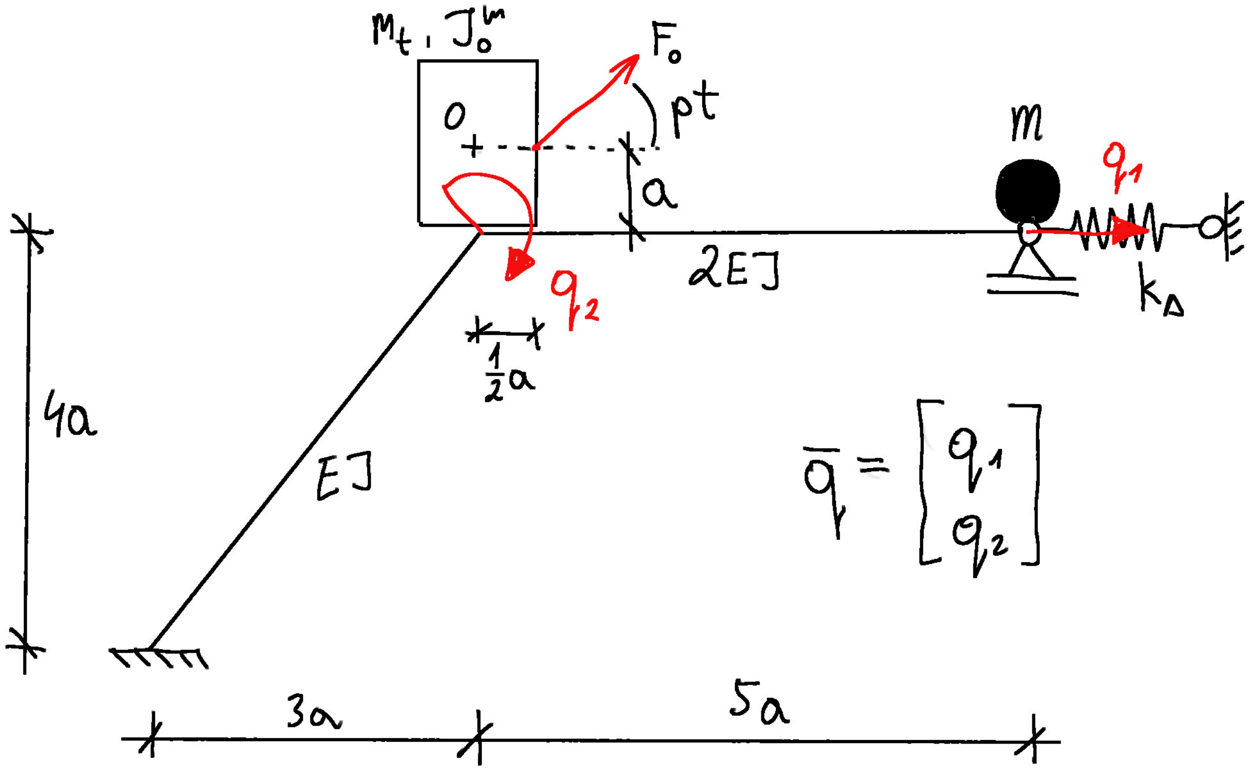
Przykład $EJ,\ a,\ m,\ F_0,\ p = 0{,}5\sqrt{\frac{EJ}{ma^3}},\ \gamma = 0{,}1$

