Diagram labels: $9\sqrt{5}$, M_A, H_A, V_A, A, B, C, $30\sqrt{2}$, $45°$, D, E, F, G, R_G, R_F, 1m

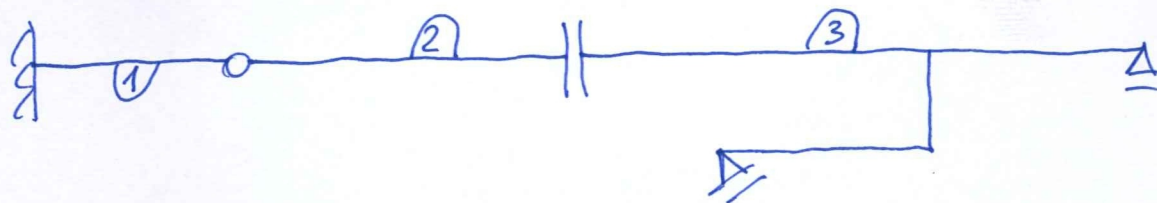Dimensions: 2m, 4m, 1m, 1m, 2m, 2m

1 Spr. GN i SW

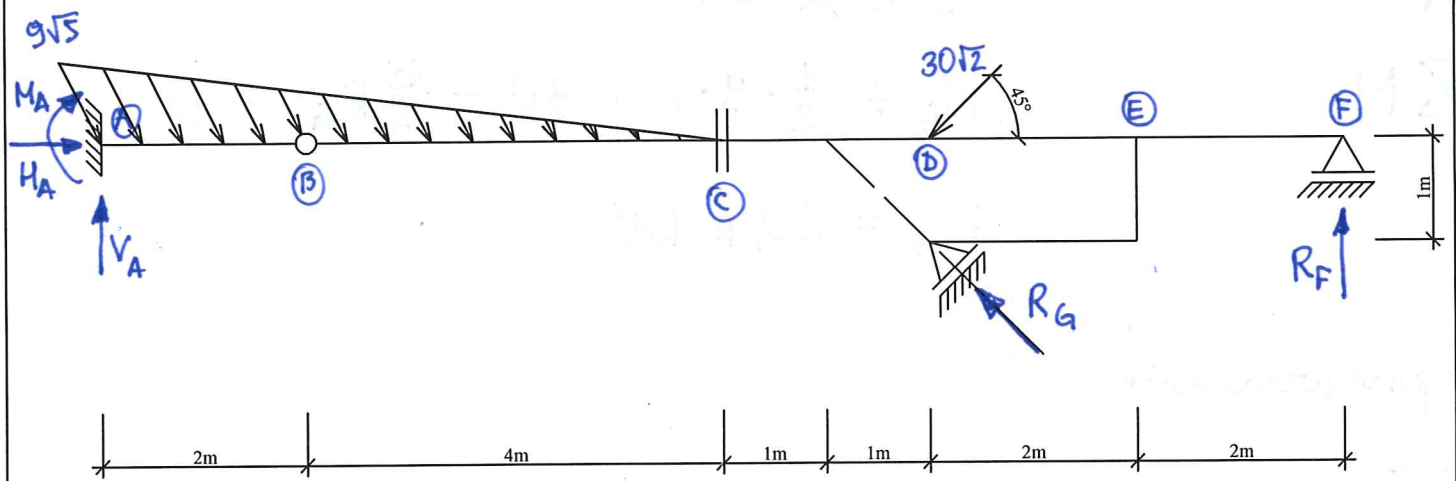* war ilościowy

$$n = e - 3t$$
$$n = 9 - 3 \cdot 3 = 0$$

+ war jakościowy

Z tw. o 2 tarczach ① GN.

Z tw. o 3 tarczach ② ③ GN

Układ jest geometrycznie niezmienny i statycznie wyznaczalny.

2 Reakcje

War. konstrukcyjny $\quad \sum y_i^c = 0$

$$-V_A + \frac{1}{2} \cdot 6 \cdot 9\sqrt{5} \cdot \frac{2}{\sqrt{5}} = 0$$

$$V_A = 54 kN$$

War. konstrukcyjny $\quad \sum M_B(L) = 0$

$$M_A + 2V_A - 2\cdot 12 \cdot 1 - \frac{1}{2} \cdot 6 \cdot 2 \cdot \frac{4}{3} = 0$$

$$M_A = -76 \, kNm$$

$$\sum M_F = 0$$

$$M_A + 12 V_A - \frac{1}{2} \cdot 6 \cdot 18 \cdot 10 - 4 \cdot 30 + 5 \cdot \frac{\sqrt{2}}{2} R_G = 0$$

$$\frac{5\sqrt{2}}{2} R_G = 88$$

$$R_G = 24,89 \, kN$$

$$R_{Gx} = R_{Gy} = 17,6 \, kN$$

War. konstrukcyjny $\quad \sum y_P^c = 0$

$$30 - R_{Gy} - R_F = 0 \quad \Rightarrow \quad R_F = 12,4 kN$$

$$\sum x = 0$$

$$M_A + \frac{1}{2} \cdot 9 \cdot 6 - 30 - \frac{\sqrt{2}}{2} R_G = 0$$

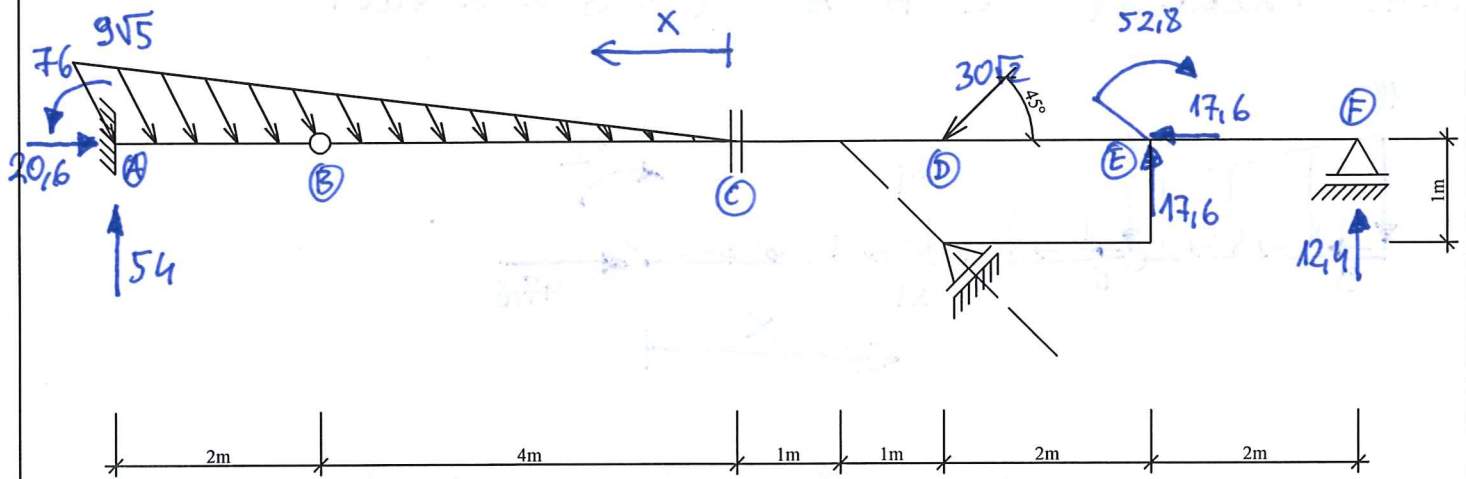$$M_A = 20,6 \text{ kN}$$

Sprawdzenie

$$\sum M_E = 0$$

$$10 V_A + M_A - \frac{1}{2} \cdot 18 \cdot 6 \cdot 8 - 2 \cdot 30 + 3 \cdot R_{Gy} - 2 R_F =$$

$$= 10 \cdot 54 - 76 - 432 - 60 + 3 \cdot 17,6 - 2 \cdot 12,4 =$$

$$= 592,8 - 592,8 = 0 \qquad \leftarrow \text{ok!}$$

SCHEMAT OBLICZENIOWY
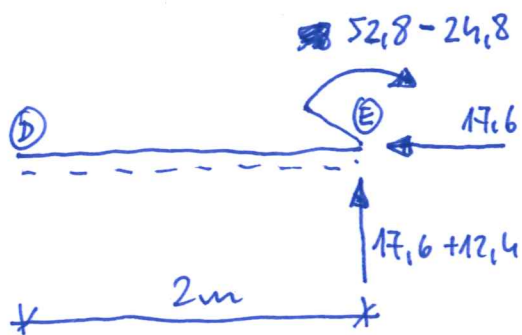
3. Siły przekrojowe

3.1. Przekrój F-E

$$M_{FE} = 0 \quad , \quad M_{EF} = 2 \cdot 12{,}4 = 24{,}8 \text{ kNm}$$

$$T_{FE} = T_{EF} = -12{,}4 \text{ kN}$$
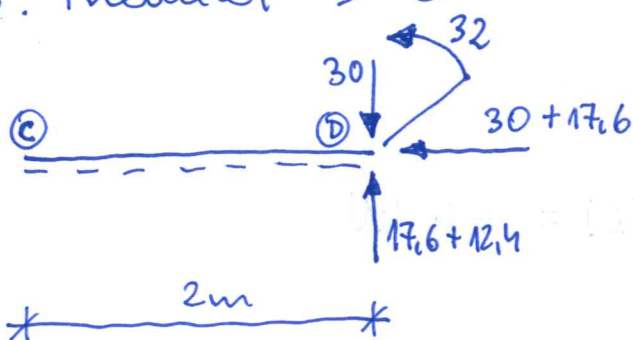
$$N_{FE} = N_{EF} = 0$$

3.2. Przekrój E-D

$$M_{ED} = -52{,}8 + 24{,}8 = -28 \text{ kNm}$$

$$M_{DE} = -28 + 2(17{,}6 + 12{,}4) =$$
$$= 32 \text{ kNm}$$

$$T_{DE} = T_{ED} = -(17{,}6 + 12{,}4) = -30 \text{ kN}$$

$$N_{DE} = N_{ED} = -17{,}6 \text{ kN}$$
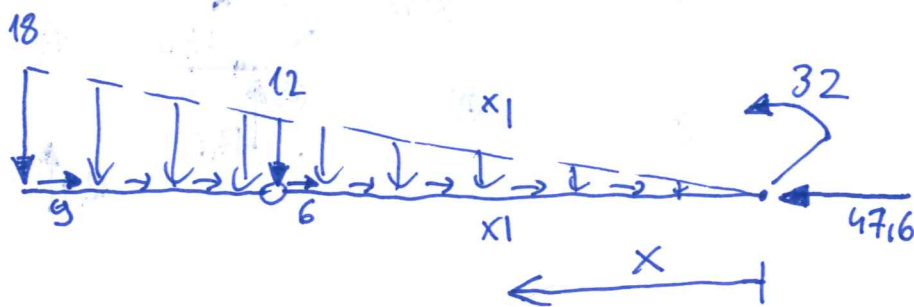
3.3. Przekrój D-C

$$M_{DC} = 32 \text{ kNm}$$

$$M_{CD} = 32 - 2 \cdot 30 + 2 \cdot (17{,}6 + 12{,}4) =$$
$$= 32 \text{ kNm}$$

$$T_{DC} = T_{CD} = 30 - 17{,}6 - 12{,}4 = 0$$

$$N_{DC} = N_{CD} = -30 - 17{,}6 = -47{,}6 \text{ kN}$$

3.4. Przedział C-B-A $(0 \leq x \leq 6m)$

$$M(x) = 32 - \frac{1}{2} \cdot x \cdot \frac{18}{6} x \cdot \frac{x}{3} = -\frac{1}{2} x^3 + 32$$

$$T(x) = +\frac{1}{2} \cdot x \cdot \frac{18}{6} x = 1{,}5 x^2$$

$$N(x) = -47{,}6 + \frac{1}{2}\frac{9}{6} x \cdot x = 0{,}75 x^2 - 47{,}6$$

$$T(x_0) = 0 \quad \Longleftrightarrow \quad 1{,}5 x^2 = 0 \quad \Longrightarrow \quad x_0 = 0$$

Na przedziale nie
występuje ekstremum momentu

$M(0) = 32\,kNm$

$M(2) = 28\,kNm$

$M(4) = 0 \quad$ – przgub

$M(6) = -76\,kNm$

$T(0) = 0$

$T(2) = 6\,kN$

$T(4) = 24\,kN$

$T(6) = 54\,kN$

$N(0) = -47{,}6\,kN \qquad N(6) = 6{,}4\,kN$

$N(2) = -44{,}6\,kN$

$N(4) = -35{,}6\,kN$

M [kNm]

76

28

0

0

28

32

32

24,8

T [kN]

54

24

+

6

0   0

30

12,4

N [kN]

