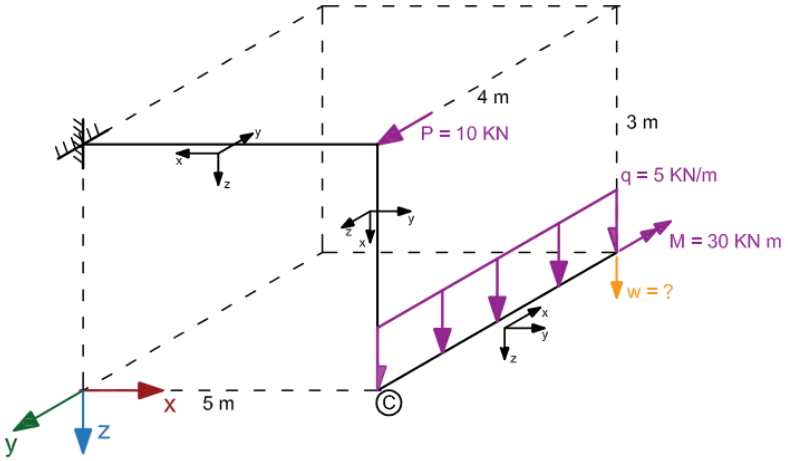
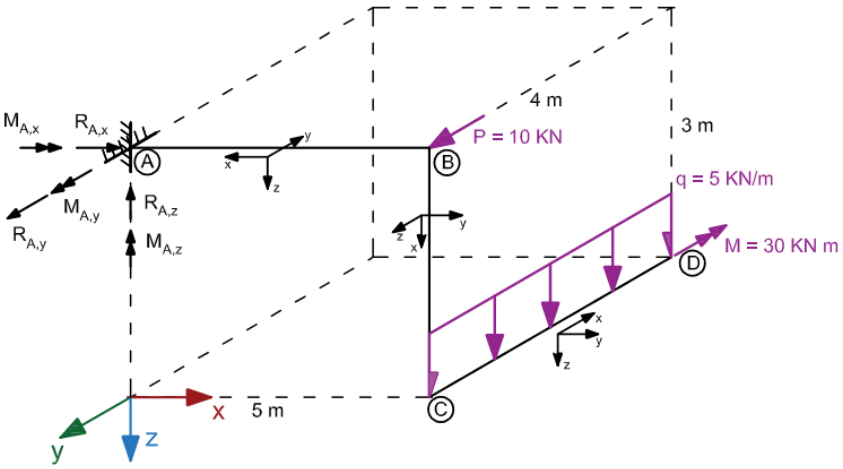
Dany jest hiperstatyczny układ przestrzenny o schemacie statycznym i obciążeniu jak na rysunku. Zakładamy, że $GI_s = 0{,}5\ EI_y$ oraz $EI_y = EI_z$. Obliczyć wskazane przemieszczenie w:



1. Obliczenie reakcji od obciążenia danego



$\sum X = 0 \rightarrow R_{A,x} = 0,$

$\sum Y = 0 \rightarrow R_{A,y} + P = 0 \rightarrow R_{A,y} = -P \rightarrow R_{A,y} = -10\ kN,$

$\sum Z = 0 \rightarrow R_{A,z} - q \cdot 4\ m \rightarrow R_{A,z} = q \cdot 4\ m \rightarrow R_{A,z} = 5\ \frac{kN}{m} \cdot 4\ m \rightarrow R_{A,z} = 20\ kN,$

$\sum M_{xA} = 0 \rightarrow M_{A,x} - q \cdot 4\ m \cdot 2\ m = 0 \rightarrow M_{A,x} = q \cdot 4\ m \cdot 2\ m = 5\ \frac{kN}{m} \cdot 4\ m \cdot 2\ m \rightarrow M_{A,x} = 40\ kN\ m,$

$\sum M_{yA} = 0 \rightarrow M_{A,y} - q \cdot 4\ m \cdot 5\ m - M = 0 \rightarrow M_{A,y} = q \cdot 4\ m \cdot 5\ m + M = 5\ \frac{kN}{m} \cdot 4\ m \cdot 5\ m +$
$30\ kN\ m \rightarrow\ M_{A,y} = 130\ kN\ m,$

