB}^F = \bar{M}_{yBE}^F = 0$
$\bar{M}_{yDC}^F = \bar{M}_{xCD}^F = 0$
$\bar{M}_{yAB}^F = 0$
$\bar{M}_{yBA}^F = -\bar{R}_{xA}^F \cdot 6\,\text{m} = 36\,\text{kNm}$
$\bar{M}_{yCB}^F = 0$
$\bar{M}_{yBC}^F = q \cdot 4\,\text{m} \cdot 2\,\text{m} = 48\,\text{kNm}$

$\bar{M}_{zEB}^F = 0$
$\bar{M}_{zBE}^F = \bar{R}_{xE}^F \cdot 3\,\text{m} = 30\,\text{kNm}$
$\bar{M}_{zDC}^F = \bar{M}_{zCD}^F = 0$
$\bar{M}_{zAB}^F = -\bar{M}_{xA}^F = 60\,\text{kNm}$
$\bar{M}_{zBA}^F = -\bar{M}_{xA}^F - \bar{R}_{yA}^F \cdot 6\,\text{m} = 0$
$\bar{M}_{yCB}^F = \bar{M}_{zBC}^F = -M = -30\,\text{kNm}$





5. Rozwiązanie układu podstawowego metody sił od X_1=1kN



Obliczenie reakcji:

$$\sum \mathrm{Y} = 0 \Rightarrow \bar{R}_{yA}^1 + X_1 = 0 \Rightarrow \bar{R}_{yA}^1 = -1\,\mathrm{kN}$$

$$\sum \mathrm{Z} = 0 \Rightarrow \bar{R}_{zA}^1 = 0$$

$$\sum M_x^A = 0 \Rightarrow \bar{M}_{xA}^1 - X_1 \cdot 3\,\mathrm{m} = 0 \Rightarrow \bar{M}_{xA}^1 = 3\,\mathrm{kNm}$$

$$\sum M_z^A = 0 \Rightarrow \bar{R}_{xE}^1 \cdot 5\,\mathrm{m} + X_1 \cdot 4\,\mathrm{m} = 0 \Rightarrow \bar{R}_{xE}^{F1} = -0{,}8\,\mathrm{kN}$$

$$\sum M_y^A = 0 \Rightarrow \bar{R}_{xE}^1 \cdot 6\,\mathrm{m} + \bar{M}_{yE}^1 = 0 \Rightarrow \bar{M}_{yE}^1 = 4{,}8\,\mathrm{kNm}$$

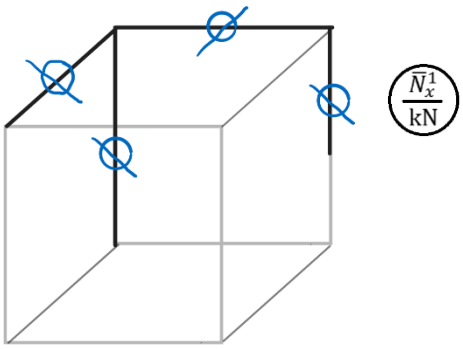
$$\sum \mathrm{X} = 0 \Rightarrow \bar{R}_{xA}^1 + \bar{R}_{xE}^1 = 0 \Rightarrow \bar{R}_{xA}^1 = 0{,}8\,\mathrm{kN}$$

Obliczenie sił wewnętrznych

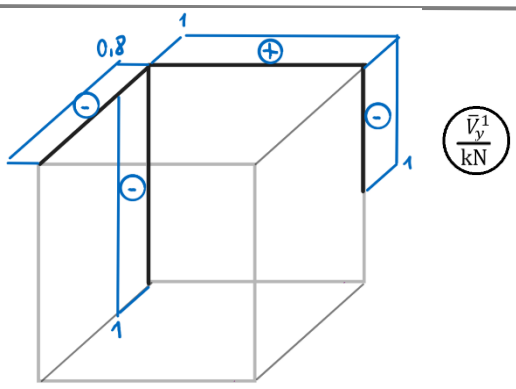
$$\bar{N}_{xEB}^1 = \bar{N}_{xBE}^1 = 0$$
$$\bar{N}_{xAB}^1 = \bar{N}_{xBA}^1 = -\bar{R}_{zA}^1 = 0$$
$$\bar{N}_{xDC}^1 = \bar{N}_{xCD}^1 = 0$$
$$\bar{N}_{xCB}^1 = \bar{N}_{xBC}^1 = 0$$

$$\bar{V}_{yEB}^1 = \bar{V}_{yBE}^1 = \bar{R}_{xE}^F = -0{,}8\,\mathrm{kN}$$
$$\bar{V}_{yDC}^F = \bar{V}_{yCD}^F = X_1 = 1\,\mathrm{kN}$$
$$\bar{V}_{yCB}^1 = \bar{V}_{yBC}^1 = X_1 = 1\,\mathrm{kN}$$
$$\bar{V}_{yAB}^1 = \bar{V}_{yBA}^1 = \bar{R}_{yA}^1 = -1\,\mathrm{kN}$$