47,6

44,6

35,6

20,6

17,6

0

$9\sqrt{5}$

H

$30\sqrt{2}$

$45°$

2m 4m 1m 1m 2m 2m

1m

Mechanism „H"

(1,0)
∞

(2,3)
∞

A"   B"   D" C''' E"
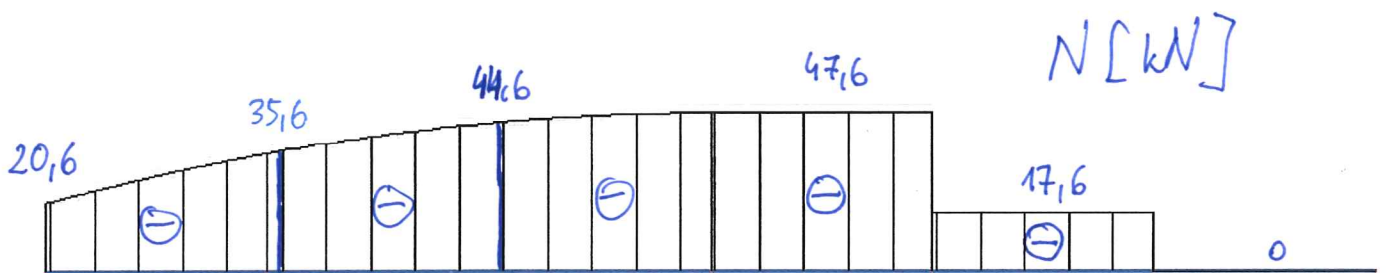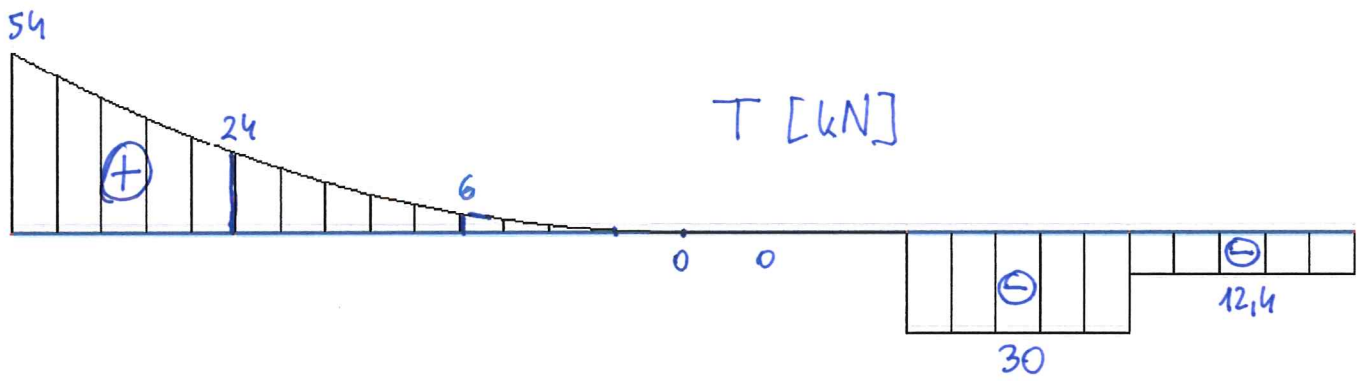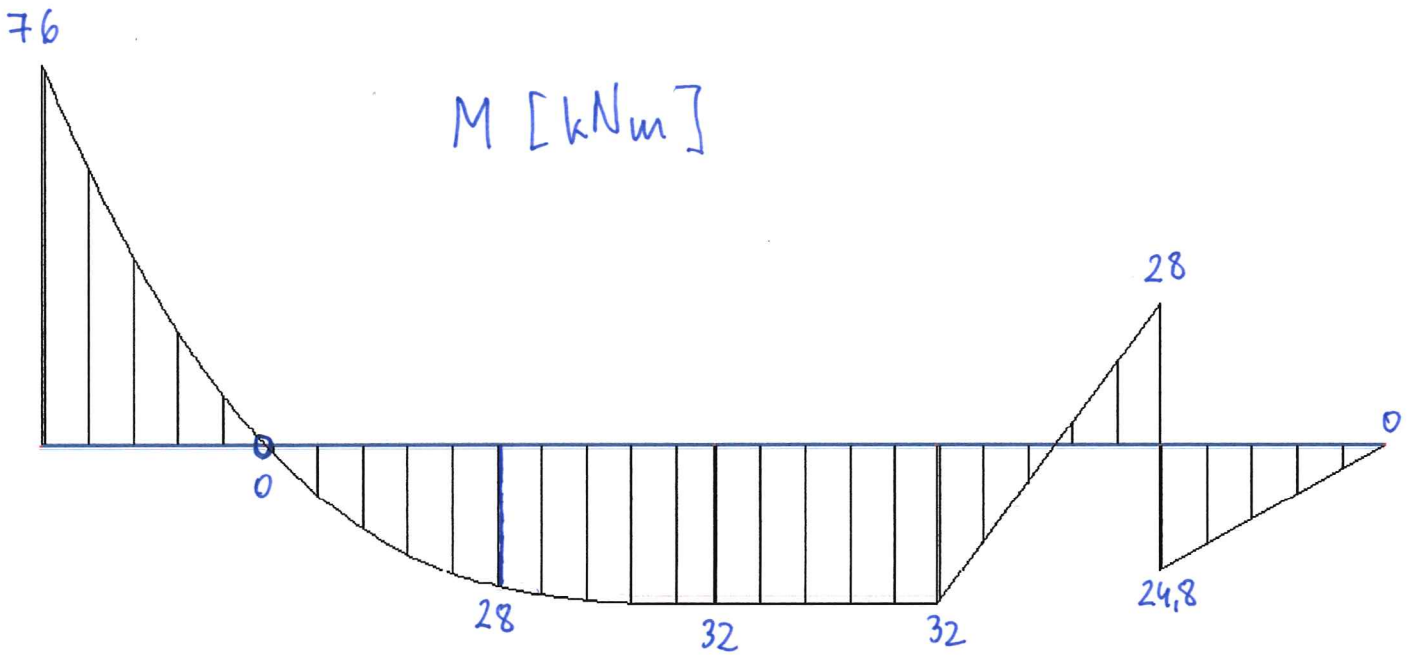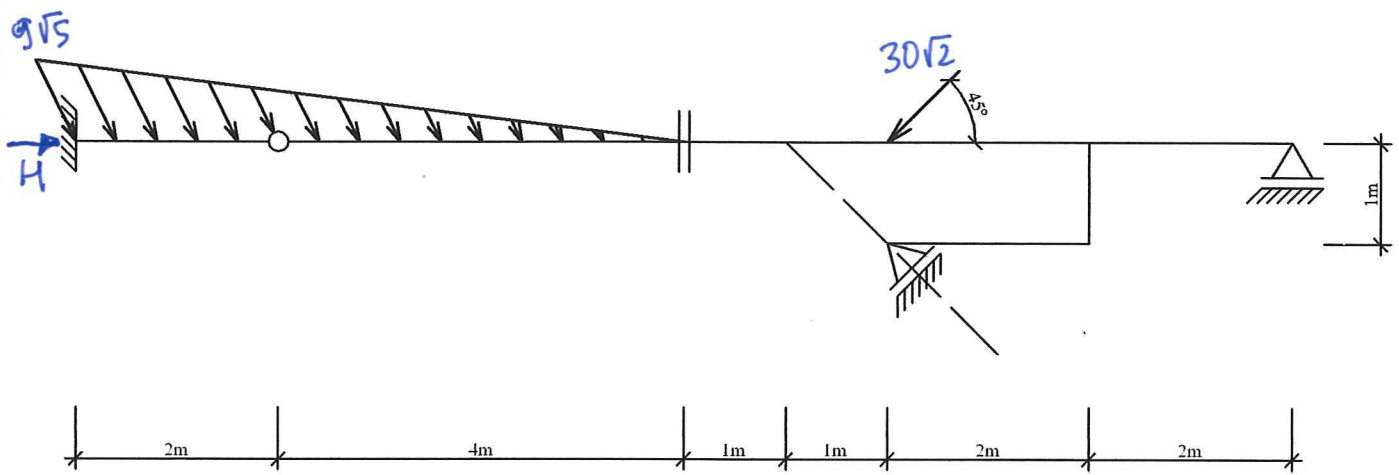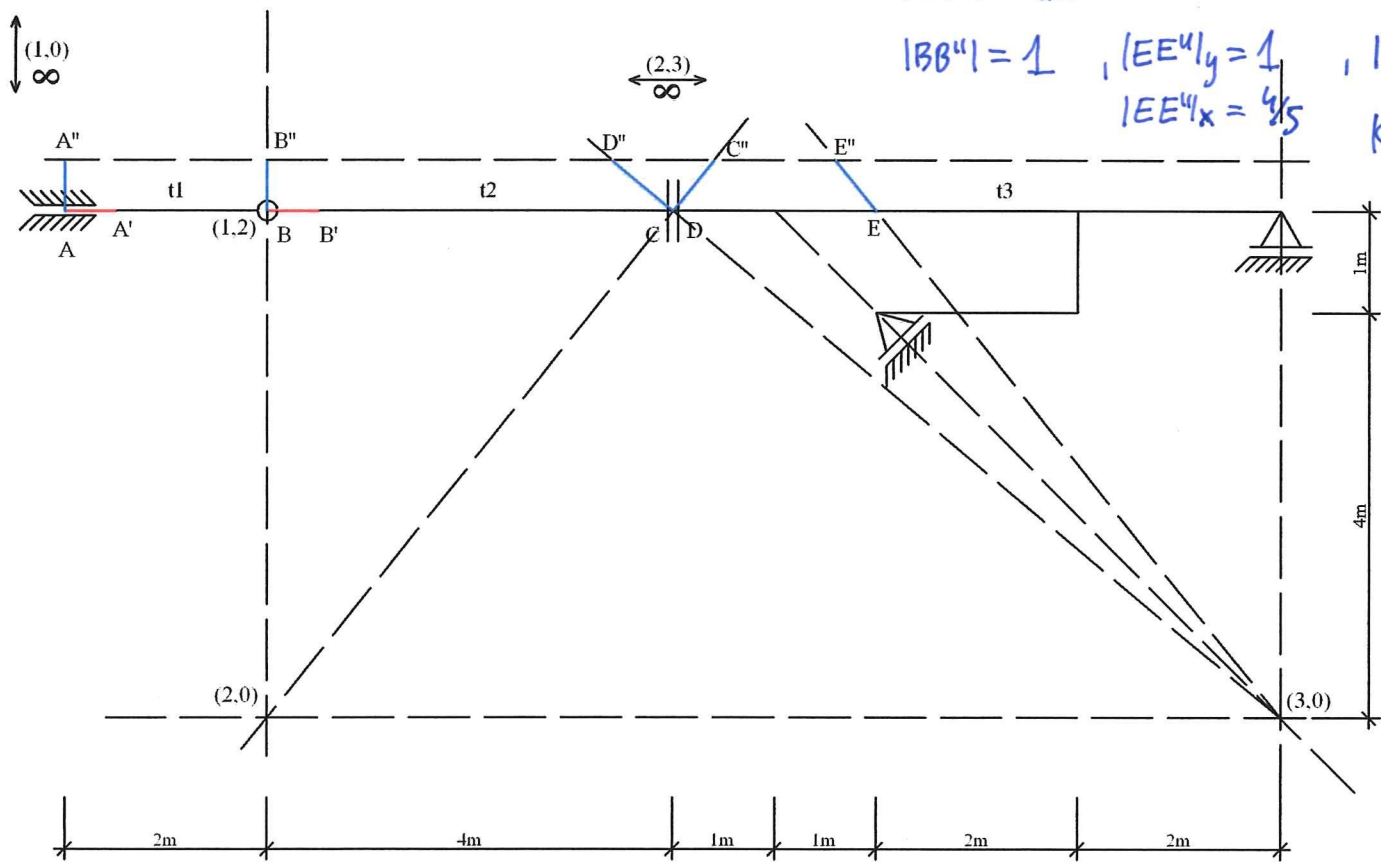
$|AA''| = 1$ — nastane

$|BB''| = 1$, $|EE''|_y = 1$, $|DD''|_x = \frac{6}{5}$

$|EE''|_x = \frac{4}{5}$, $|CC''|_x = \frac{4}{5}$

t1   t2   t3

A'   (1,2) B B'   C D   E

A

(2,0)   (3,0)