$k_\Delta = \frac{EJ}{a^3},\quad m_t = 2m,\quad J_0^m = 3ma^2$



1. Określenie $n_h,\ n_g,\ d,\ n_{gd}$

$n_h = 5 - 3\cdot 1 = 2$

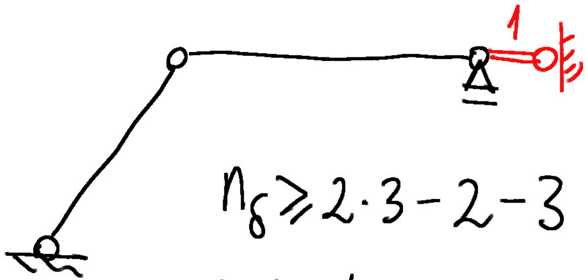
$n_g = 1\varphi + 1\delta = 2$

$d = 2$

$n_{gd} = n_g - d = 2 - 2 = 0$

$n_h = 2 > 0 = n_{gd}$

$\Downarrow$



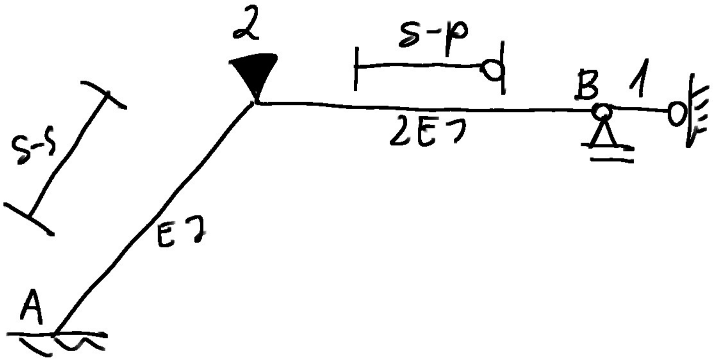
$n_\delta \geq 2\cdot 3 - 2 - 3$

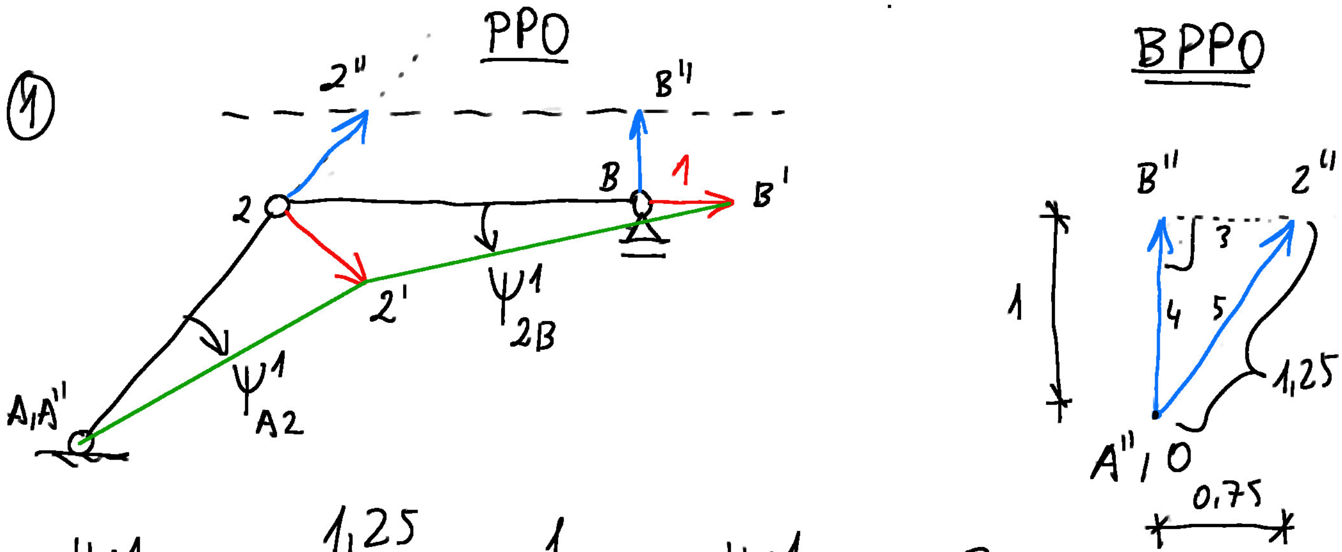
$n_\delta \geq 1$

GN przy $n_\delta = 1$

MP

2. Wyznaczenie macierzy sztywności $\mathbb{K}$

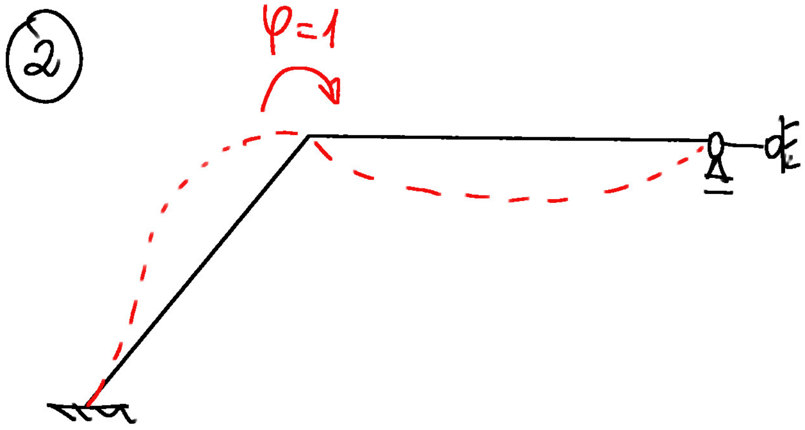




$$\psi^1_{A2} = + \frac{1,25}{5a} = \frac{1}{4a} = 0,25\frac{1}{a} \quad , \quad \psi^1_{2B} = -\frac{3}{4\cdot 5a} = -0,15\frac{1}{a}$$

$$M^1_{A2} = \frac{EJ}{5a}\left(-6\cdot\frac{1}{4a}\right) = -0,3\frac{EJ}{a^2} = M^1_{2A}$$

$$M^1_{2B} = \frac{2EJ}{5a}\left(-3\cdot\left(-0,15\frac{1}{a}\right)\right) = 0,18\frac{EJ}{a^2} \quad , \quad M^1_{B2} = 0$$



$$M^2_{A2} = \frac{EJ}{5a}(2\cdot 1) = 0,4\frac{EJ}{a}$$

