Wrocław University of Technology

DYNAMICS LECTURE 1

Dr hab. inż. Zbigniew Wójcicki, prof. WUT

LECTURE 1

- Aims, scope and plan of the subject.
- Overview of the engineering problems in structural dynamics.
- Dynamic degrees of freedom and generalized coordinates.
- Continuous and discrete dynamic models of deformable bar structures.
- Examples of determining the number of dynamic degrees of freedom of discrete bar systems.
- Degree of static and geometric (kinematic) indeterminacy.
- Geometric indeterminacy in the dynamic sense.

Dynamic degrees of freedom and generalized coordinates

Degrees of Freedom

Degrees of freedom are the ways in which the space configuration of a mechanical system may change, i.e. the independent movements the system can possibly undergo.

Generalized coordinate

Generalized coordinates uniquely define any possible configuration of the system relative to the reference configuration. The generalized coordinates are chosen to be independent of one another.

Continuous and discrete dynamic models of deformable bar structures

- In reality, structures are not built of separate mass points, but consist of a continuous mass also called distributed mass.
- Such systems have an infinite number of degrees of freedom. However, it is virtually impossible to find dynamical solutions to any but the most simple of such systems.
- In general, it is necessary to discretize systems, i.e. replace infinite-number-of-degrees-of-freedom systems with simplified models - finite-number-of-degrees-of-freedom systems which are also called Multiple-Degree-of-Freedom (MDOF) systems.

Discrete dynamic models of structures (MDOF Systems)

- A model which contains a finite number of degrees of freedom is called a discrete model.
- Discretization concerns the process of transferring continuous models and equations into discrete ones.
- Discretization can be realized as a mathematical approximation or as a granulation of masses.
- Discrete models can consist of clearly distinguishable, separate masses, called lumped masses.
- Lumped mass models are created from continuous structures by replacing the distributed mass elements with a given number of lumped masses - the larger the number of masses, the better the approximation to the real structure.
- The masses (or lumped masses) may be interconnected by rigid elements - in such cases, the whole group acts as one rigid body possessed of both mass and moment of inertia.
- Mass points have translational degrees of freedom only, while the rigid bodies additionally have rotational degrees of freedom.
- The number of masses that may be used to represent a system is unlimited.

Number of Degrees of Freedom

- The number of degrees of freedom of a mechanical system is equal to the minimum number of independent coordinates required to define completely the position of all parts of the system (configuration of a mechanical system) at any instant in time.
- In general, it is equal to the number of possible independent displacements.

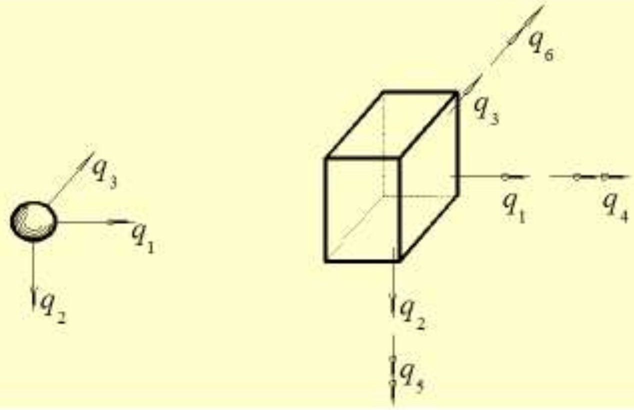
$$d = d_{\Delta} + d_{\varphi}$$

where

d_{Δ} - number of translational degrees of freedom

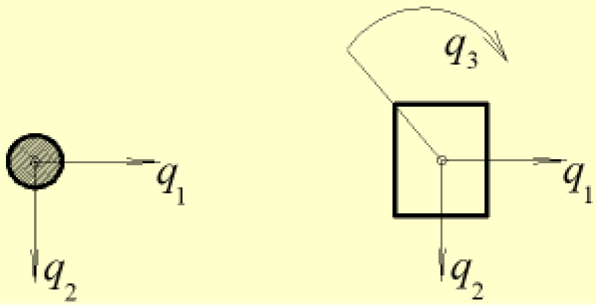
d_{φ} - number of rotational degrees of freedom

The numbers of degrees of freedom of a free (unconstrained) point and a free rigid body in space



Wroclaw University of Technology

The number of degrees of freedom of a free point and a free rigid body in a plane



Systems of Coordinates

In a dynamical analysis of MDOF systems three types of coordinates are used:

- **External Coordinates**
- **Local Coordinates**
- **Generalized Coordinates**

Wroclaw University of Technology

External Coordinates

The system of external coordinates is a fixed inertial set of reference axes (for instance the Cartesian coordinate system x_1, x_2, x_3 or x, y, z) useful for defining the configuration of a dynamic structure in a static equilibrium state.