2m 4m 1m 1m 2m 2m

1m

4m

$$-H \cdot |AA''| - \frac{9+6}{2} \cdot 2 \cdot |AA''| - \frac{1}{2} \cdot 6 \cdot 4 \cdot |BB''| + 30 \cdot |EE''|_x + 30 |EE''|_y +$$

$$- \frac{1}{2} \cdot 12 \cdot 4 \cdot \frac{1}{3} \cdot |CC''|_x = 0$$

$$-H - 15 - 12 + 30 \cdot \frac{4}{5} + 30 - 8 \cdot \frac{4}{5} = 0$$
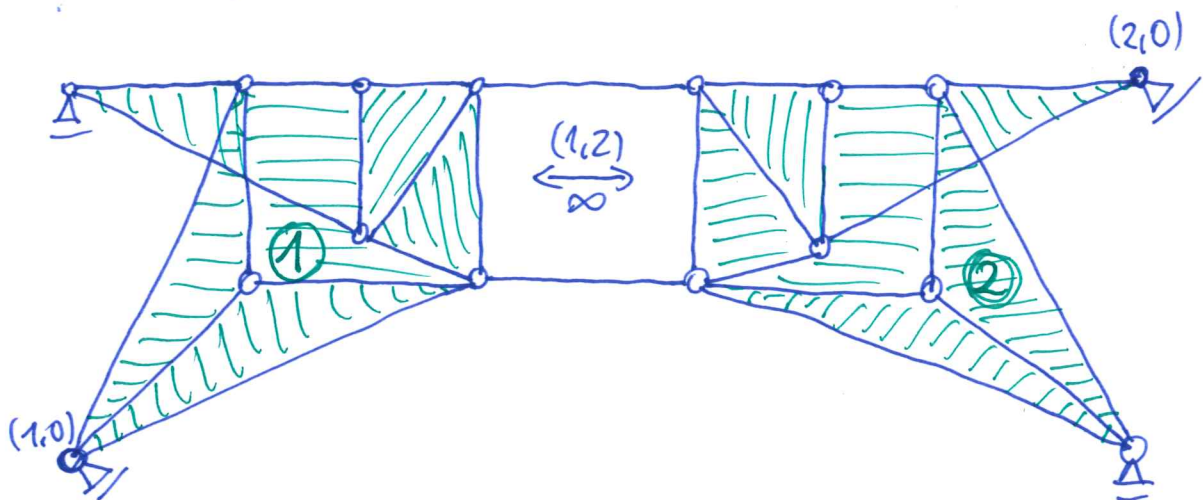
$$H = 20,6 \text{ kN}$$
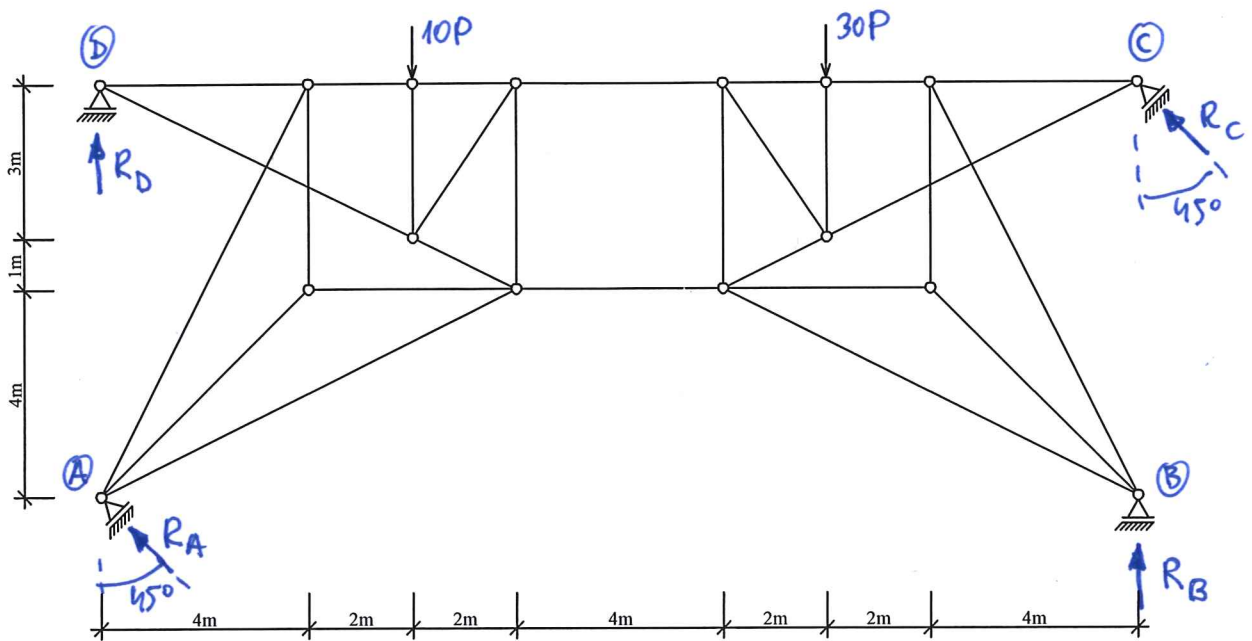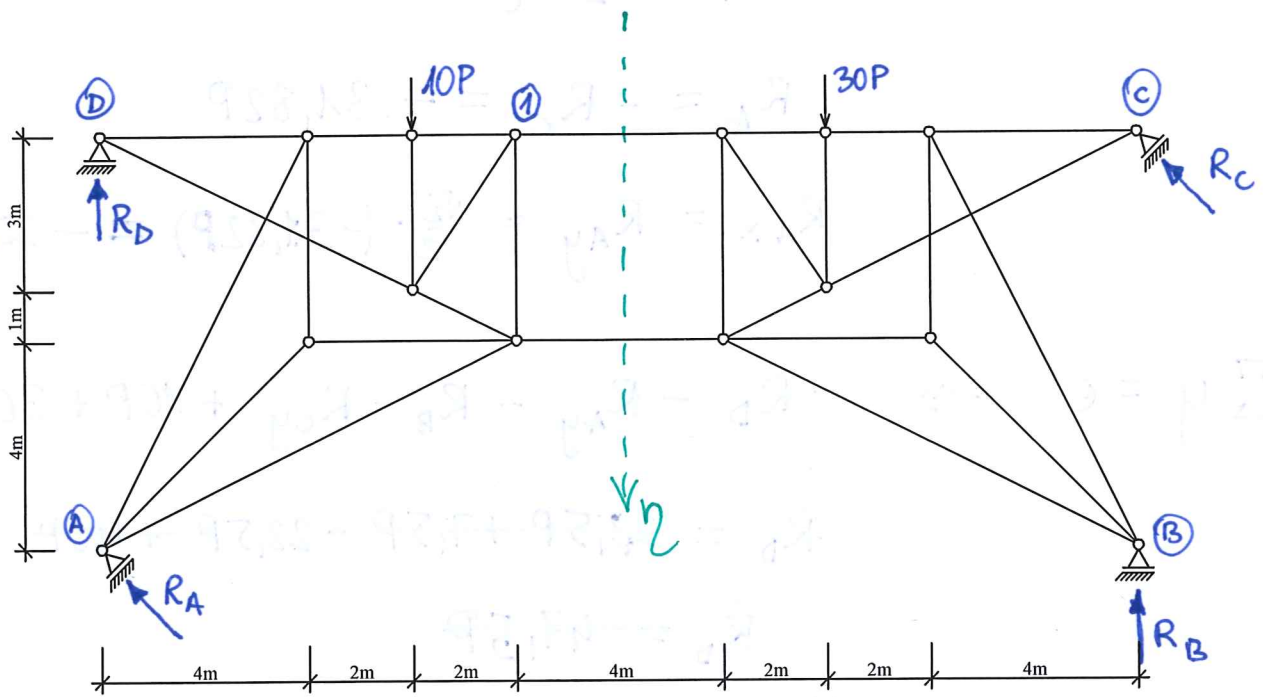
1. Sprawdzenie GN i SW

* warunek ilościowy

$$n = 2w - p - r$$
$$n = 2 \cdot 16 - 28 - 4 = 0$$

* warunek jakościowy

W prętach kratownicy można wyodrębnić dwie
tarcze, które tworzą z ostoją (tarcza 0) układ
GN na mocy tw. o 3 tarczach

2. Reakcje

$$\sum M_A = 0 \quad \rightarrow \quad -20 \cdot \frac{\sqrt{2}}{2} R_C - 8 \cdot \frac{\sqrt{2}}{2} R_C - 20 R_B + 6 \cdot 10P +$$

$$+ \ 14 \cdot 30P = 0$$

War. konstrukcyjny $\quad \sum \eta_{(p)} = 0$

$$30P - \frac{\sqrt{2}}{2} R_C - R_B = 0$$

$$R_B = -\frac{\sqrt{2}}{2} R_C + 30P$$

Stąd:

$$-10\sqrt{2} R_C - 4\sqrt{2} R_C + 10\sqrt{2} R_C - 600P + 60P + 420P = 0$$

$$R_C = -21,213P$$

$$R_{Cx} = R_{Cy} = -\frac{\sqrt{2}}{2} \cdot 21,213P = -15P$$

$$R_B = +15P + 30P = 45P$$

$$\sum x = 0 \rightarrow -\frac{\sqrt{2}}{2} R_A - \frac{\sqrt{2}}{2} R_c = 0$$

$$R_A = -R_c = 21,213 \, P$$

$$R_{Ax} = R_{Ay} = \frac{\sqrt{2}}{2} \cdot (21,213 P) = \mathbf{15P}$$

$$\sum y = 0 \rightarrow -R_D - R_{Ay} - R_B - R_{cy} + 10P + 30P = 0$$

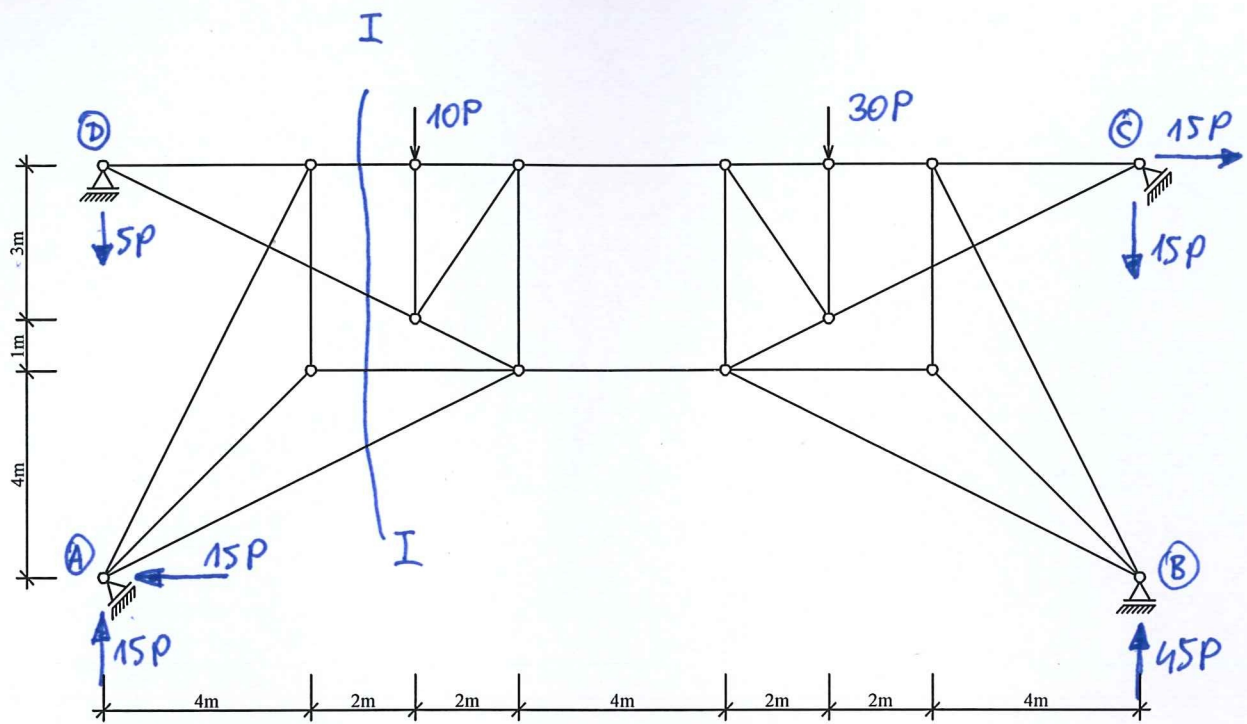$$R_D = -15P - 45P - (-15P) + 40P$$

$$R_D = -5P$$

Sprawdzenie

$$\sum M_1 = 0$$

$$8 R_{Ay} + 8 R_{Ax} + 8 R_D - 2 \cdot 10P + 6 \cdot 30P - 12 R_{cy} - 12 R_B =$$
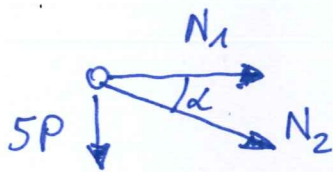
$$= 8 \cdot 15P + 8 \cdot 15P + 8 \cdot (-5P) - 20P + 180P - 12 \cdot (-15P) - 12 \cdot 45P =$$