$$M^2_{2A} = \frac{EJ}{5a}(4\cdot 1) = 0,8\frac{EJ}{a}$$

$$M^2_{2B} = \frac{2EJ}{5a}(3\cdot 1) = 1,2\frac{EJ}{a}$$

$$M^2_{B2} = 0$$

$$k_{11} = -\sum_{ij}\left(M_{ij}^{1} + M_{ji}^{1}\right)\psi_{ij}^{1} + \sum_{s} k_s^{\delta}\cdot \Delta L_s^{1}\cdot \Delta L_s^{1} =$$

$$= -\left(-0{,}3\frac{EJ}{a^2} - 0{,}3\frac{EJ}{a^2}\right)\frac{1}{4a} - \left(0{,}18\frac{EJ}{a^2} + 0\right)\left(-0{,}15\frac{1}{a}\right) +$$

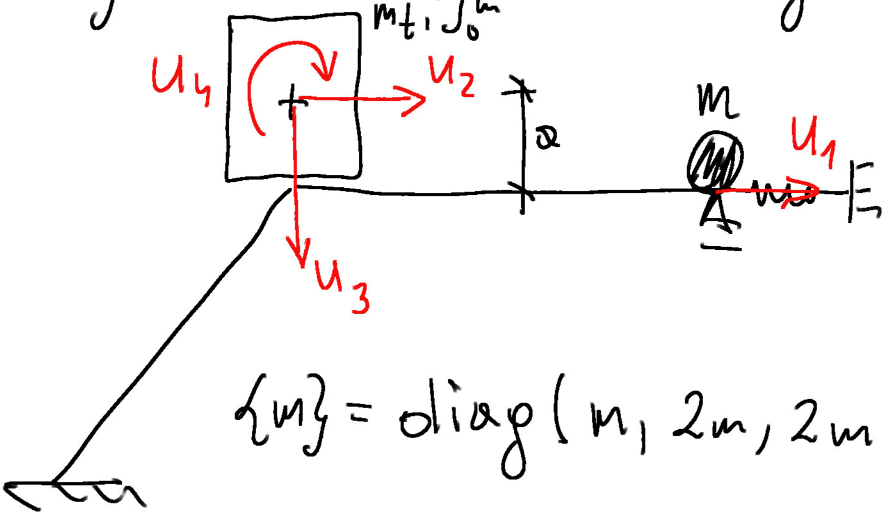
$$+ \frac{EJ}{a^3}\cdot 1\cdot 1 = 1{,}177\frac{EJ}{a^3}$$

$$k_{12} = k_{21} = M_{2A}^{1} + M_{2B}^{1} = -0{,}3\frac{EJ}{a^2} + 0{,}18\frac{EJ}{a^2} = -0{,}12\frac{EJ}{a^2}$$

$$k_{22} = M_{2A}^{2} + M_{2B}^{2} = 0{,}8\frac{EJ}{a} + 1{,}2\frac{EJ}{a} = 2\frac{EJ}{a}$$

$$\mathbb{K} = \begin{bmatrix} 1{,}177\frac{EJ}{a^3} & -0{,}12\frac{EJ}{a^2} \\ -0{,}12\frac{EJ}{a^2} & 2\frac{EJ}{a} \end{bmatrix} \qquad \det \mathbb{K} > 0 \qquad \mathbb{K}^T = \mathbb{K}$$

