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DYNAMICS
LECTURE 5

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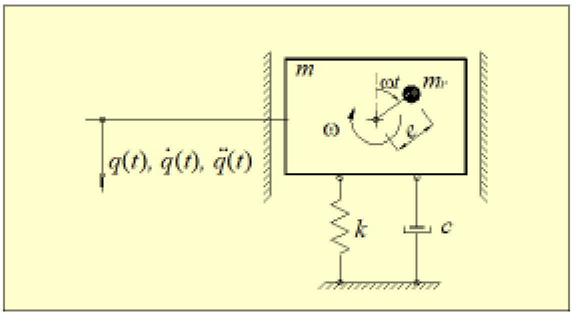
Special cases of harmonic excitation

Special cases of excitation in a one-degree-of-freedom system:

- inertial excitation and
- kinematic excitation

Vibration Due to a Rotating Eccentric weight (Inertial Excitation)

In the mass-spring-damper system shown in Fig, an unbalanced mass m_r , rotating with the angular frequency ω, is fixed to the mass m in a way that allows for rotating movement only. The mass m_r follows a circular path of radius (eccentricity radius) with respect to the pivot.



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The centrifugal force acting on the mass is described by formula

$$F_e = M_e\omega^2 = m_r e\omega^2 \qquad (2.71)$$

where

$$\Im_e = m_r e \qquad (2.72)$$

is the moment of unbalance. It must be underlined that the amplitude of the exciting force is proportional to the square of the angular velocity (frequency). The projection of the centrifugal force onto the direction of oscillation is

$$F(t) = F_e \cos\omega t = m_r e\omega^2 \cos\omega t \qquad (2.73)$$

This force ought to be substituted with the right side of Eq. (2.40) and the equation of motion can be written in form

$$m\ddot{q} + c\,\dot{q} + k\,q = m_r e\omega^2 \cos\omega t \qquad (2.74)$$

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Now, with accordance to Eq. (2.49), one can achieve

$$\mathrm{am}\, q = \nu_d \frac{F_o}{k} = \nu_d \frac{m_r e \omega^2}{k} = \eta^2 \nu_d \frac{m_r e}{m} \qquad (2.75)$$

The magnitude of the force transmitted to the foundation, in accordance with Eq. (2.71)

$$\mathrm{am}\, F_T = \nu_d \sqrt{1+(2\alpha\eta)^2}\, F_o = \eta^2 \nu_T\, m_r e \omega_o^2 \qquad (2.76)$$

It can be seen that the dynamic magnification factor, Fig. 2.18, and the transmissibility, Fig. 2.19, are now described by new formulas, respectively

$$\nu_r = \eta^2 \nu_d = \frac{\eta^2}{\sqrt{(1-\eta^2)^2+(2\alpha\eta)^2}} \qquad (2.77)$$

$$\nu_T' = \eta^2 \nu_T = \eta^2 \sqrt{\frac{1+(2\alpha\eta)^2}{(1-\eta^2)^2+(2\alpha\eta)^2}} \qquad (2.78)$$

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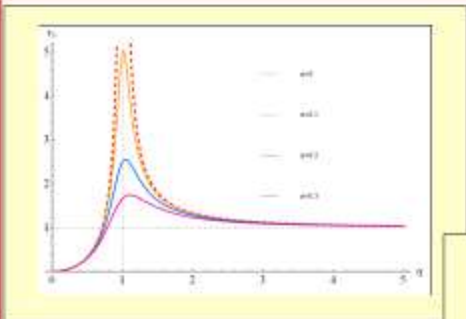
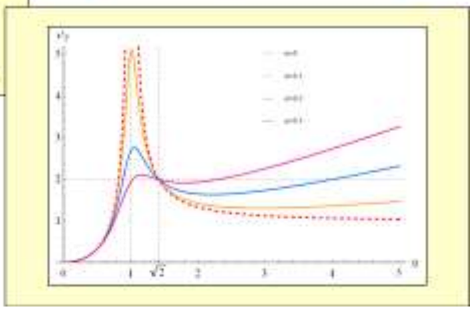


Dynamic magnification factor ν_r as a function of frequency ratio for various amounts of damping

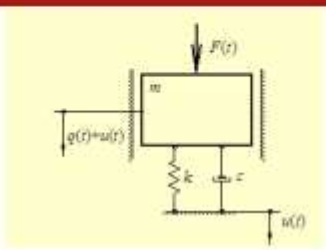
$$\nu_r = \eta^2 \nu_d = \frac{\eta^2}{\sqrt{(1-\eta^2)^2+(2\alpha\eta)^2}}$$



The transmissibility ν_T' as a function of frequency ratio for various amounts of damping

$$\nu_T' = \eta^2 \nu_T = \eta^2 \sqrt{\frac{1+(2\alpha\eta)^2}{(1-\eta^2)^2+(2\alpha\eta)^2}}$$

Kinematically Forced Vibration



Single-degree-of-freedom system with viscous damper, excited in forced vibration by foundation motion

- The foundation or support of a structure undergoes motion which varies in time.
- The movement of the foundations may have to be considered in the analysis of the dynamic response of structures subjected to ground motion by:
  - seismic (earthquakes)
  - paraseismic excitation (mining tremors, action of machines).