$$= 600P - 600P = 0$$

3. Wyznaczenie sił w zaznaczonych prętach

3.1. Równowaga węzła Ⓓ

$$\cos\alpha = \frac{2}{\sqrt{5}}$$
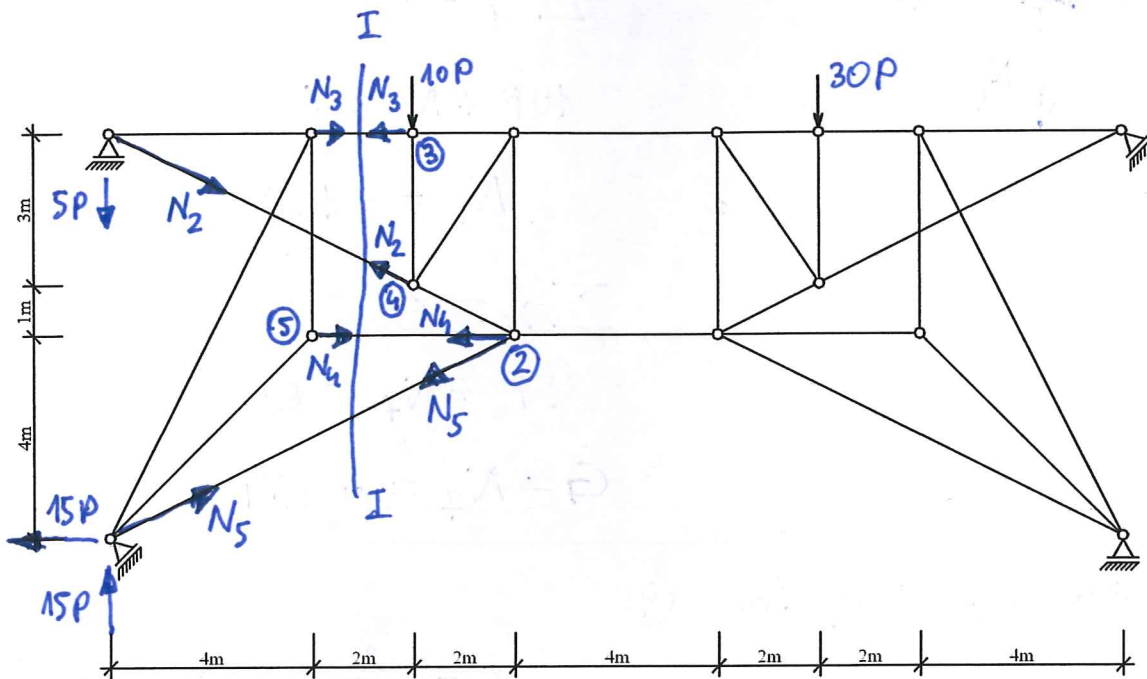$$\sin\alpha = \frac{1}{\sqrt{5}}$$

$$\Sigma_y = 0 \qquad 5P + \frac{1}{\sqrt{5}} N_2 = 0$$

$$N_2 = -5\sqrt{5}\, P$$

$$\Sigma x = 0$$

$$N_1 + \frac{2}{\sqrt{5}} N_2 = 0$$

$$N_1 = -\frac{2}{\sqrt{5}} \cdot (-5\sqrt{5}P) = 10P$$

3.2. Pręcięcie Rittera I-I

$$\sum M_2(L) = 0 \qquad 8 \cdot 15P + 4 \cdot 15P + 4 \cdot N_3 - 8 \cdot 5P = 0$$

$$N_3 = -35P$$

$$\sum y(L) = 0 \qquad -15P + 5P + \frac{1}{\sqrt{5}} N_2 - \frac{1}{\sqrt{5}} N_5 = 0$$

$$-10P + \frac{1}{\sqrt{5}} \cdot (-5\sqrt{5}P) = \frac{1}{\sqrt{5}} N_5$$

$$N_5 = -15\sqrt{5}P = -33,541P$$
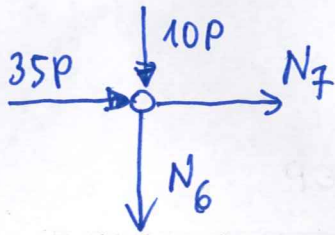
$$\sum x(L) = 0 \qquad -15P + \frac{2}{\sqrt{5}} \cdot N_5 + \frac{2}{\sqrt{5}} \cdot N_2 + N_3 + N_4 = 0$$

$$-15P + \frac{2}{\sqrt{5}} \cdot (-15\sqrt{5}P) + \frac{2}{\sqrt{5}} \cdot (-5\sqrt{5}) - 35P + N_4 = 0$$

$$N_4 = 90P$$

3.3. Równowaga węzła ③
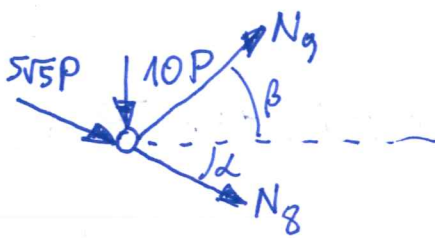

$$\sum y = 0$$
$$10P + N_6 = 0$$
$$N_6 = -10P$$

$$\sum x = 0$$
$$35P + N_7 = 0$$
$$G = N_7 = -35P$$

3.4. Równowaga węzła ④

$$\cos\beta = \frac{2}{\sqrt{13}} \quad , \quad \sin\beta = \frac{3}{\sqrt{13}}$$

$$\sum x = 0$$

$$5\sqrt{5}P \cdot \frac{2}{\sqrt{5}} + \frac{2}{\sqrt{5}}N_8 + \frac{2}{\sqrt{13}}N_9 = 0$$

$$\sum y = 0$$

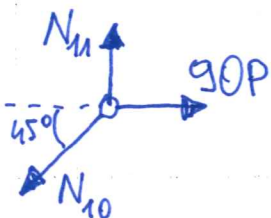$$-\frac{3}{\sqrt{13}}N_9 + \frac{1}{\sqrt{5}}N_8 + 10P + \frac{1}{\sqrt{5}} \cdot 5\sqrt{5}P$$

$$N_9 = \frac{\sqrt{13}}{3\sqrt{5}}N_8 + \frac{15\sqrt{13}}{3}P$$

$$0 = 10P + \frac{2}{\sqrt{5}}N_8 + \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{3\sqrt{5}}N_8 + \frac{2}{\sqrt{13}} \cdot \frac{15\sqrt{13}}{3}P$$

$$1,1926 N_8 = -20P$$

$$N = N_8 = -16,77P$$
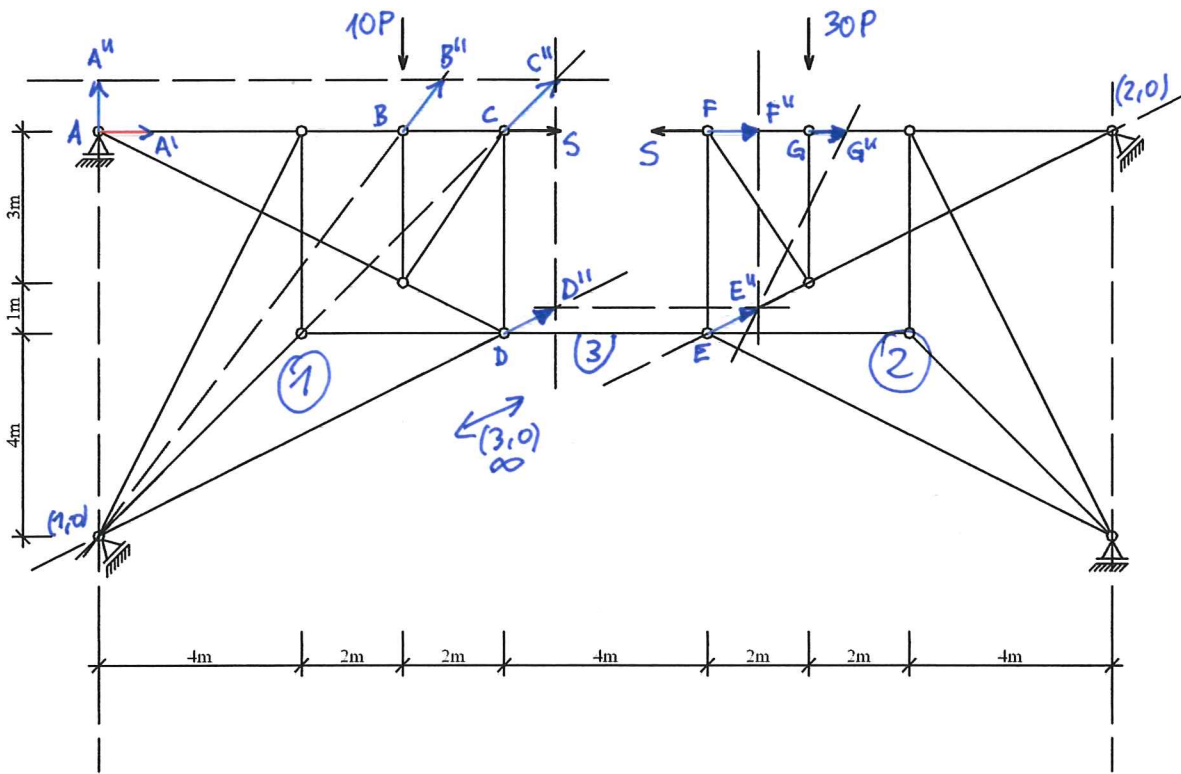
3.5. Równowaga węzła ⑤

$$\sum x = 0$$

$$-\frac{\sqrt{2}}{2} \cdot N_{10} + 90P = 0$$

$$K = N_{10} = +127,279P$$

Mechanism „S"

$|AA''| = 1 -$ prou medone

$|BB''|_x = \dfrac{3}{4}$

$|CC''|_y = 1$, $|CC''|_x = 1$

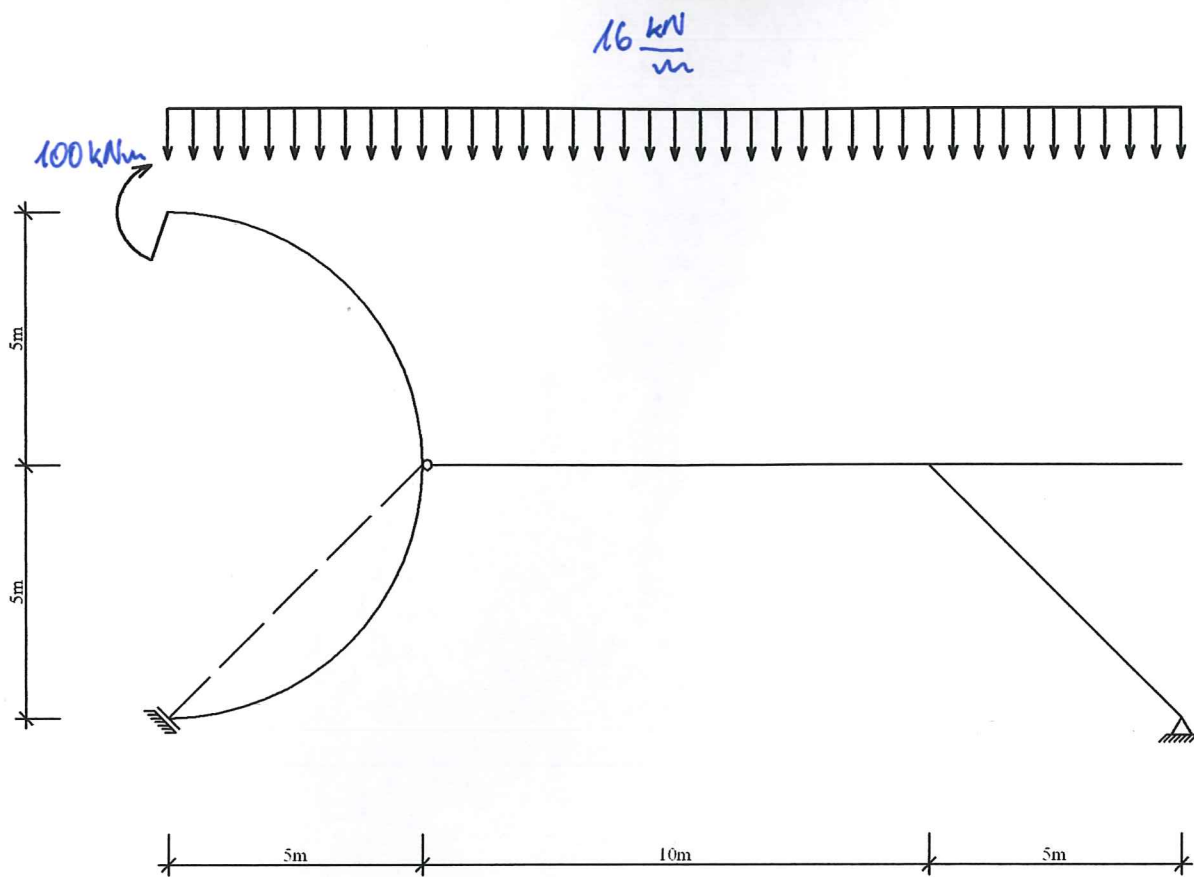$|DD''|_x = 1$, $|DD''|_y = \dfrac{1}{2}$