3. Wyznaczenie macierzy bezwładności $\mathbb{B}$

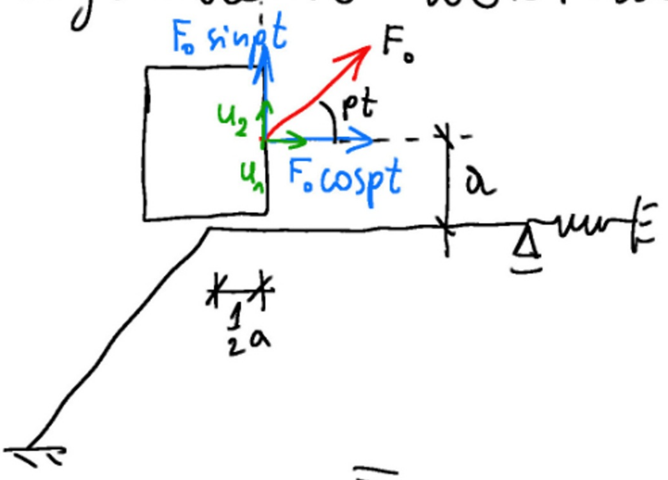


$$\{m\} = \mathrm{diag}(m, 2m, 2m, 3ma^2)$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & a \\ 3/4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$\mathbb{B}=\begin{bmatrix}1 & 1 & 0{,}75 & 0\\ 0 & a & 0 & 1\end{bmatrix}\begin{bmatrix}m & 0 & 0 & 0\\ 0 & 2m & 0 & 0\\ 0 & 0 & 2m & 0\\ 0 & 0 & 0 & 3ma^2\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & a\\ 0{,}75 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}4{,}125m & 2ma\\ 2ma & 5ma^2\end{bmatrix}$$

4. Wyznaczenie wektora wzbudzenia



$$\begin{bmatrix}u_1\\ u_2\end{bmatrix}=\underbrace{\begin{bmatrix}1 & a\\ -3/4 & -\frac{1}{2}a\end{bmatrix}}_{A_p}\begin{bmatrix}q_1\\ q_2\end{bmatrix}$$

$$\bar{F}(t)=A_p^T\cdot\bar{P}\quad,\quad \bar{P}=\begin{bmatrix}F_0\cos pt\\ F_0\sin pt\end{bmatrix}$$

$$\bar{F}(t)=\begin{bmatrix}1 & -3/4\\ a & -\frac{1}{2}a\end{bmatrix}\begin{bmatrix}F_0\cos pt\\ F_0\sin pt\end{bmatrix}=\begin{bmatrix}F_0\cos pt-\frac{3}{4}F_0\sin pt\\ F_0 a\cos pt-\frac{1}{2}F_0 a\sin pt\end{bmatrix}$$

$$\bar{F}(t)=\bar{F}_s(t)+\bar{F}_c(t)=\begin{bmatrix}-\frac{3}{4}F_0\sin pt\\ -\frac{1}{2}F_0 a\sin pt\end{bmatrix}+\begin{bmatrix}F_0\cos pt\\ F_0 a\cos pt\end{bmatrix}=$$

$$=\underbrace{\begin{bmatrix}-\frac{3}{4}F_0\\ -\frac{1}{2}F_0 a\end{bmatrix}}_{\bar{F}_s}\sin pt+\underbrace{\begin{bmatrix}F_0\\ F_0 a\end{bmatrix}}_{\bar{F}_c}\cos pt$$

5. Macierz tłumienia według hipotezy V-K

$$\mathbb{C}=\mathcal{H}\,\mathbb{K}\quad,\quad \boxed{\mathcal{H}=\frac{\gamma}{p}}$$