$\sum M_{z,A} = 0 \rightarrow M_{A,z} = 0.$

1. Obliczenie sił wewnętrznych i sporządzenie wykresów

Siły osiowe:

$N_{x,DC} = N_{x,CD} = 0,$

$N_{x,CB} = N_{x,BC} = q \cdot 4\ m = 5\ \frac{kN}{m} \cdot 4\ m = 20\ kN\ m,$

$N_{x,BA} = N_{x,AB} = 0.$

Siły tnące:

$V_{y,DC} = V_{y,CD} = 0,$

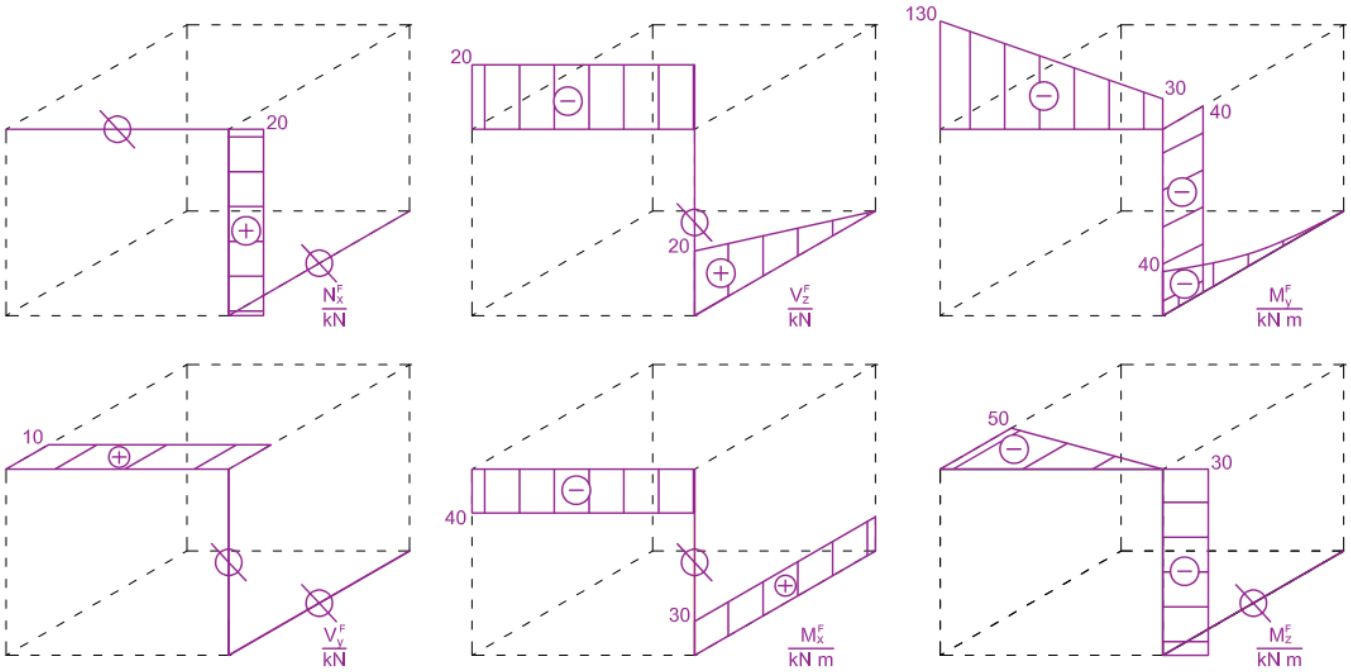
$$V_{y,CB} = V_{y,BC} = 0,$$
$$V_{z,BA} = V_{z,AB} = P = 10\ kN,$$
$$V_{z,DC} = 0,$$
$$V_{z,CD} = q \cdot 4\ m = 5\ \frac{kN}{m} \cdot 4\ m = 20\ kN\ m,$$
$$V_{z,CB} = V_{z,BC} = 0,$$
$$V_{z,BA} = V_{z,AB} = -q \cdot 4\ m = -5\ \frac{kN}{m} \cdot 4\ m = -20\ kN\ m.$$

Momenty skręcające:

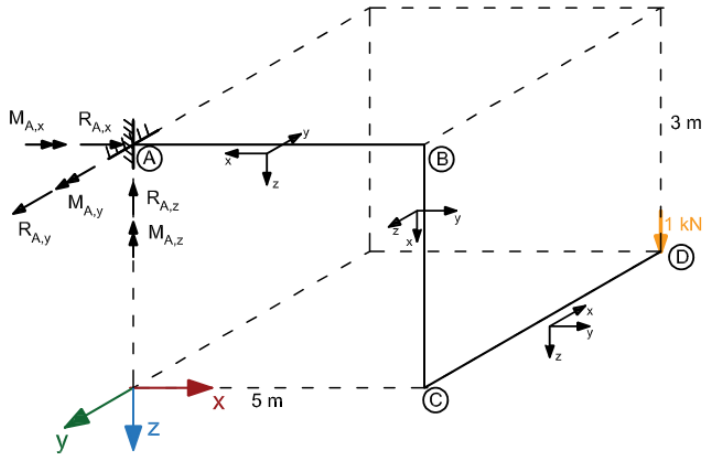
$$M_{x,DC} = M_{x,CD} = M = 30\ kN\ m,$$
$$M_{x,CB} = M_{x,BC} = 0,$$
$$M_{x,BA} = M_{x,AB} = -q \cdot 4\ m \cdot 2\ m = -5\ \frac{kN}{m} \cdot 4\ m \cdot 2\ m = -40\ kN\ m.$$

Momenty zginające:

$$M_{y,DC} = 0,$$
$$M_{y,CD} = -q \cdot 4\ m \cdot 2\ m = -5\ \frac{kN}{m} \cdot 4\ m \cdot 2\ m = -40\ kN\ m,$$
$$M_{y,CB} = M_{y,BC} = -q \cdot 4\ m \cdot 2\ m = -5\ \frac{kN}{m} \cdot 4\ m \cdot 2\ m = -40\ kN\ m,$$
$$M_{y,BA} = -M = -30\ kN\ m,$$
$$M_{x,AB} = -M - q \cdot 4\ m \cdot 5\ m = -30\ kN\ m - 5\ \frac{kN}{m} \cdot 4\ m \cdot 5\ m = -130\ kN\ m.$$



2. Obliczenie reakcji od obciążenia jednostkowego



$$\sum X = 0 \rightarrow R_{A,x} = 0,$$
$$\sum Y = 0 \rightarrow R_{A,y} = 0,$$
$$\sum Z = 0 \rightarrow R_{A,z} = 1\ kN,$$
$$\sum M_{xA} = 0 \rightarrow M_{A,x} - 1\ kN \cdot 4\ m = 0 \rightarrow M_{A,x} = 4\ kN\ m,$$
$$\sum M_{yA} = 0 \rightarrow M_{A,y} - 1\ kN \cdot 5\ m = 0 \rightarrow M_{A,x} = 5\ kN\ m,$$
$$\sum M_{z,A} = 0 \rightarrow M_{A,z} = 0.$$

Siły osiowe:

$$N_{x,DC} = N_{x,CD} = 0,$$
$$N_{x,CB} = N_{x,BC} = 1\ kN,$$
$$N_{x,BA} = N_{x,AB} = 0.$$

Siły tnące:

$$V_{y,DC} = V_{y,CD} = 0,$$
$$V_{y,CB} = V_{y,BC} = 0,$$
$$V_{z,BA} = V_{z,AB} = 0,$$
$$V_{z,DC} = V_{z,CD} = 1\ kN,$$
$$V_{z,CB} = V_{z,BC} = 0,$$
$$V_{z,BA} = V_{z,AB} = -1kN.$$

Momenty skręcające:

$$M_{x,DC} = M_{x,CD} = 0,$$
$$M_{x,CB} = M_{x,BC} = 0,$$
$$M_{x,BA} = M_{x,AB} = -1kN \cdot 4\ m = -4\ kN\ m.$$