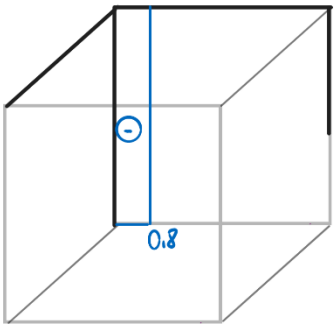
Opracowała:
dr inż. Olga Szyłko-Bigus

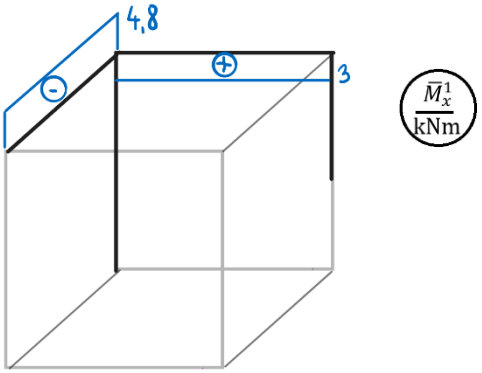


$\bar{V}_{zEB}^1 = \bar{V}_{zBE}^1 = \bar{V}_{zDC}^1 = \bar{V}_{zCD}^1 = 0$
$\bar{V}_{zAB}^1 = \bar{V}_{zBA}^1 = -\bar{R}_{xA}^1 = -0{,}8\,\text{kN}$
$V_{zCB}^1 = \bar{V}_{zBC}^1 = 0$



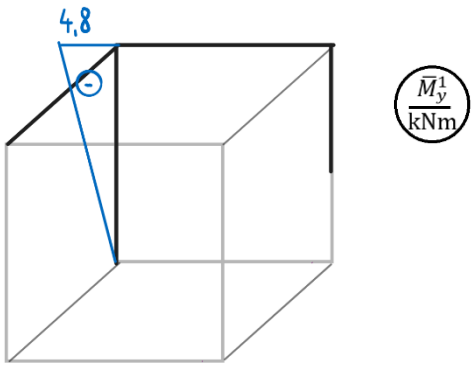
$\bar{M}_{xEB}^1 = \bar{M}_{xBE}^1 = \bar{M}_{yE}^F = -4{,}8\,\text{kNm}$
$\bar{M}_{xDC}^1 = \bar{M}_{xCD}^1 =$
$\bar{M}_{xAB}^1 = \bar{M}_{xBA}^1 = 0$
$\bar{M}_{xBC}^1 = \bar{M}_{xCB}^1 = X_1 \cdot 3\,\text{m} = 3\,\text{kNm}$

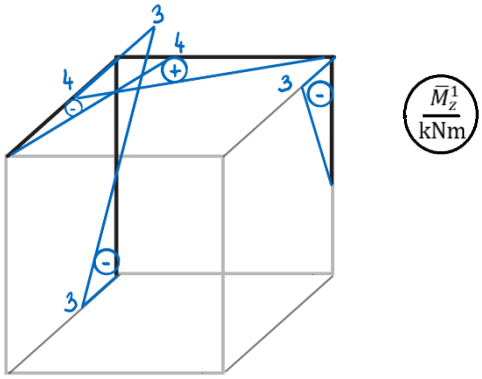




$\bar{M}_{yEB}^1 = \bar{M}_{yBE}^1 = 0$
$\bar{M}_{yDC}^1 = \bar{M}_{xCD}^1 = 0$
$\bar{M}_{yAB}^1 = 0$
$\bar{M}_{yBA}^1 = -\bar{R}_{xA}^1 \cdot 6\,\text{m} =$
$-0{,}8\,\text{kN} \cdot 6\,\text{m} = -4{,}8\,\text{kNm}$
$\bar{M}_{yCB}^1 = \bar{M}_{yBC}^1 = 0$

$\bar{M}_{zEB}^1 = 0$
$\bar{M}_{zBE}^1 = \bar{R}_{xE}^F \cdot 5\,\text{m} =$
$-0{,}8\,\text{kN} \cdot 5\,\text{m} = -4\,\text{kNm}$
$\bar{M}_{zDC}^1 = 0$
$\bar{M}_{zCD}^1 = -X_1 \cdot 3\,\text{m} = -3\,\text{kNm}$
$\bar{M}_{zAB}^1 = -\bar{M}_{xA}^1 = -3\,\text{kNm}$
$\bar{M}_{zBA}^1 = -\bar{M}_{xA}^1 - \bar{R}_{yA}^1 \cdot 6\,\text{m} =$
$-3\,\text{kNm} - (-1)\,\text{kN} \cdot 6\,\text{m} = 3\,\text{kNm}$
$\bar{M}_{yCB}^1 = 0$
$\bar{M}_{zBC}^1 = X_1 \cdot 4\,\text{m} = 4\,\text{kNm}$