6. Równanie ruchu układu dynamicznego

$$\begin{bmatrix} 4{,}125m & 2ma \\ 2ma & 5ma^2 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \frac{\gamma}{p}\begin{bmatrix} 1{,}177\frac{EJ}{a^3} & -0{,}12\frac{EJ}{a^2} \\ -0{,}12\frac{EJ}{a^2} & 2\frac{EJ}{a} \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} +$$

$$+\begin{bmatrix} 1{,}177\frac{EJ}{a^3} & -0{,}12\frac{EJ}{a^2} \\ -0{,}12\frac{EJ}{a^2} & 2\frac{EJ}{a} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} F_0\cos pt - \frac{3}{4}F_0\sin pt \\ F_0 a\cos pt - \frac{1}{2}F_0 a\sin pt \end{bmatrix}$$

Zał. rozw. harmonicznego

$$\bar{q}(t) = \begin{bmatrix} q_{1s} \\ q_{2s} \end{bmatrix}\sin pt + \begin{bmatrix} q_{1c} \\ q_{2c} \end{bmatrix}\cos pt$$

$$\begin{bmatrix} (\mathbb{K}-p^2\mathbb{B}) & -\gamma\cdot\mathbb{K} \\ \gamma\mathbb{K} & (\mathbb{K}-p^2\mathbb{B}) \end{bmatrix}\begin{bmatrix} \bar{q}_s \\ \bar{q}_c \end{bmatrix} = \begin{bmatrix} \bar{F}_s \\ \bar{F}_c \end{bmatrix}$$

7. Rozw. równania algebraicznego

$$\mathbb{C} = \frac{\gamma}{p}\mathbb{K}$$

$$\begin{pmatrix} \frac{0.14575\,ej}{a^3} & -\frac{0.62\,ej}{a^2} & -\frac{0.1177\,ej}{a^3} & \frac{0.012\,ej}{a^2} \\ -\frac{0.62\,ej}{a^2} & \frac{0.75\,ej}{a} & \frac{0.012\,ej}{a^2} & -\frac{0.2\,ej}{a} \\ \frac{0.1177\,ej}{a^3} & -\frac{0.012\,ej}{a^2} & \frac{0.14575\,ej}{a^3} & -\frac{0.62\,ej}{a^2} \\ -\frac{0.012\,ej}{a^2} & \frac{0.2\,ej}{a} & -\frac{0.62\,ej}{a^2} & \frac{0.75\,ej}{a} \end{pmatrix}\begin{bmatrix} q_{1s} \\ q_{2s} \\ q_{1c} \\ q_{2c} \end{bmatrix} = \begin{bmatrix} -\frac{3}{4}F_0 \\ -\frac{1}{2}F_0 a \\ F_0 \\ F_0 a \end{bmatrix}$$

$$\begin{bmatrix} q_{1s} \\ q_{2s} \\ q_{1c} \\ q_{2c} \end{bmatrix} = \begin{pmatrix} \frac{4.49959\,a^3\,F0}{ej} \\ \frac{2.71892\,a^2\,F0}{ej} \\ -\frac{2.52138\,a^3\,F0}{ej} \\ -\frac{1.40406\,a^2\,F0}{ej} \end{pmatrix}$$

$$\operatorname{am} q_1 = \sqrt{q_{1s}^2 + q_{1c}^2} = \sqrt{\left(4{,}49959 \frac{F_0 a^3}{EJ}\right)^2 + \left(-2{,}52138 \frac{F_0 a^3}{EJ}\right)^2}$$

$$\boxed{\operatorname{am} q_1 = 5{,}15788 \frac{F_0 a^3}{EJ}}$$

$$\operatorname{am} q_2 = \sqrt{q_{2s}^2 + q_{2c}^2} = \sqrt{\left(2{,}71892 \frac{F_0 a^2}{EJ}\right)^2 + \left(-1{,}40406 \frac{F_0 a^2}{EJ}\right)^2}$$