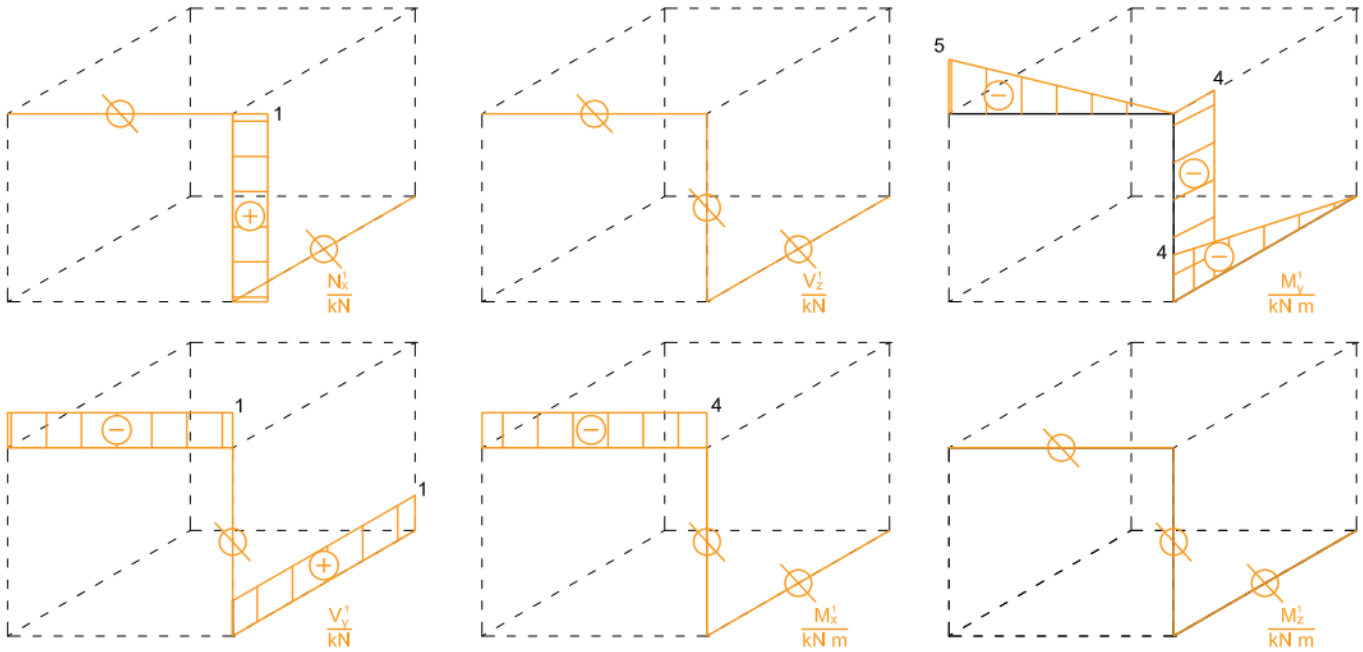
Momenty zginające:

$$M_{y,DC} = 0,$$
$$M_{y,CD} = -1kN \cdot 4\ m = -4\ kN\ m,$$
$$M_{y,CB} = M_{y,BC} = -1kN \cdot 4\ m = -4\ kN\ m,$$
$$M_{y,BA} = 0,$$
$$M_{y,AB} = -1kN \cdot 5\ m = -5\ kN\ m.$$

$M_{z,DC} = M_{z,CD} = 0,$

$M_{z,CB} = M_{z,BC} = 0,$

$M_{z,BA} = M_{z,AB} = 0.$



3. Obliczenie szukanego przemieszczenia

$$1kN \cdot w = \int \frac{M_y^1 M_y^F}{EI_y} dx + \int \frac{M_z^1 M_z^F}{EI_z} dx + \int \frac{M_x^1 M_x^F}{GI_x} dx = \frac{1}{GI_x} \cdot 5m \cdot (-40kN \cdot m) \cdot (-4kN \cdot m)$$

$$+ \frac{4m}{6EI_y} [0 \cdot 0 + 4 \cdot (-10kN \cdot m) \cdot (-2kN \cdot m) + (-40kN \cdot m) \cdot (-4kN \cdot m)]$$

$$+ \frac{1}{EI_y} 3m \cdot (-40kN \cdot m) \cdot (-4kN \cdot m)$$

$$+ \frac{5m}{6EI_y} [0 \cdot (-30kN \cdot m) + 4 \cdot (-80kN \cdot m) \cdot (-2{,}5kN \cdot m) + (-150kN \cdot m) \cdot (-5kN \cdot m)],$$

$w = 3448{,}3 \frac{kN \cdot m^3}{EI_y}.$