$|EE''|_x = 1$, $|EE''|_y = \dfrac{1}{2}$

$|FF''| = 1$, $|GG''| = \dfrac{3}{4}$

$L = -10P \cdot |BB''|_x - S \cdot |CC''|_y - 30P \cdot |GG''| = 0$

$\qquad -10P \cdot \dfrac{3}{4} - S - 30P \cdot \dfrac{3}{4} = 0$

$\qquad\qquad S = -30P$

$16 \frac{kN}{m}$

$100 \, kNm$

5m

5m

5m

10m

5m

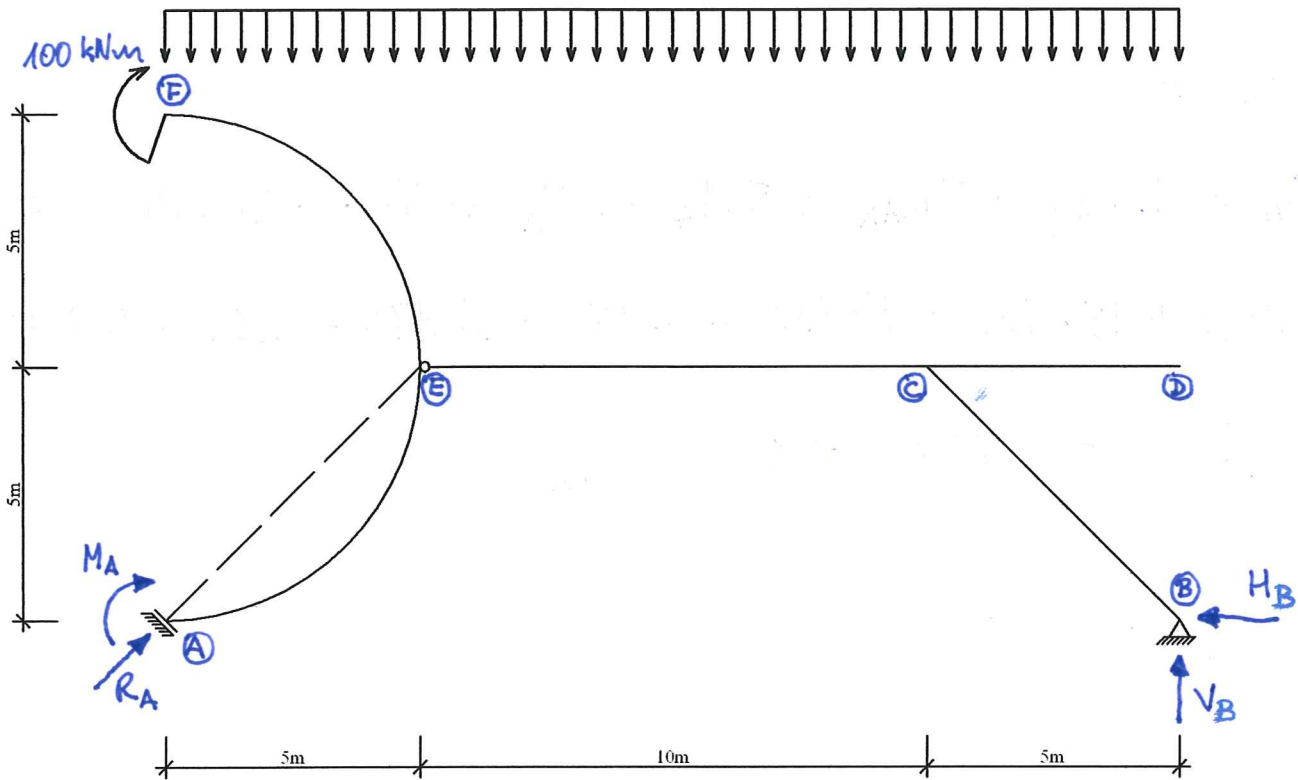1 Sprawdzenie GN i SW

* warunek ilościowy

$$n = e - 3t$$
$$n = 6 - 3 \cdot 2 = 0$$

* warunek jakościowy

Na mocy twierdzenia o 3 tarczach uzasadnia
się geometryczną niezmienność układu.

Rama jest GN i SW ($n = 0$).

2. Reakcje

Warunek konstrukcyjny : $\sum M_E^{(L)} = 0$

$$M_A + 100 - 16 \cdot 5 \cdot 2{,}5 = 0$$

$$M_A = 100 \ kNm$$

$$\sum M_A = 0 \rightarrow -20 V_B + 20 \cdot 16 \cdot 10 + 100 + 100 = 0$$

$$V_B = 170 \ kN$$

$$\sum y = 0 \rightarrow -\frac{\sqrt{2}}{2} R_A + 16 \cdot 20 - V_B = 0$$
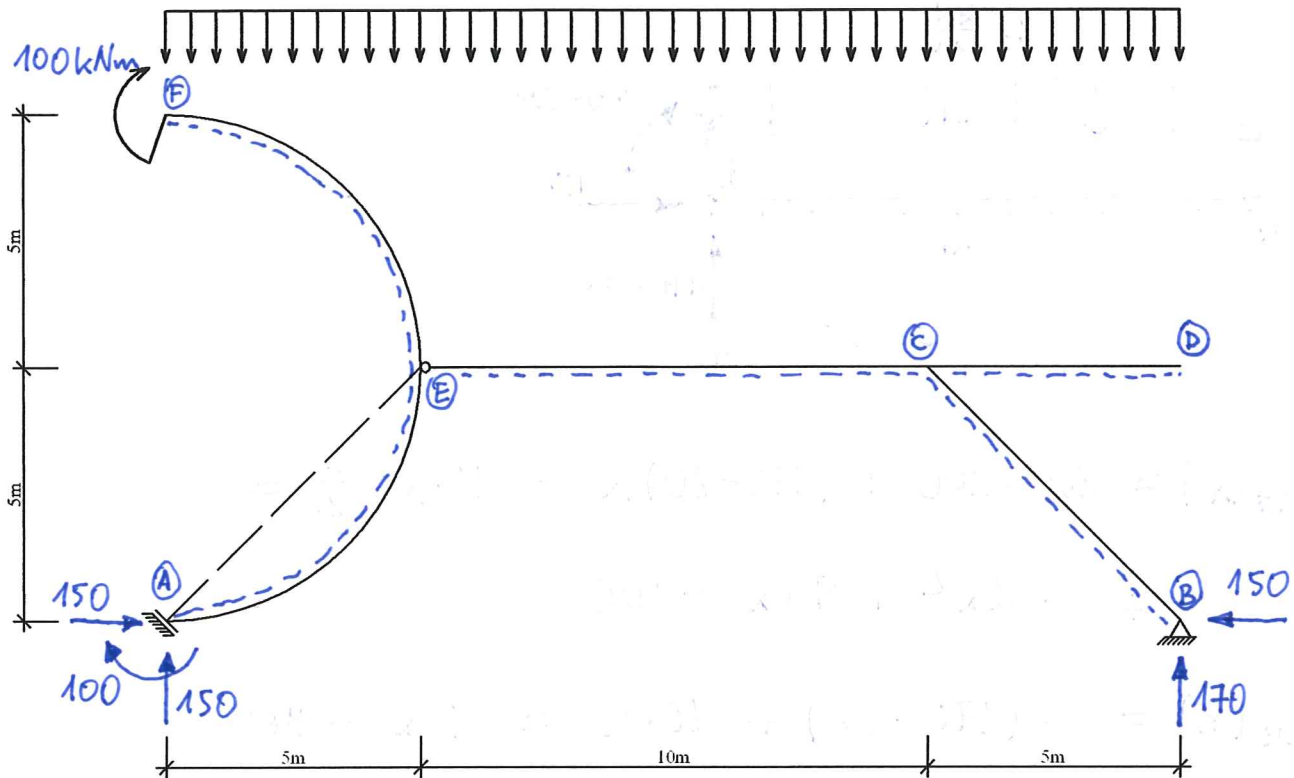
$$R_A = 212{,}132 \ kN$$

$$R_{AX} = R_{Ay} = 150 \ kN$$

$$\sum x = 0 \rightarrow R_{AX} - H_B = 0 \rightarrow H_B = 150 \ kN$$

Sprawdzenie:

$$\sum M_c = 0$$

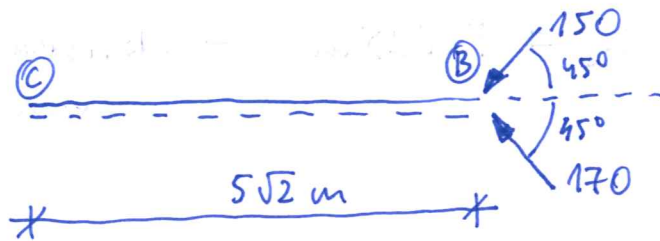$$M_A + 15 R_{Ay} - 5 R_{Ax} + 5 H_A - 5 V_A - 5 \cdot 20 \cdot 16 + 100 =$$

$$= 100 + 15 \cdot 150 - 5 \cdot 150 + 5 \cdot 150 - 5 \cdot 170 - 1600 + 100 =$$

$$= 3200 - 3200 = 0 \qquad ok$$

3. Siły przekrojowe
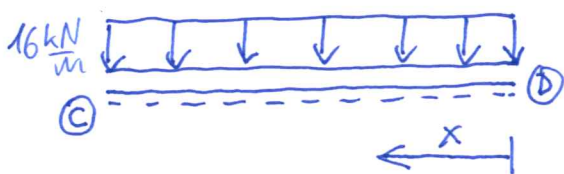
3.1. Przedział B-C

$$M_{BC} = 0$$

$$M_{CB} = \frac{\sqrt{2}}{2} \cdot 170 \cdot 5\sqrt{2} -$$

$$- \frac{\sqrt{2}}{2} \cdot 150 \cdot 5\sqrt{2} =$$

$$= 100 \text{ kNm}$$

$$T_{BC} = T_{CB} = \frac{\sqrt{2}}{2} \cdot 150 - \frac{\sqrt{2}}{2} \cdot 170 = -14,142 \text{ kN}$$

$$N_{BC} = N_{CB} = -\frac{\sqrt{2}}{2} \cdot 150 - \frac{\sqrt{2}}{2} \cdot 170 = -226,274 \text{ kN}$$

3.2. Przedział D-C   ($0 \leq x \leq 5m$)

$$M_{DC}(x) = -16 \cdot x \cdot \frac{x}{2} = -8x^2$$
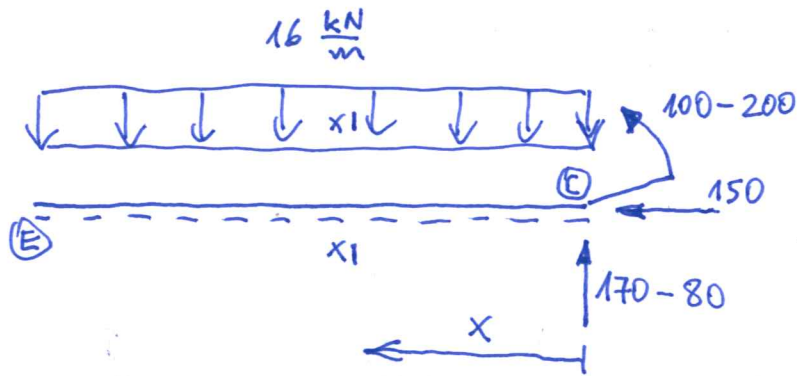
$$T_{DC}(x) = 16 \cdot x$$

$$N_{DC}(x) \equiv 0$$

$$M_{DC}(0) = 0 \ , \ M_{DC}(2,5m) = -50 \text{ kNm} \ , \ M_{DC}(5m) = -200 \text{ kNm}$$

$$T_{DC}(0) = 0 \ , \ T_{DC}(2,5m) = 40 \text{ kN} \ , \ T_{DC}(5m) = 80 \text{ kN}$$