$$\boxed{\operatorname{am} q_2 = 3{,}06005 \frac{F_0 a^2}{EJ}}$$

Obciążenie kinetyczne - hipoteza V-K

$$\bar{Q}(t) = \bar{F}(t) - \mathbb{B}\ddot{\bar{q}}$$

$$\bar{q}(t) = \bar{q}_s \sin pt + \bar{q}_c \cos pt$$

$$\ddot{\bar{q}}(t) = -p^2 \bar{q}(t)$$

$$\bar{Q}(t) = \bar{F}(t) + p^2 \mathbb{B} \bar{q}(t)$$

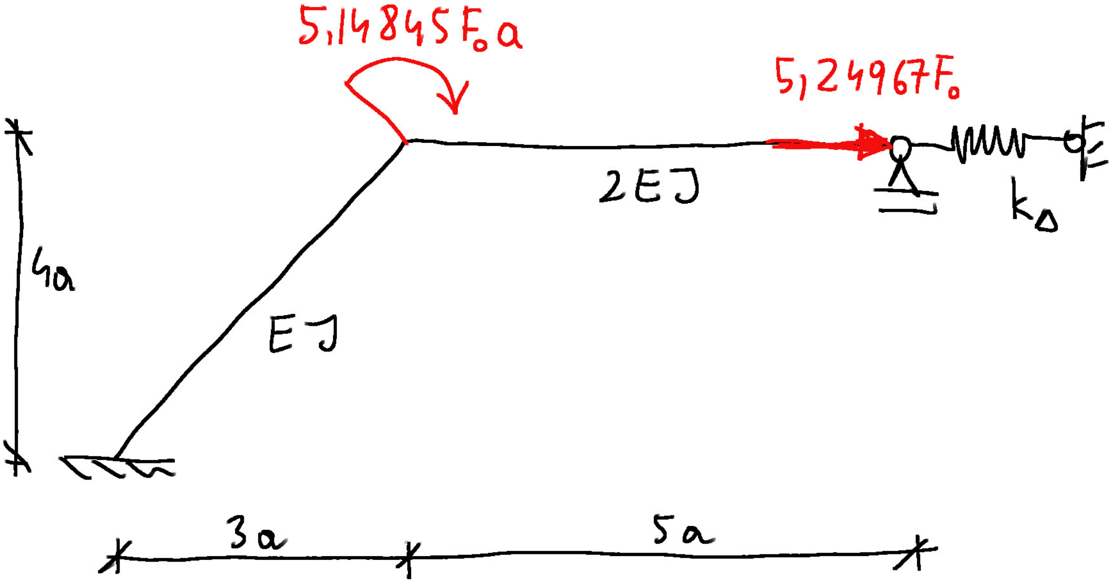
$$\bar{Q}_s = \bar{F}_s + p^2 \mathbb{B} \bar{q}_s$$

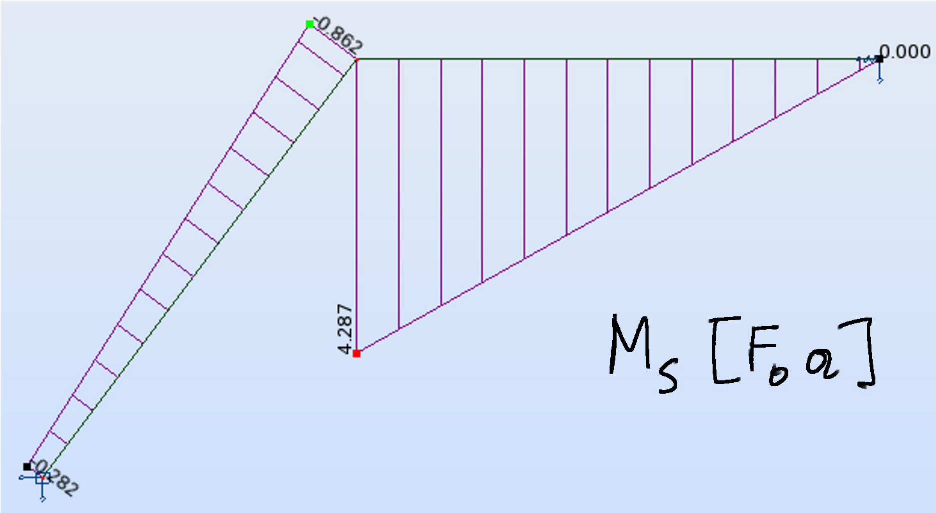
$$\bar{Q}_c = \bar{F}_c + p^2 \mathbb{B} \bar{q}_c$$