Opracowała:
dr inż. Olga Szyłko-Bigus

6. Obliczenie współczynników i wyrazów wolnych układu równań metody sił

$$\delta_{11} \cdot \mathrm{X}_1 + \delta_{1F} = \Delta_{1R} = 0$$

$$\bar{1}\mathrm{kN} \cdot \delta_{11} = \sum_p \int \frac{\bar{M}_x^1 \bar{M}_x^1}{GI_x} dx + \sum_p \int \frac{\bar{M}_y^1 \bar{M}_y^1}{EI_y} dx + \sum_p \int \frac{\bar{M}_z^1 \bar{M}_z^1}{EI_z} dx + \sum_S \frac{\bar{S}_\delta^1 \bar{S}_\delta^1}{k_\delta}$$

$$\bar{1}\mathrm{kN} \cdot \delta_{1F} = \sum_p \int \frac{\bar{M}_x^1 \bar{M}_x^F}{GI_x} dx + \sum_p \int \frac{\bar{M}_y^1 \bar{M}_y^F}{EI_y} dx + \sum_p \int \frac{\bar{M}_z^1 \bar{M}_z^F}{EI_z} dx + \sum_S \frac{\bar{S}_\delta^1 \bar{S}_\delta^F}{k_\delta}$$

Ustalenie jednostek współczynników:

$$[\delta_{11}] = [\delta_{1F}] = \frac{1}{\mathrm{kN}} \frac{\mathrm{kNm} \cdot \mathrm{kNm} \cdot \mathrm{m}}{EI_y} == \frac{1}{\mathrm{kN}} \frac{\mathrm{kNm} \cdot \mathrm{kNm} \cdot \mathrm{m}}{\frac{\mathrm{kN}}{\mathrm{m}^2} \cdot \mathrm{m}^4} = \mathrm{m}$$

Obliczenie współczynników i wyrazów wolnych układu równań metody sił:

$$\delta_{11} = \frac{1}{GI_x}[5 \cdot (-4{,}8) \cdot (-4{,}8) + 4 \cdot 3 \cdot 3]\mathrm{kNm}^3 + \frac{1}{EI_y}\left[\frac{1}{2} \cdot 6 \cdot (-4{,}8) \cdot \frac{2}{3} \cdot (-4{,}8)\right]\mathrm{kNm}^3$$
$$+ \frac{1}{EI_z}\left[\frac{1}{2} \cdot 5 \cdot (-4) \cdot \frac{2}{3} \cdot (-4) + \frac{1}{2} \cdot 3 \cdot (-3) \cdot \frac{2}{3} \cdot (-3) + \frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4 + \frac{1}{2} \cdot 3 \cdot (-3) \cdot \frac{2}{3} \cdot (-3) + \frac{1}{2} \cdot 3 \cdot 3 \cdot \frac{2}{3} \cdot 3\right]\mathrm{kNm}^3$$
$$= \frac{1}{0{,}769EI_y}[115{,}2 + 36]\mathrm{kNm}^3 + \frac{1}{EI_y}[46{,}08]\mathrm{kNm}^3$$
$$+ \frac{1}{EI_y}[26, (6) + 9 + 21, (3) + 9 + 9]\mathrm{kNm}^3 = 317{,}6990\frac{\mathrm{kNm}^3}{EI_y}$$

$$\delta_{1F} = \frac{1}{GI_x}[5 \cdot (-4{,}8) \cdot 84]\mathrm{kNm}^3 + \frac{1}{EI_y}\left[\frac{1}{2} \cdot 6 \cdot (-4{,}8) \cdot \frac{2}{3} \cdot 36\right]\mathrm{kNm}^3$$
$$+ \frac{1}{EI_z}\left[\frac{1}{2} \cdot 5 \cdot (-4) \cdot \frac{2}{3} \cdot 30 + \frac{1}{2} \cdot 4 \cdot 4 \cdot (-30) + \frac{1}{2} \cdot 6 \cdot 60 \cdot \frac{1}{3} \cdot (-3)\right]\mathrm{kNm}^3$$
$$= \frac{1}{0{,}769EI_y}[-2016]\mathrm{kNm}^3 + \frac{1}{EI_y}[-345{,}6]\mathrm{kNm}^3$$
$$+ \frac{1}{EI_z}[-200 - 240 - 180]\mathrm{kNm}^3 = -3587{,}1865\frac{\mathrm{kNm}^3}{EI_y}$$

7. Postać szczegółowa układu równań metody sił i jego rozwiązanie

$$317{,}6990\frac{\mathrm{kNm}^3}{EI_y} \cdot \mathrm{X}_1 - 3587{,}1865\frac{\mathrm{kNm}^3}{EI_y} = 0$$

$$\mathrm{X}_1 = 11{,}2911$$

8. Obliczenie sił wewnętrznych i sporządzenie wykresów

$$R_{xij}^F = \bar{R}_{xij}^1 \cdot X_1 + R_{xij}^F$$

$$N_{xij}^F = \bar{N}_{xij}^1 \cdot X_1 + \bar{N}_{xij}^F \qquad V_{yij}^F = \bar{V}_{yij}^1 \cdot X_1 + \bar{V}_{yij}^F$$

$$V_{zij}^F = \bar{V}_{zij}^1 \cdot X_1 + \bar{V}_{zij}^F \qquad V_{zij}^F = \bar{V}_{zij}^1 \cdot X_1 + \bar{V}_{zij}^F$$
$$M_{yij}^F = \bar{M}_{yij}^1 \cdot X_1 + \bar{M}_{yij}^F \qquad M_{zij}^F = \bar{M}_{zij}^1 \cdot X_1 + \bar{M}_{zij}^F$$