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The motion of mass m is described by a sum of the foundation motion $u(t)$ and the relative motion $q(t)$ between the mass m and the support, i.e.

$$z(t) = u(t) + q(t) \tag{2.79}$$

The equation of motion, then, can be written in form

$$m(\ddot{u} + \ddot{q}) + c\,\dot{q} + k\,q = 0 \tag{2.80}$$

or in a more common way

$$m\ddot{q} + c\,\dot{q} + k\,q = -m\ddot{u} \tag{2.81}$$

The factor on the right side of Eq. (2.81) has the same function as the excitation force in Eq. (2.40). If the function $u(t)$ is known, the right side of Eq. (2.81) is definite, then for some types of this function (especially if it is harmonic) this equation can be solved analytically. Let us assume that $u(t)$ is a harmonic function

$$u(t) = u_o \sin \omega t \tag{2.82}$$

After two times differentiation of the function in Eq. (2.82), it can be substituted to Eq. (2.81), and one can achieve

$$m\ddot{q} + c\,\dot{q} + k\,q = m\omega^2 u_o \sin \omega t \tag{2.83}$$

As has been stated earlier, the right hand side of the Eq. (2.83) may be interpreted as a force (equivalent acting force). The acting force can therefore be written in form

$$F(t) = F_o \sin \omega t \tag{2.84}$$

where

$$F_o = m u_o \omega^2 \tag{2.85}$$

where

$$\text{am}\, q = \nu_d \frac{F_o}{k} = \nu_d \frac{m u_o \omega^2}{k} = \eta^2 \nu_d u_o = \nu_r u_o \tag{2.87}$$

The magnitude of the force transmitted to the foundation, in accordance with Eq. (2.71)

$$\text{am}\, F_T = \nu_d \sqrt{1 + (2\alpha\eta)^2}\, F_o = \nu_T \eta^2 k u_o = \nu_T' k u_o \tag{2.88}$$

It can be seen that the dynamic magnification factor and transmissibility (Eqs. (2.87) and (2.88)) are now described by the same formulas as in the case of the eccentric rotating mass – see Eq. (2.77) and (2.78), respectively.

The total displacement of the mass m is

$$z(t) = u(t) + q(t) = u_o \sin \omega t + \nu_r u_o \sin(\omega t - \psi) \tag{2.89}$$

The amplitude of displacement of mass m is

$$\text{am } z(t) = u_o\sqrt{1+\nu_r^2+2\nu_r\cos\psi} \tag{2.90}$$

Since

$$\cos\psi = \frac{1}{\sqrt{1+\tan^2\psi}} = (1-\eta^2)\nu_d \tag{2.91}$$

using definition of ν_d, Eq. (2.50), the final formula, of the total displacement of the mass m is

$$\text{am } z(t) = \nu_d\sqrt{1+(2\alpha\eta)^2}\,u_o = \nu_T u_o \tag{2.92}$$

It ought to be underlined that exactly the same function describes the transmissibility of motion (from the foundation to the structure, Eq. (2.92)), and of force (from the structure to the foundation, Eq. (2.66)).

It is interesting to note that, if $\omega >> \omega_n$, $\nu_r = \eta^2\nu_d \to 1$, $\psi = \pi$, $\nu_T \to 0$, and then

$$q(t) = -u_o\sin\omega t = -u(t) \quad \text{and} \quad z(t) = u(t) + q(t) = 0 \tag{2.93}$$

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Conclusions

In case of inertial and kinematical excitation:

- The amplitudes of displacement and force transmitted to the foundation are described now by new functions:
  - dynamic magnification factor $\nu_r = \eta^2\nu_d$ instead the ν_d
  - transmissibility $\nu_T^r = \eta^2\nu_T$ instead the ν_T
- The important features distinguish these new functions with comparison to the first ones:
  - if $\eta = \omega/\omega_n \to 0$ the values of functions approach zero ($\nu_r \to 0$) instead approaching a limit one ($\nu_d \to 1$)
  - if $\eta = \omega/\omega_n \to \infty$ the value of the ν_r approaches a limit one ($\nu_r \to 1$) instead approaching zero ($\nu_d \to 0$)
  - if $\eta = \omega/\omega_n \to \infty$ also ν_T^r approaches infinity ($\nu_T^r \to \infty$) asymptotically to the line $2\alpha\eta$;
    - for low tuning the response of the system can be greater than in the resonance (see Fig. 2.19)
    - for greater values of damping ratio α the values of the transmissibility ν_T^r grow faster
- The damping ratio still has a very large influence on the amplitude and phase angle delay in the frequency region near resonance when $\eta = \omega/\omega_n \approx 1$.
- The phase angle is very sensitive to the ratio $\eta = \omega/\omega_n$ in the region of near-resonance for small damping.
- The maximum magnification factor occurs now for $\eta = \omega/\omega_n \to \infty$.

Conclusions

For kinematically forced vibration

- When the frequency ratio $\eta = \omega/\omega_n >> 1$, dynamic magnification factor $\nu_r = \eta^2 \nu_d \to 1$, the transmissibility $\nu_T \to 0$ the phase angle delay ψ approaches π.
    - The mass remains at rest in an external inertial frame of reference $\boxed{z(t) = u(t) + q(t) = 0}$
    - The relative motion of the mass reflects in antiphase the kinematic excitation caused by the movement of the ground $\boxed{q(t) = -u_o \sin \omega t = -u(t)}$