$$\bar{Q}_s = \begin{pmatrix} 5.24967\, F0 \\ 5.14845\, a\, F0 \end{pmatrix}$$

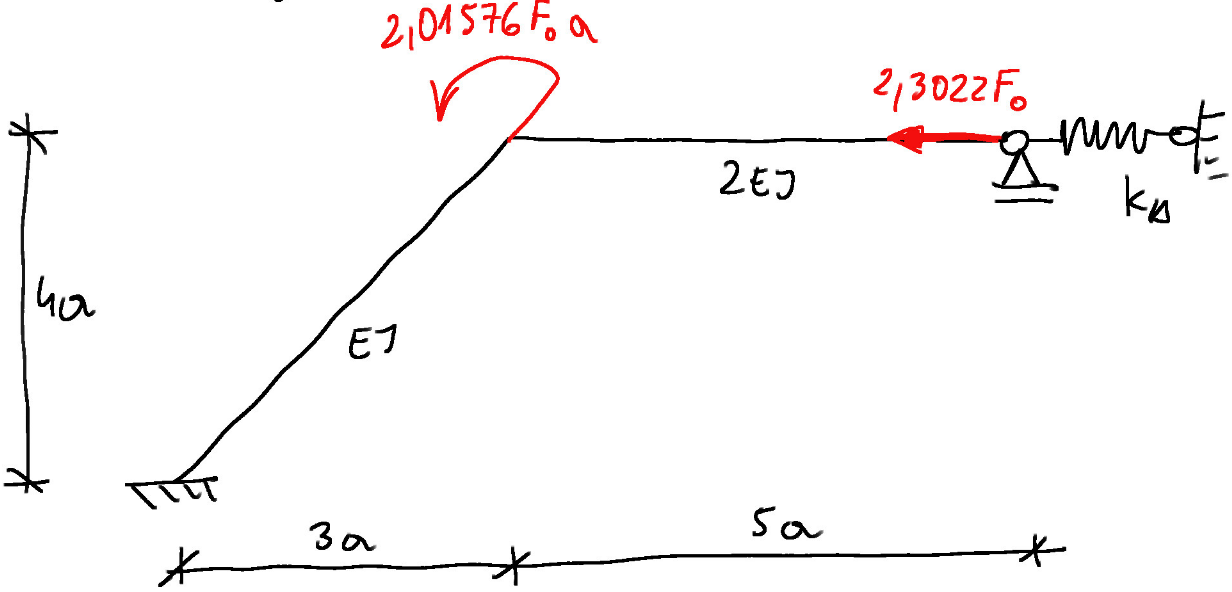
$$\bar{Q}_c = \begin{pmatrix} -2.3022\, F0 \\ -2.01576\, a\, F0 \end{pmatrix}$$

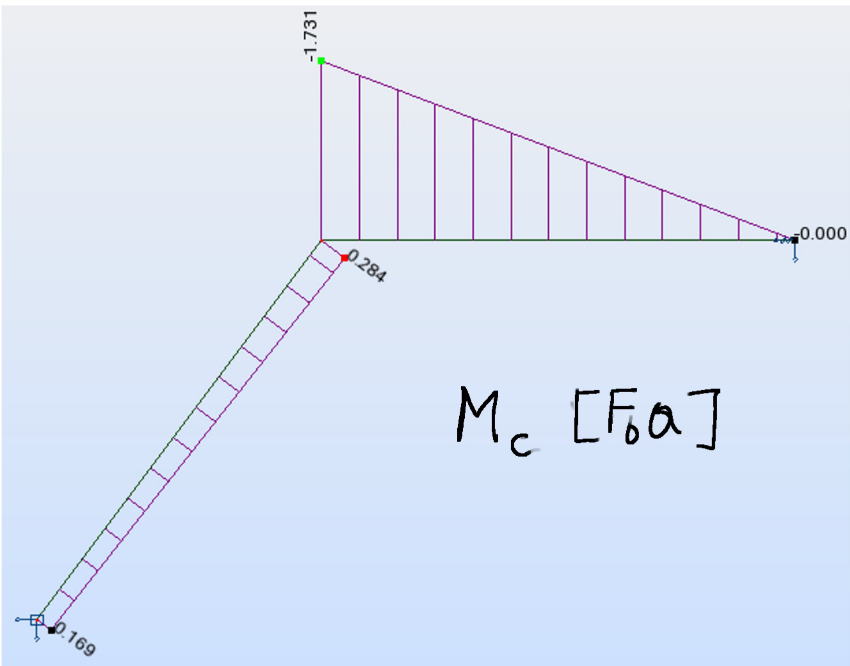
* Momenty dynamiczne - składowe sinusowe