Obliczenie reakcji:

$R_{xA}^F = \bar{R}_{xA}^1 \cdot X_1 + R_{xA}^F = 0{,}8\text{kN} \cdot 11{,}2911 + (-6)\text{kN} = 3{,}0329\text{kN}$

$R_{yA}^F = \bar{R}_{yA}^1 \cdot X_1 + R_{yA}^F = -1\text{kN} \cdot 11{,}2911 + 10\text{kN} = -1{,}2911\text{kN}$

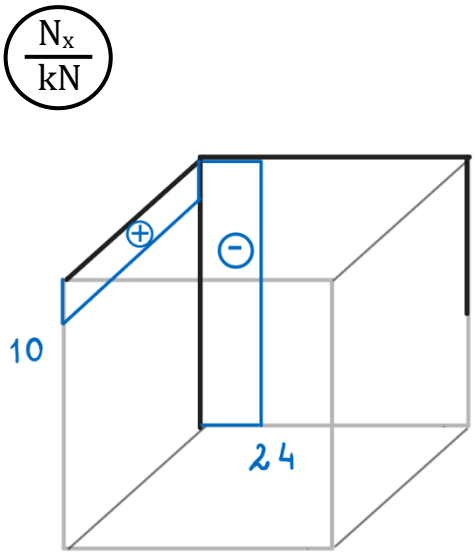
$R_{zA}^F = \bar{R}_{zA}^1 \cdot X_1 + R_{zA}^F = 0 \cdot 11{,}2911 + 24\text{kN} = 24\text{kN}$

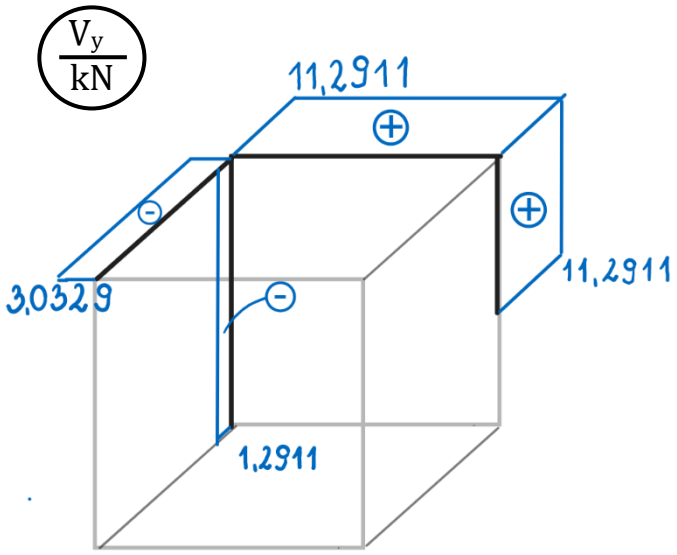
$M_{xA}^F = \bar{M}_{xA}^1 \cdot X_1 + M_{xA}^F = 3\text{kNm} \cdot 11{,}2911 + (-60)\text{kNm} = -26{,}1266\text{kNm}$

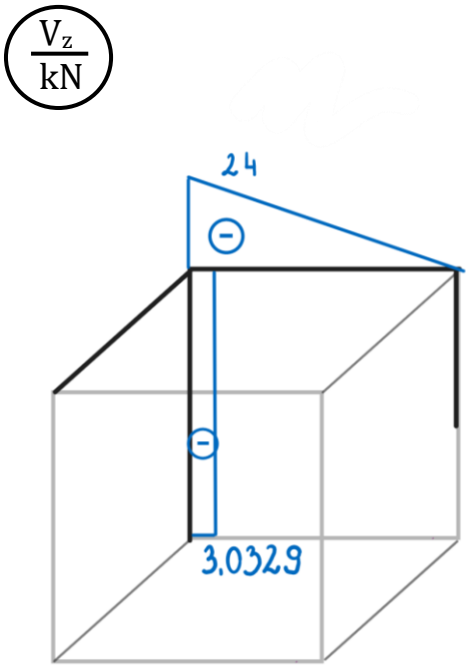
$R_{xE}^F = \bar{R}_{xE}^1 \cdot X_1 + R_{xE}^F = -0{,}8\text{kN} \cdot 11{,}2911 + 6\text{kN} = -3{,}0329\text{kN}$