3.3. Przedział C-E ($0 \leq x \leq 10m$)

$$M_{CE}(x) = 100 - 200 + (170 - 80)x - 16 \cdot x \cdot \frac{x}{2} =$$

$$= -8x^2 + 90x - 100$$

$$T_{CE}(x) = -(170 - 80) + 16 \cdot x = 16x - 90$$

$$N_{CE}(x) = -150$$

$$T_{CE}(x_o) = 0 \quad \Longleftrightarrow \quad 16x - 90 = 0$$

$$x_o = 5,625 \, m \quad - \text{ ekstremum}$$
$$\text{momentu}$$

$$M_{CE}(0) = -100 \, kNm$$

$$M_{CE}(5,625m) = M_{CE}^{max} = 153,125 \, kNm$$

$$M_{CE}(10m) = 0 \quad - \text{ w przegubie}$$

$$T_{CE}(0) = -90 \, kN$$
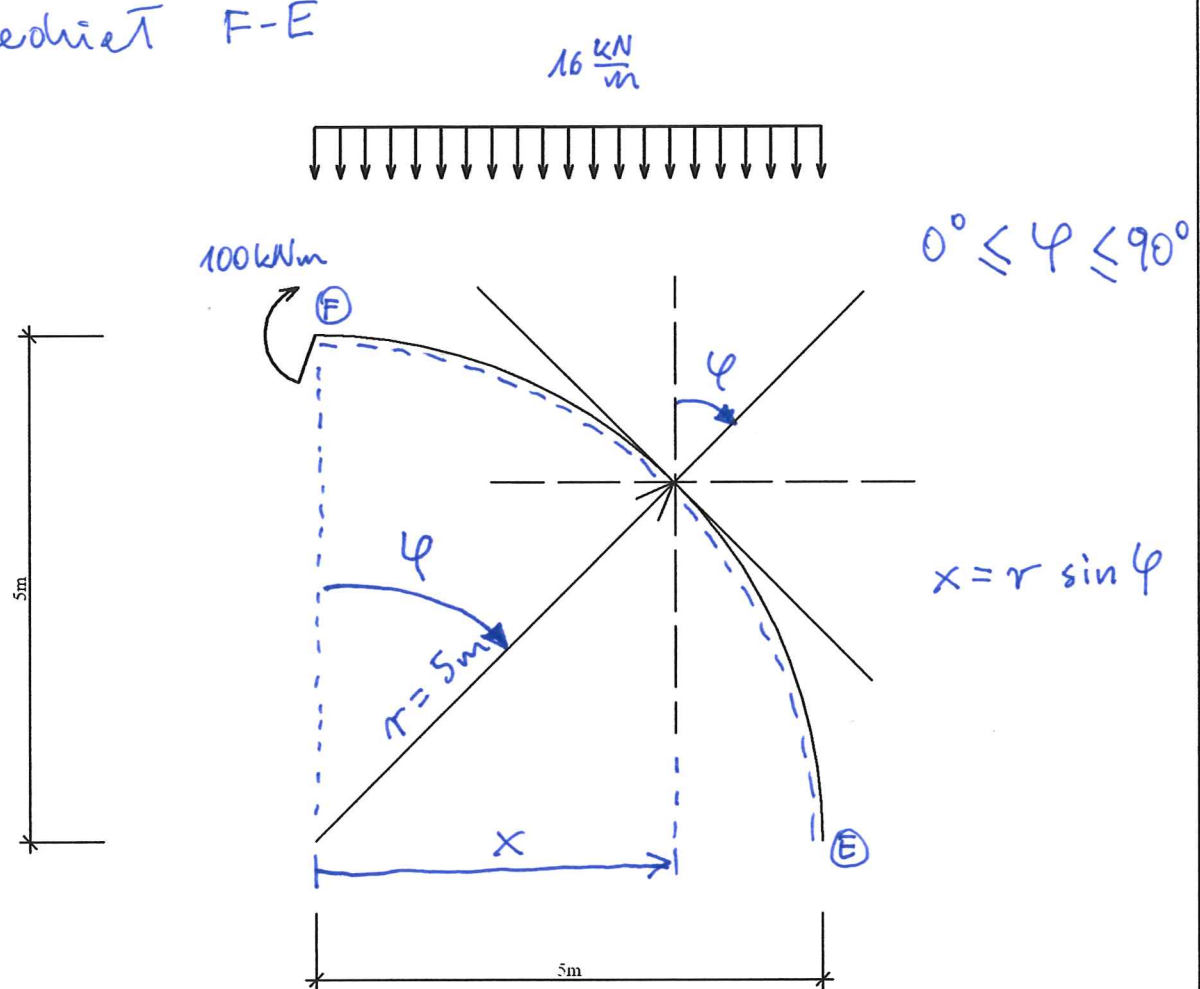
$$T_{CE}(10m) = 70 \, kN$$

$$N_{CE}(0) = -150 \, kN$$

$$N_{CE}(10m) = -150 \, kN$$

3.4. Priedmet F-E

$$M_{FE}(\varphi) = 100 - 16 \cdot x \cdot \frac{x}{2} = 100 - 8 \cdot 5^2 \cdot \sin^2\varphi =$$

$$= -200 \sin^2\varphi + 100$$

$$T_{FE}(\varphi) = -16 \cdot x \cdot \cos\varphi = -16 \cdot 5 \cdot \sin\varphi \cdot \cos\varphi =$$

$$= -80 \sin\varphi \cdot \cos\varphi$$

$$N_{FE}(\varphi) = -16 \cdot x \cdot \sin\varphi = -16 \cdot 5 \cdot \sin\varphi \cdot \sin\varphi =$$

$$= -80 \sin^2\varphi$$

φ [deg]	$M(\varphi)$ [kNm]	$T(\varphi)$ [kN]	$N(\varphi)$ [kN]
0	100,000	0,000	0,000
15	86,603	-20,000	-5,359
30	50,000	-34,641	-20,000
45	0,000	-40,000	-40,000
60	-50,000	-34,641	-60,000
75	-86,603	-20,000	-74,641
90	-100,000	0,000	-80,000

3.5. Predict A–E

$0° \leq \varphi \leq 90°$

$$y = r - r \cos\varphi$$
$$x = r \sin\varphi$$
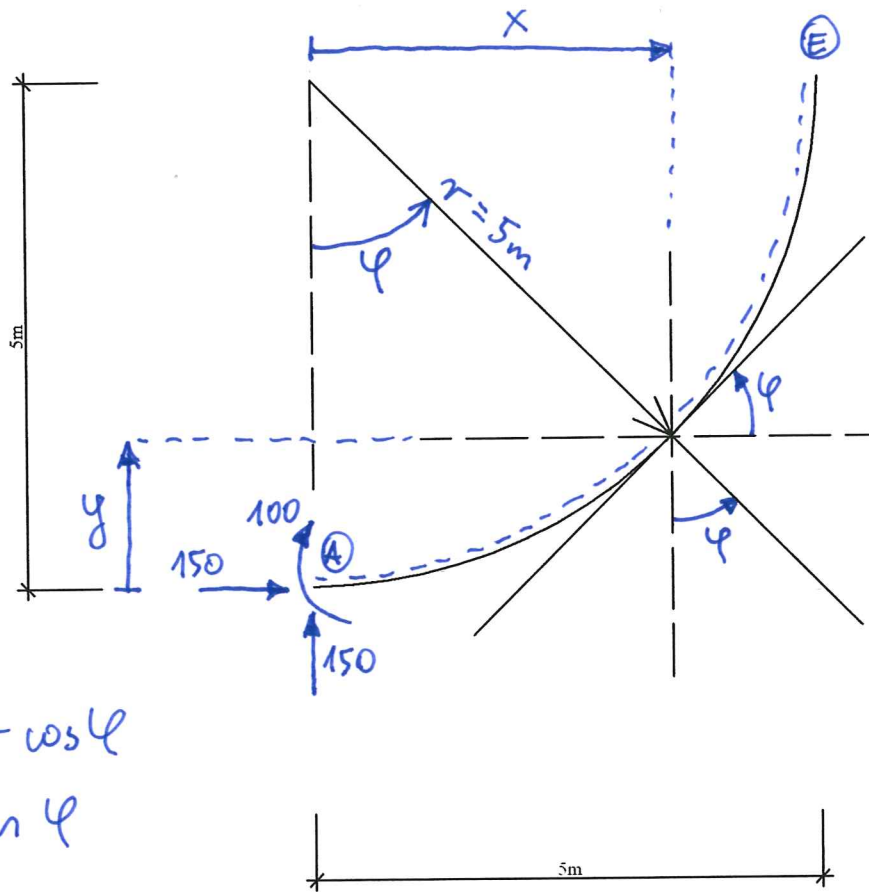
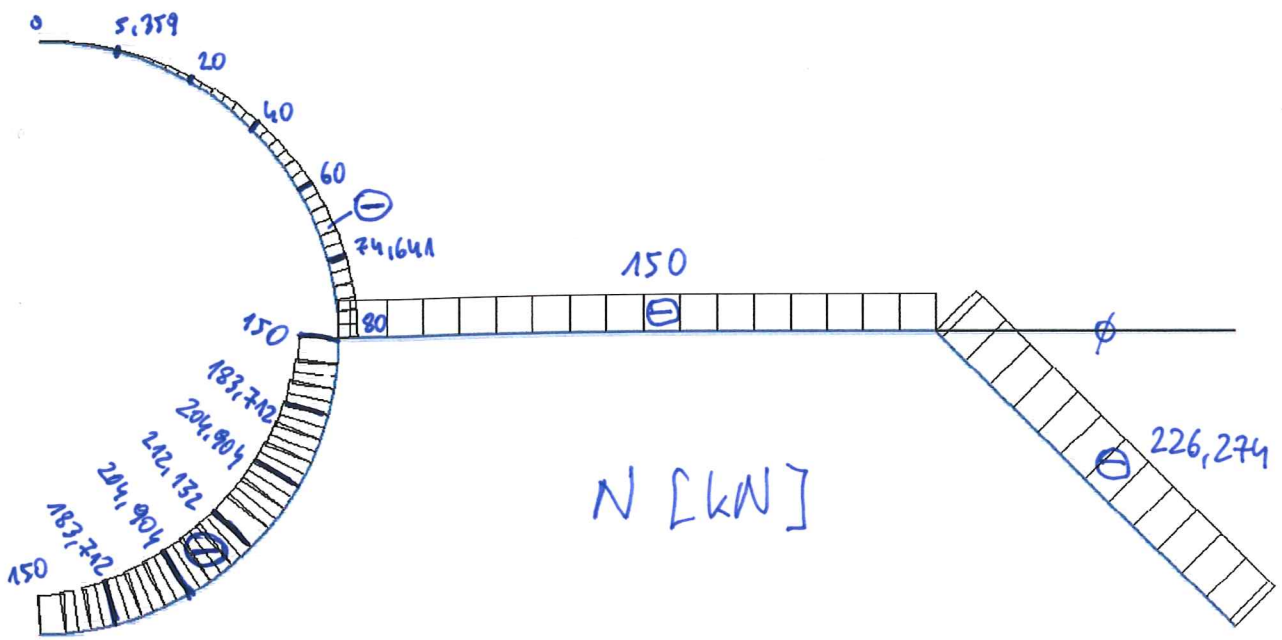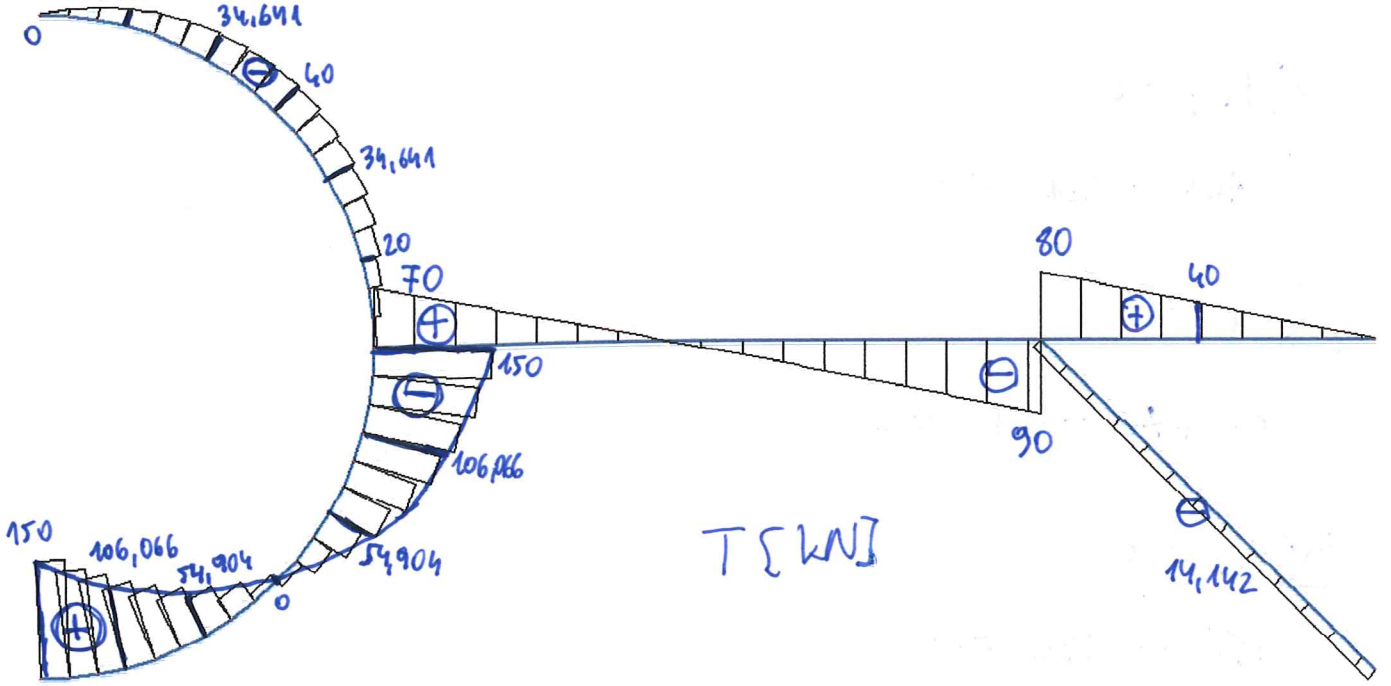$$M_{AE}(\varphi) = -100 - 150x + 150y =$$
$$= -100 - 150 \cdot 5 \cdot \sin\varphi + 150(5 - 5\cos\varphi) =$$
$$= 650 - 750 \sin\varphi - 750 \cos\varphi$$