* Momenty dynamiczne – składowe cosinusowe





* Amplitudalne momenty dynamiczne - amM

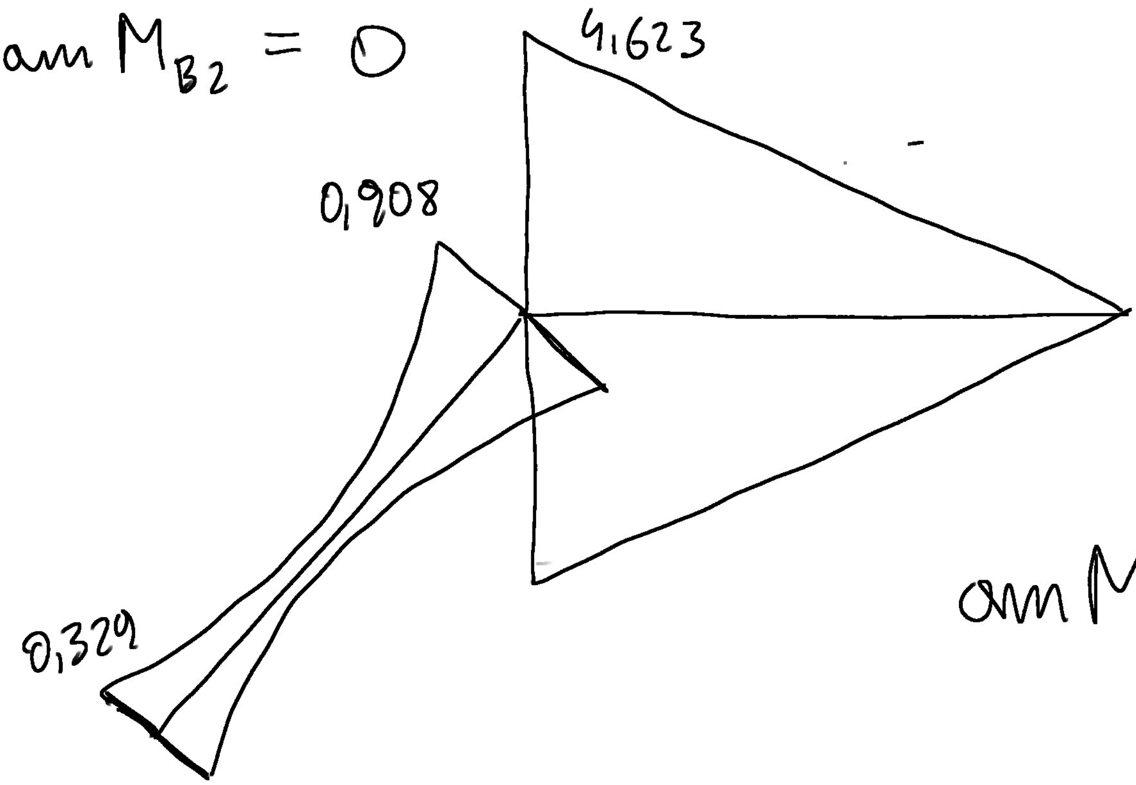
$$amM_i = \sqrt{M_{is}^2 + M_{ic}^2}$$

$$amM_{A2} = \sqrt{(0{,}282 F_o a)^2 + (0{,}169 F_o a)^2} = 0{,}329 F_o a$$

$$amM_{2A} = \sqrt{(0{,}862 F_o a)^2 + (0{,}284 F_o a)^2} = 0{,}908 F_o a$$

$$amM_{2B} = \sqrt{(4{,}287 F_o a)^2 + (1{,}731 F_o a)^2} = 4{,}623 F_o a$$

$$amM_{B2} = 0$$



$amM\ [F_o a]$