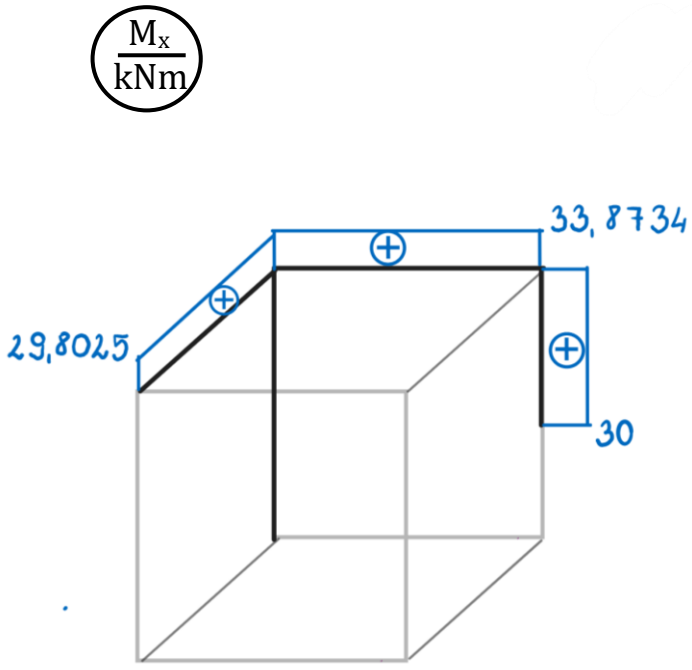
$M_{yE}^F = \bar{M}_{yE}^1 \cdot X_1 + M_{yE}^F = 4{,}8\text{kNm} \cdot 11{,}2911 - 84\text{kNm} = -29{,}8025\text{kNm}$

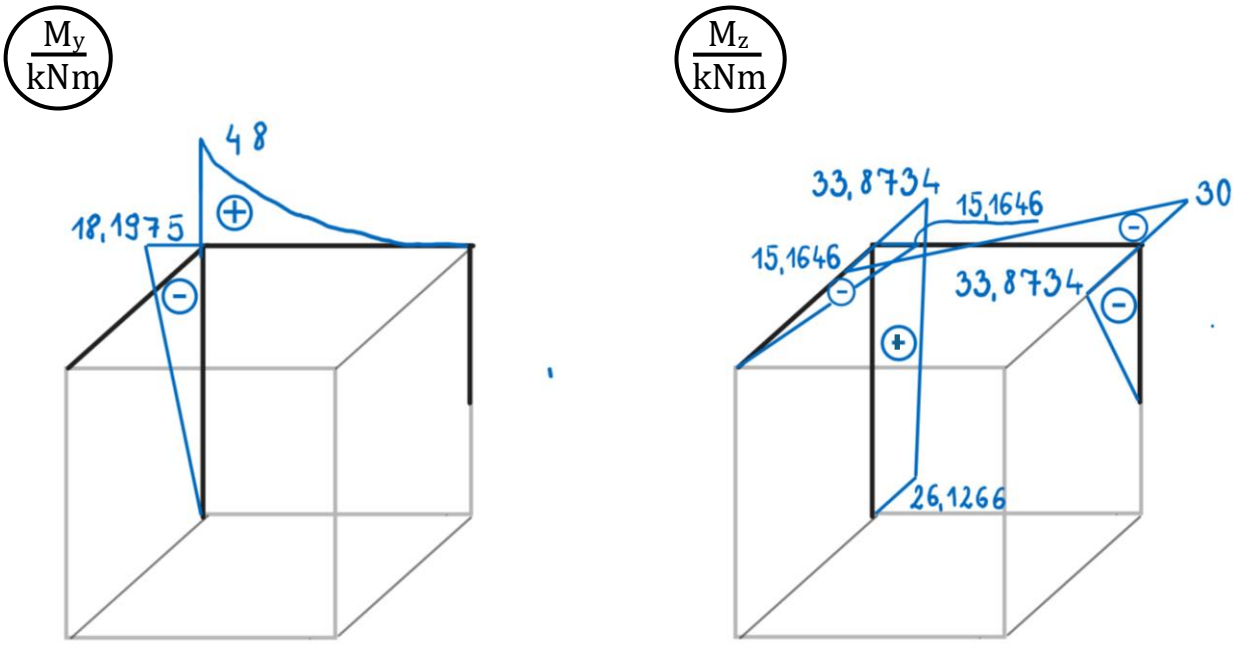
Sporządzenie wykresów:





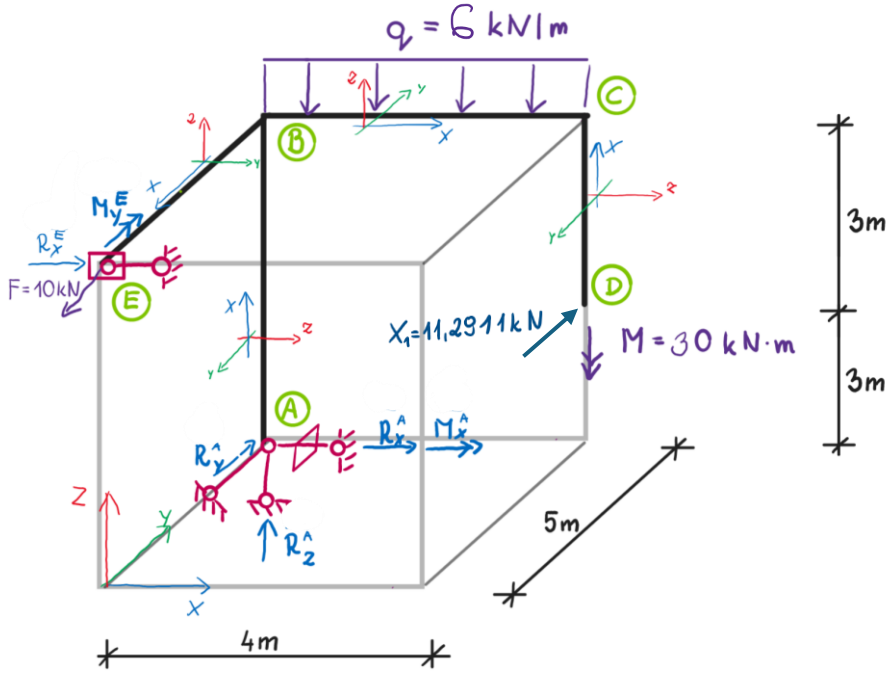






9. Kontrola statyczna rozwiązania

<u>Globalna równowaga sił</u>



$$\sum \mathrm{X} = 0 \Rightarrow R_{xA}^{1} + R_{xE}^{1} = 3{,}0329\mathrm{kN} + (-3{,}0329)\mathrm{kN} = 0$$

$$\sum \mathrm{Y} = 0 \Rightarrow R_{yA} + X_1 - F = -1{,}2911\mathrm{kN} + 11{,}2911\mathrm{kN} - 10\mathrm{kN} = 0$$

$$\sum \mathrm{Z} = 0 \Rightarrow R_{zA} - q \cdot 4\,\mathrm{m} = 24\mathrm{kN} - 6\frac{\mathrm{kN}}{\mathrm{m}} \cdot 4\,\mathrm{m} = 0$$

$$\sum M_x^A = 0 \Rightarrow M_{xA} - X_1 \cdot 3\,\mathrm{m} + F \cdot 6\,\mathrm{m} = -26{,}1266\mathrm{kNm} - 11{,}2911\mathrm{kN} \cdot 3\,\mathrm{m} + 10kN \cdot 6\,\mathrm{m} = 0$$

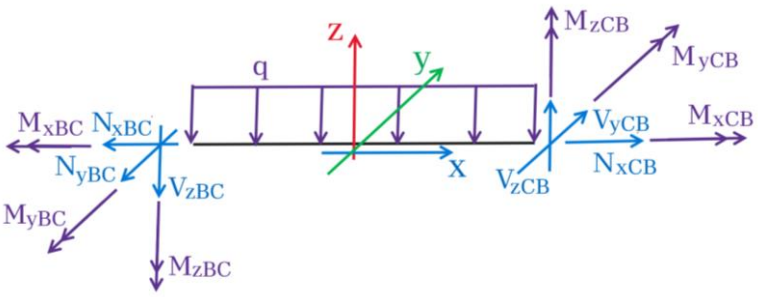
Opracowała:
dr inż. Olga Szyłko-Bigus

$$\sum M_y^A = 0 \Rightarrow R_{xE} \cdot 6\,\mathrm{m} + M_{yE} - q \cdot 4\,\mathrm{m} \cdot 2\,\mathrm{m} =$$

$$-3{,}0329\mathrm{kN} \cdot 6\,\mathrm{m} - 1{,}2911\mathrm{kN} + 6\frac{\mathrm{kN}}{\mathrm{m}} \cdot 4\,\mathrm{m} \cdot 2\,\mathrm{m} = 0$$

$$\sum M_z^A = 0 \Rightarrow R_{xE} \cdot 5\,\mathrm{m} + X_1 \cdot 4\,\mathrm{m} - M = -3{,}0329\mathrm{kN} \cdot 5\,\mathrm{m} + 11{,}2911 \cdot 4\,\mathrm{m} - 30kNm = 0$$

Równowaga sił w pręcie BC (liczona względem lokalnego układu współrzędnych):



$$\sum \mathrm{X} = 0 \Rightarrow N_{xCB} - N_{zBC} = 0 + 0 = 0$$

$$\sum \mathrm{Y} = 0 \Rightarrow V_{yCB} - V_{yBC} = 11{,}2911\mathrm{kN} - 11{,}2911\mathrm{kN} = 0$$

$$\sum \mathrm{Z} = 0 \Rightarrow \mathrm{V_{zCB}} - \mathrm{V_{zBC}} - \mathrm{q} \cdot 4\mathrm{m} = 0 - (-24)\mathrm{kN} - 6\frac{\mathrm{kN}}{\mathrm{m}} \cdot 4\mathrm{m} = 0$$

$$\sum \mathrm{M_{x,B}} = 0 \Rightarrow M_{xCB} - M_{xBC} = 33{,}8734\mathrm{kNm} - 33{,}8734\mathrm{kNm} = 0$$