$$T_{AE}(\varphi) = 150 \cos\varphi - 150 \sin\varphi$$

$$N_{AE}(\varphi) = -150 \cos\varphi - 150 \sin\varphi$$

φ [deg]	$M(\varphi)$ [kNm]	$T(\varphi)$ [kN]	$N(\varphi)$ [kN]
0	-100,000	150,000	-150,000
15	-268,559	106,066	-183,712
30	-374,519	54,904	-204,904
45	-410,660	0,000	-212,132
60	-374,519	-54,904	-204,904
75	-268,559	-106,066	-183,712
90	-100,000	-150,000	-150,000

3.6. WYKRESY

M [kNm]

100
86,603
50
0
50
86,603
100
180
268,559
374,519
410,66
268,559 374,519
100
20
200
100
50
0
100
153,125

T [kN]

0
20
34,641
40
34,641
20
70
150
106,66
150
106,066
54,904
54,904
0
80
40
90
14,142

N [kN]

0
5,359
20
40
60
34,641
150
80
150
183,712
204,904
212,132
204,904
183,712
150
150
226,274
0
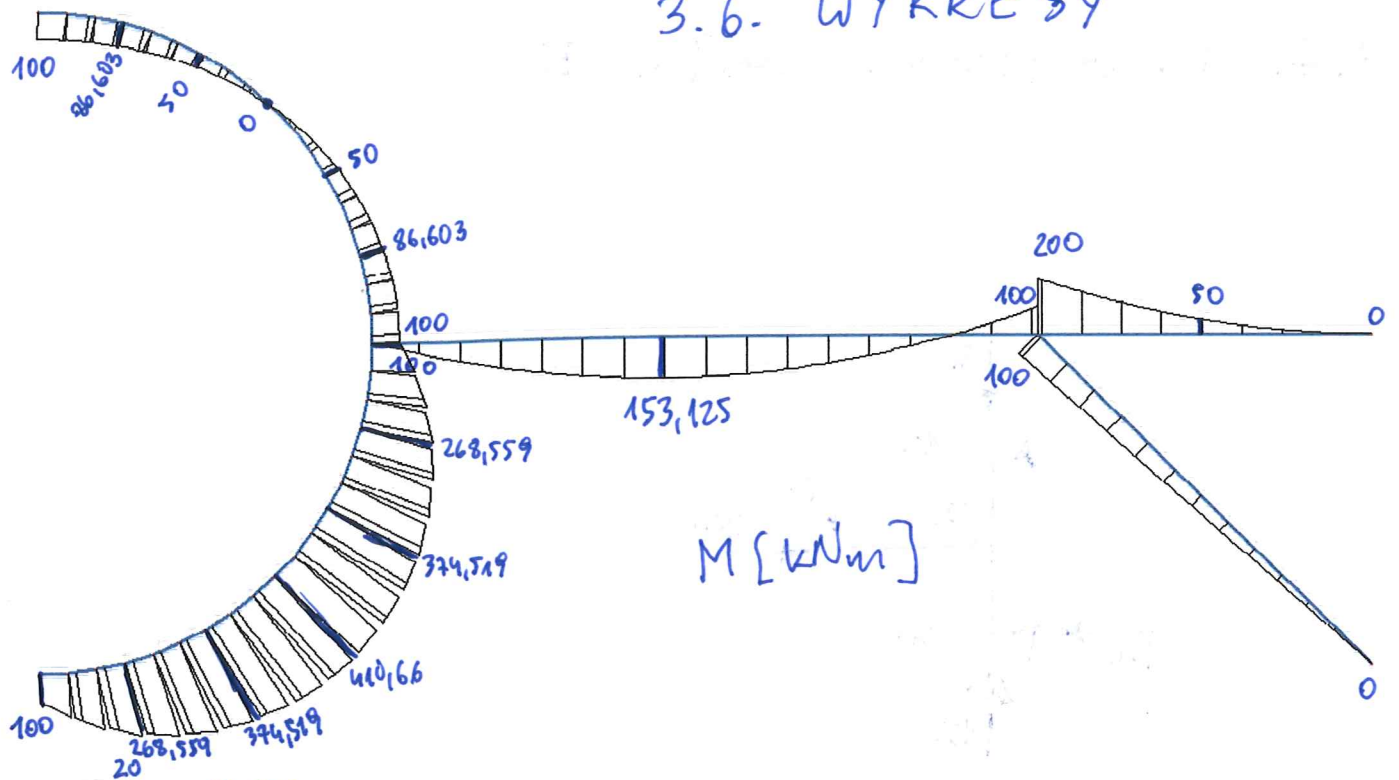
3.7. Równowaga węzła Ⓔ
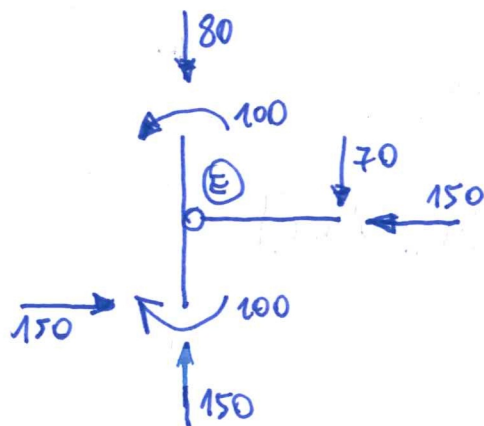
$$\sum M_E = 0$$

$$100 - 100 = 0$$

$$\sum y = 0$$

$$80 + 70 - 150 = 0$$

$$\sum x = 0$$

$$150 - 150 = 0$$

$16 \frac{kN}{m}$

$100\,kNm$

5m

5m

$\frac{M_\alpha}{10}$

M_α

$\frac{M_\alpha}{5}$

$\frac{M_\alpha}{10}$

$\frac{M_\alpha}{5}$

5m   10m   5m
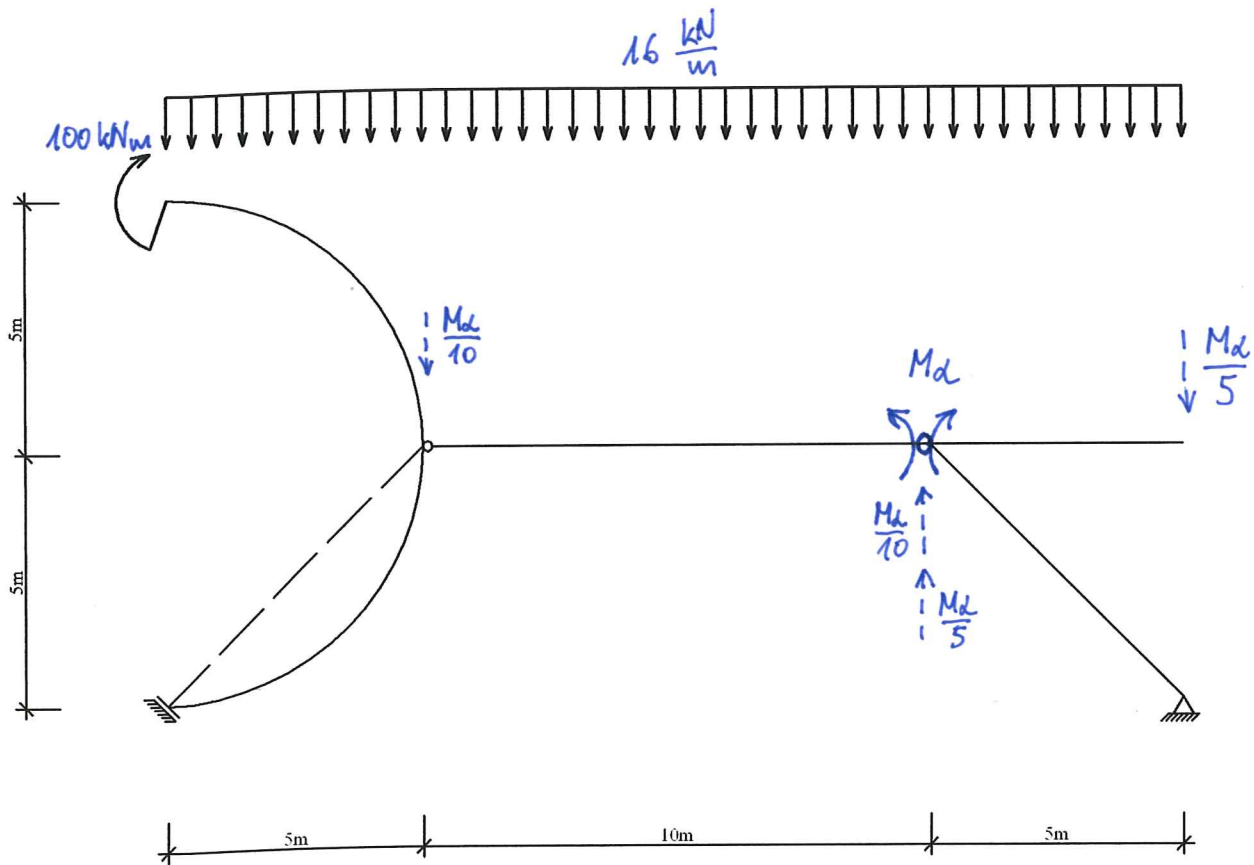
Mechanizm M_α

$$L = -16 \cdot 5 \cdot \frac{\sqrt{2}}{2} \cdot 1 - 16 \cdot \frac{10}{2} \cdot \frac{\sqrt{2}}{2} \cdot 1 + 16 \cdot \frac{10}{2} \cdot \frac{\sqrt{2}}{2} \cdot 1 + 16 \cdot \frac{5}{2} \cdot \frac{\sqrt{2}}{2} \cdot 1 - \frac{M_\alpha}{10} \cdot \frac{\sqrt{2}}{2} \cdot 1 -$$

$$- \frac{M_\alpha}{10} \cdot \frac{\sqrt{2}}{2} \cdot 1 - \frac{M_\alpha}{5} \cdot \frac{\sqrt{2}}{2} \cdot 1 = 0$$

$$M_\alpha = -100\,kNm$$

C''
C
C'
(2,0)
t1
B''
(1,2)
B
B'
t2
D'' D'
(2,3) D
E''
E
E'
(1,0)
∞
A''
A
A'
t3
(3,0)