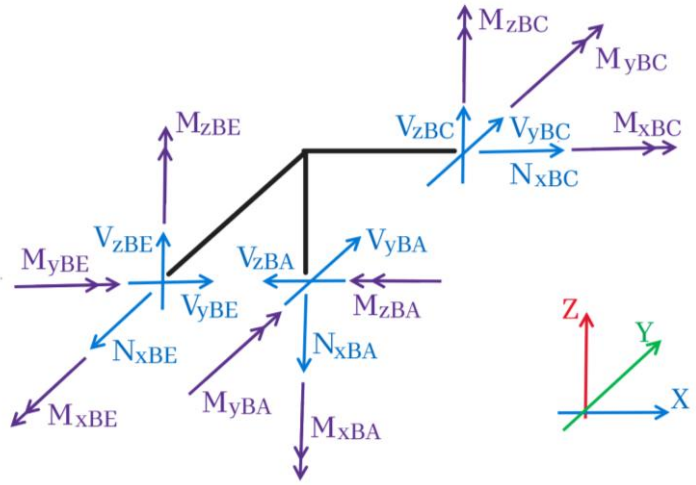
$$\sum \mathrm{M_{y,B}} = 0 \Rightarrow M_{yCB} - M_{zBC} - V_{zCB} \cdot 4\mathrm{m} + \mathrm{q} \cdot 4\mathrm{m} \cdot 2\mathrm{m}$$

$$= 0 - 48\mathrm{kNm} - 0 \cdot 4\mathrm{m} + 6\frac{\mathrm{kN}}{\mathrm{m}} \cdot 4\mathrm{m} \cdot 2\mathrm{m} = 0$$

$$\sum \mathrm{M_{z,B}} = 0 \Rightarrow M_{zCB} - M_{zBC} + V_{yCB} \cdot 4\mathrm{m}$$

$$= -30\mathrm{kNm} - 15{,}1646\mathrm{kNm} + 11{,}2911\mathrm{kNm} \cdot 4\mathrm{m} = 0$$

Równowaga sił w węźle B (liczona względem globalnego układu współrzędnych):



$$\sum X = 0 \Rightarrow N_{xBC} - V_{zBA} + V_{yBE} = 0 - (-3{,}0329)\text{kN} + (-3{,}0329)\text{kN} = 0$$

$$\sum Y = 0 \Rightarrow V_{yBC} + V_{yBA} - N_{xBE} = 11{,}2911\text{kN} + (-1{,}2911)\text{kN} - 10\text{kN} = 0$$

$$\sum Z = 0 \Rightarrow V_{zBC} - N_{xBA} + V_{zBE} = -24\text{kN} - (24)\text{kN} + 0 = 0$$

$$\sum M_x = 0 \Rightarrow M_{xBC} - M_{zBA} + M_{yBE} = 33{,}8734\text{kNm} - (33{,}8734)\text{kNm} + 0 = 0$$

$$\sum M_y = 0 \Rightarrow M_{yBC} + M_{yBA} - M_{xBE} = 48\text{kNm} + (-18{,}1975)\text{kNm} - 29{,}8025\text{kNm} = 0$$

$$\sum M_z = 0 \Rightarrow M_{zBC} - M_{xBA} + M_{zBE} = 15{,}1646\text{kNm} - 0 + (-15{,}1646)\text{kNm} = 0$$

10. Kontrola kinematyczna rozwiązania

$$\bar{1}\text{kN} \cdot \delta_{1F} = \sum_p \int \frac{\bar{M}_x^1 M_x^F}{GI_x} dx + \sum_p \int \frac{\bar{M}_y^1 M_y^F}{EI_y} dx + \sum_p \int \frac{\bar{M}_z^1 M_z^F}{EI_z} dx + \sum_s \frac{\bar{S}_\delta^1 S_\delta^F}{k_\delta}$$

$$= \frac{1}{GI_x}[5 \cdot (-4{,}8) \cdot 29{,}8025 + 4 \cdot 3 \cdot 33{,}8734]\text{kNm}^3$$

$$+ \frac{1}{EI_y}\left[\frac{1}{2} \cdot 6 \cdot (-4{,}8) \cdot \frac{2}{3} \cdot (-18{,}1975)\right]\text{kNm}^3$$

$$+ \frac{1}{EI_z}\left\{\frac{1}{2} \cdot 5 \cdot (-4) \cdot \frac{2}{3} \cdot (-15{,}1646) + \frac{1}{2} \cdot 3 \cdot (-3) \cdot \frac{2}{3} \cdot (-33{,}8734) + \frac{4}{6}\right.$$

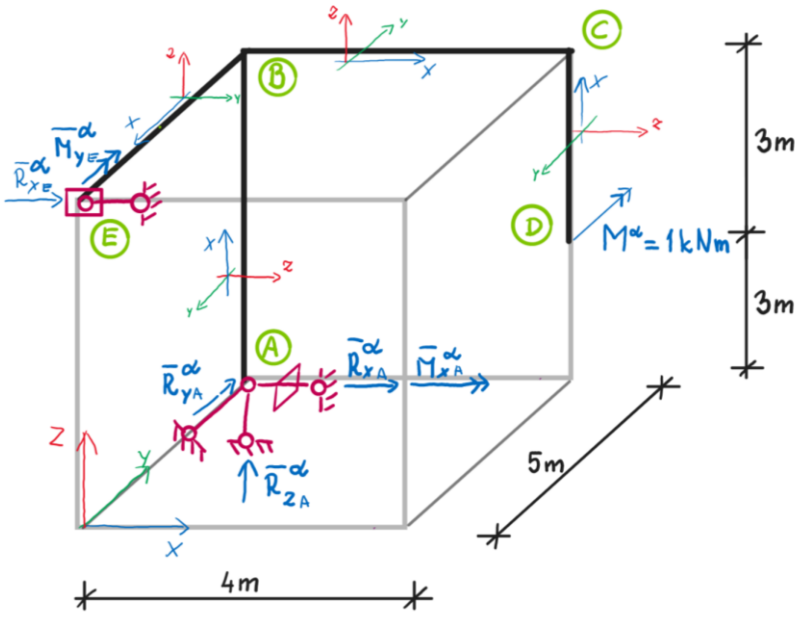
$$\cdot \left[15{,}1646 \cdot 4 + 4 \cdot \frac{15{,}1646 - 30}{2} \cdot 2 + (-30) \cdot 0\right] + \frac{6}{6}$$

$$\left. \cdot \left[(-26{,}1266) \cdot 3 + 4 \cdot \frac{-15{,}1646 - 26{,}1266}{2} \cdot 0 + (-33{,}8734) \cdot (-3)\right]\right\}\text{kNm}^3$$

$$= \frac{1}{0{,}769EI_y}[-715{,}26 + 406{,}4807]\text{kNm}^3 + \frac{1}{EI_y}[174{,}696]\text{kNm}^3$$

$$+ \frac{1}{EI_y}[101{,}0973 + 101{,}6202 + 0{,}8763 + 23{,}2404]\text{kNm}^3 = -0{,}0034\frac{\text{kNm}^3}{EI_y}$$

11. Obliczenie szukanego przemieszczenia



Opracowała:
dr inż. Olga Szyłko-Bigus

Obliczenie reakcji:

$$\sum \mathrm{Y} = 0 \Rightarrow \bar{R}^{\alpha}_{yA} = 0$$

$$\sum \mathrm{Z} = 0 \Rightarrow \bar{R}^{\alpha}_{zA} = 0$$

$$\sum M^{A}_{x} = 0 \Rightarrow \bar{M}^{\alpha}_{xA} = 0$$

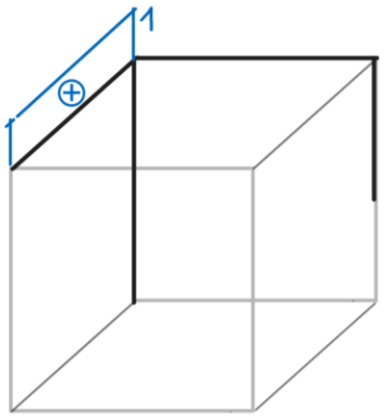
$$\sum M^{A}_{z} = 0 \Rightarrow \bar{R}^{\alpha}_{xE} = 0$$

$$\sum M^{A}_{y} = 0 \Rightarrow \bar{R}^{\alpha}_{xE} \cdot 6\,\mathrm{m} + \bar{M}^{\alpha}_{yE} + M^{\alpha} = 0 \Rightarrow \bar{M}^{\alpha}_{yE} = -1\,\mathrm{kNm}$$

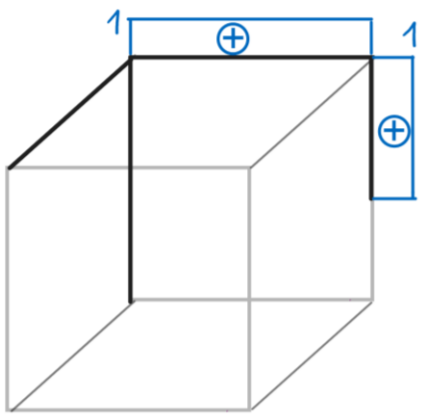
$$\sum \mathrm{X} = 0 \Rightarrow \bar{R}^{\alpha}_{xA} + \bar{R}^{\alpha}_{xE} = 0 \Rightarrow \bar{R}^{\alpha}_{xA} = 0$$

Sporządzenie wykresów:

$\frac{\bar{M}_x}{\mathrm{kNm}}$



$\frac{\bar{M}_y}{\mathrm{kNm}}$



Obliczenie szukanego przemieszczenia:

$$\bar{1}\mathrm{kN} \cdot \delta_{\alpha F} = \sum_{p} \int \frac{\bar{M}^{\alpha}_{x} M^{F}_{x}}{GI_x} dx + \sum_{p} \int \frac{\bar{M}^{\alpha}_{y} M^{F}_{y}}{EI_y} dx + \sum_{p} \int \frac{\bar{M}^{\alpha}_{z} M^{F}_{z}}{EI_z} dx + \sum_{S} \frac{\bar{S}^{\alpha}_{\delta} S^{F}_{\delta}}{k_{\delta}}$$

$$= \frac{1}{GI_x}[5 \cdot 1 \cdot 29{,}8025]\mathrm{kNm}^3 + \frac{1}{EI_y}\left[\frac{4}{6} \cdot [1 \cdot 48 + 4 \cdot 1 \cdot 12 + 1 \cdot 0]\right]\mathrm{kNm}^3$$

$$= \frac{1}{0{,}769EI_y} 149{,}0125\mathrm{kNm}^3 + \frac{1}{EI_y} 64\mathrm{kNm}^3 = 257{,}7744 \frac{\mathrm{kNm}^3}{EI_y}$$