$|AA'| = 1$ — nadane

5m

5m

5m   10m   5m

$16 \frac{kN}{m}$

$100\,kNm$

T_α

5m

5m

5m

10m

5m

MECHANIZM T_α

$|AA'| = 1$ — wadane
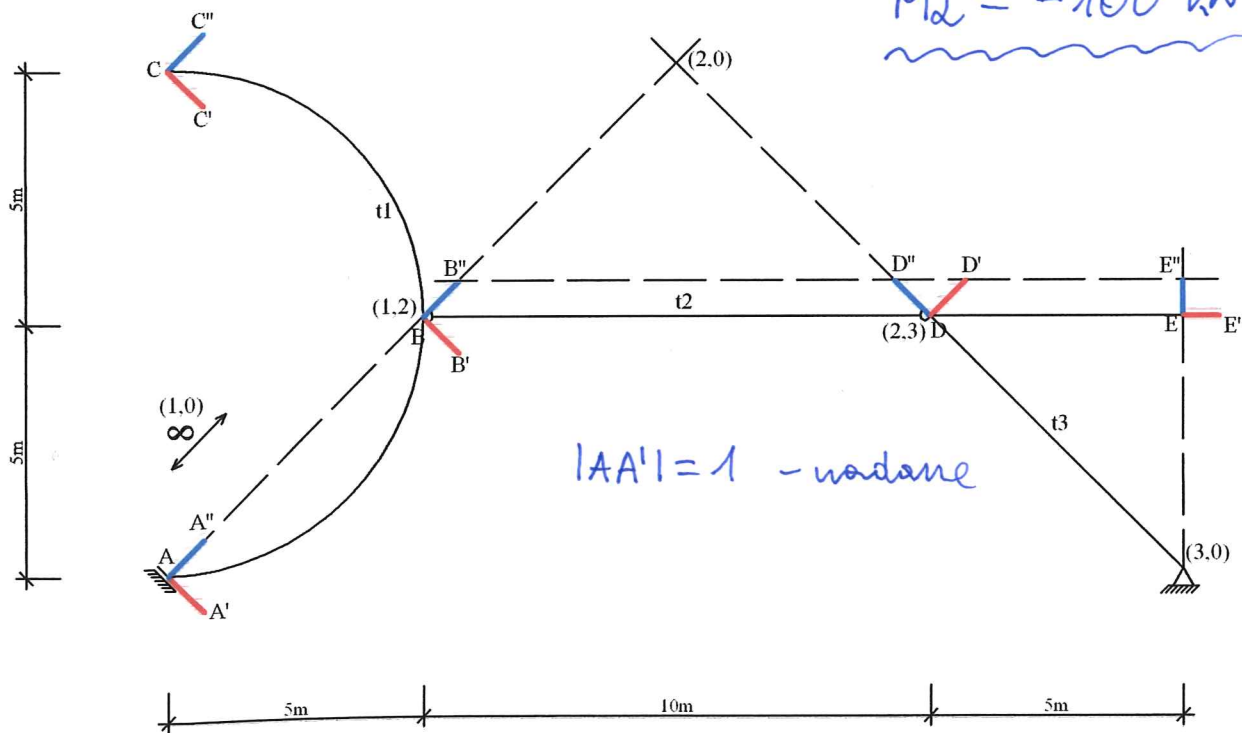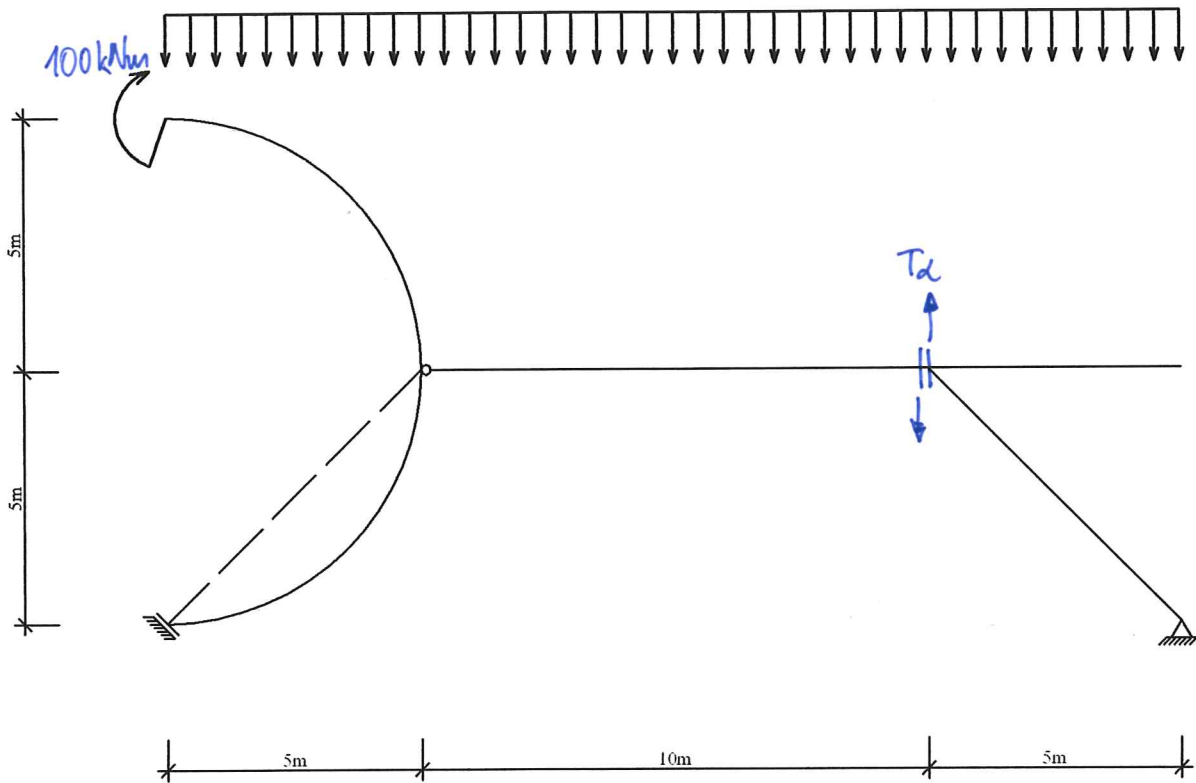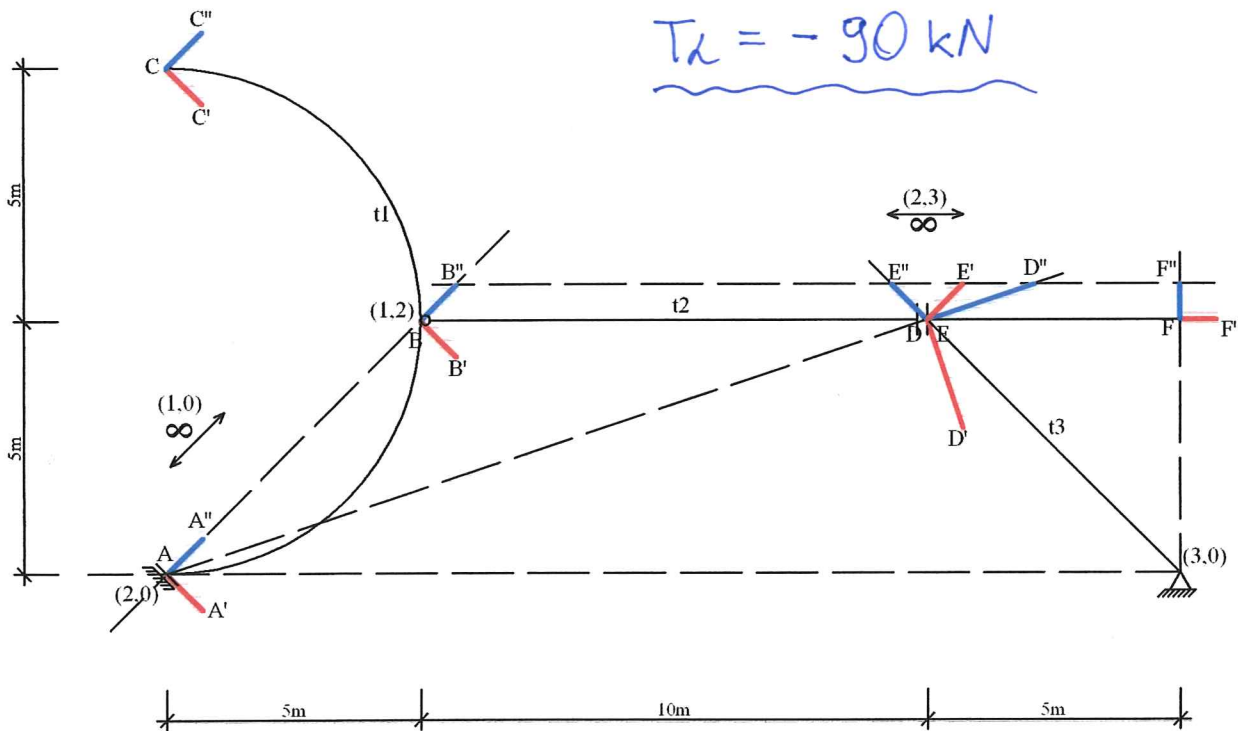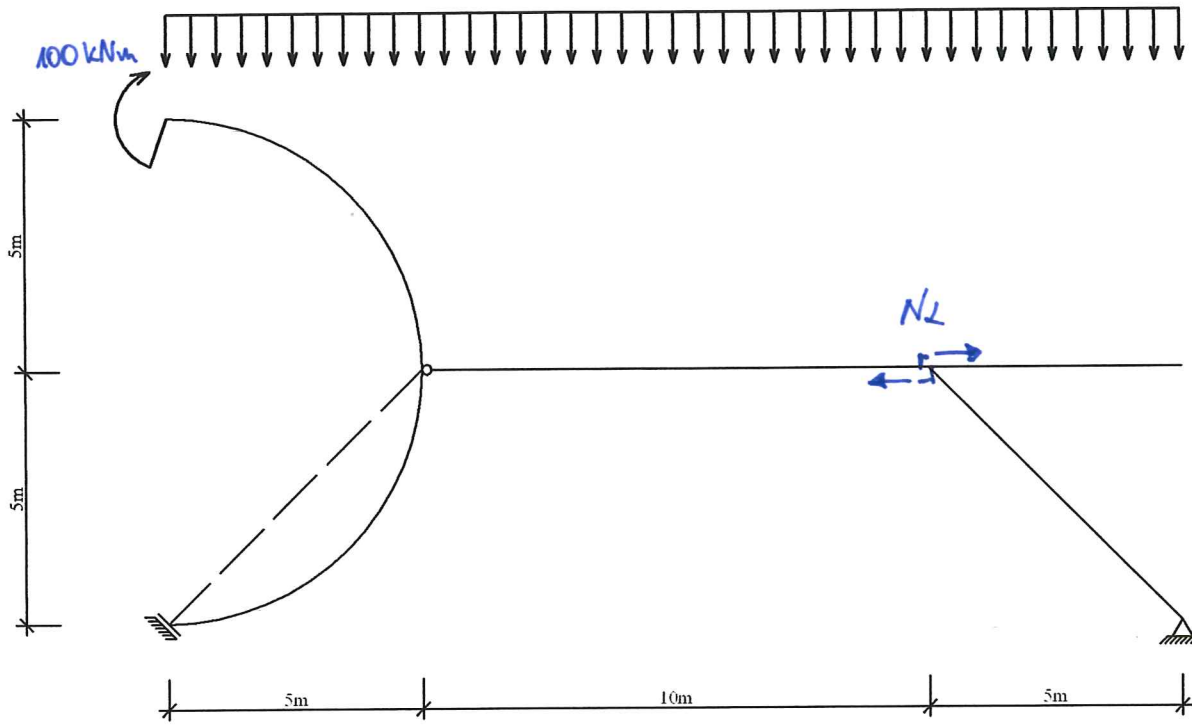
$$L = -16 \cdot 5 \cdot \frac{\sqrt{2}}{2} \cdot 1 - 16 \cdot \frac{10}{2} \cdot \frac{\sqrt{2}}{2} \cdot 1 - 16 \cdot \frac{10}{2} \cdot \frac{\sqrt{2}}{2} \cdot 3 + 16 \cdot \frac{5}{2} \cdot \frac{\sqrt{2}}{2} \cdot 1 - T_\alpha \cdot \frac{\sqrt{2}}{2} \cdot 3 - T_\alpha \cdot \frac{\sqrt{2}}{2} \cdot 1 = 0$$

$$T_\alpha = -90\,kN$$

C''
C
C'

t1

B''
(1,2)
B
B'

(2,3)
∞

E'' E' D''
D E
D'

F''
F F'

(1,0)
∞

t2

t3

A''
A
A'
(2,0)

(3,0)

5m

5m

5m

10m

5m

$16 \frac{kN}{m}$

$100\,kNm$

$N_\perp$

5m

5m

5m   10m   5m

MECHANIZM N_α

$|AA'| = 1$ medeme

$L = -16 \cdot 5 \cdot \frac{\sqrt{2}}{2} \cdot 1 - 16 \cdot 10 : 2 \cdot \frac{\sqrt{2}}{2} \cdot 1 - 16 \cdot 10 : 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{3} - 16 \cdot 5 : 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{3} - N_\alpha \cdot \frac{\sqrt{2}}{2} \cdot 1 - N_\alpha \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{3} = 0$

$N_\alpha = -150\ kN$

(2,0)

C''
C
C'

5m

t1

(2,3)
∞

B''
(1,2)
B
B'

t2

D''
D   E
E'   E''   D'
F' F
F''

(1,0)
∞

5m

A''
A
A'

t3

(3,0)

5m   10m   5m