Local Coordinates

- Local coordinates u_i are dependent on time.
- They describe the movement of system elements from the static equilibrium state.
- Usually, but not necessarily, they describe the possible displacements of elements.
- Local coordinates are associated with:
  - mass centers of masses and rigid bodies,
  - points in which the springs and dampers are connected to the structure,
  - points in which forces act on the structure and other points whose displacements are important for the dynamic description of the structure.
- Local coordinates may be of a translational or a rotational type.

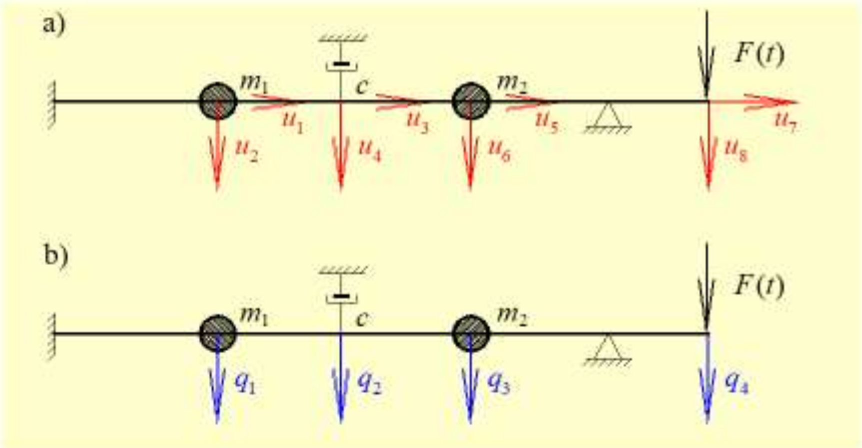
Generalized Coordinates

- Generalized coordinates (Lagrange's generalized coordinates ***q***) are dependent on time.
- They are a set of coordinates used to describe the configuration of a system relative to some reference configuration.
- The expression "generalized" is a remnant of a time when Cartesian coordinates were the standard.
- Generalized coordinates may be of a translational or a rotational type.
- A restriction for choosing a set of generalized coordinates is that they have to <u>unequivocally</u> define any possible configuration of the system relative to the reference configuration.
- That is to say, with the use of those coordinates it <u>must be possible to determine all local movements of all elements of the whole system</u>.
- The generalized coordinates are chosen to be <u>independent</u> of one another.

Number of Generalized Coordinates

- The number of independent generalized coordinates ***n*** is defined by the number of degrees of freedom of the system ***d***.
- Usually, generalized coordinates are related to the mass centers' position or rigid bodies mass centers' positions, but as a rule, they are related to points of connections of masses or mass rigid bodies to the structure.
- In general, these points do not have to be mass centers. The number of generalized coordinates is then equal to the number of dynamic degrees of freedom (minimal base ***n=d***).
- Nevertheless, there are some situations when it is more convenient to assume the number of generalized coordinates to be greater than the number of dynamic degrees of freedom ***n>d*** .
- These additional generalized coordinates are then usually related to the position of forces, springs or dampers, which are connected to the structure in points not related to the mass points. There may also be other reasons to assume ***n>d***.

Examples of determining the number of dynamic degrees of freedom of discrete bar systems



The additional generalized coordinates q_2 and q_4 not related to the mass

Generalized Coordinates

- Apart from practical reasons, all sets of generalized coordinates are equally good.
- The physics of the system are independent of the choice made between those sets.
- However, for practical reasons, some sets of coordinates are more useful than others – some are more optimally adapted to the system, and will make the solution of its equations of motion easier than others.

Degree of static and geometric (kinematic) indeterminacy

The degree of kinematic (geometric) indeterminacy is the number of kinematic constraints necessary to achieve the kinematic (geometrical) determinacy of the system. It is described by the formula

$$n_g = n_\Delta + n_\varphi$$

where

n_φ - the number of rotational constraints necessary to obtain geometric determinacy from the point of view of the Displacement Method

n_Δ - the number of translational constraints it is necessary to add to the kinematic chain in order to obtain a geometrically stable and statically determinate truss

$$n_\Delta = 2w - (p + r)$$

where

w - number of truss hinges in a kinematic chain

p - number of members in a kinematic chain

r - number of supporting constraints (links) in a kinematic chain

Degree of Kinematic (Geometric) Indeterminacy in a Dynamic Sense

The degree of kinematic (geometric) indeterminacy in a dynamic sense n_{gd} is defined as the number of degrees of kinematic indeterminacy n_g reduced by the number of degrees of freedom which are the dynamic generalized coordinate d.

Therefore, the number n_{gd} can be interpreted as the number of additional non-dynamic information, necessary only due to static solution of the structure in the Displacement Method sense.

$$n_{gd} = n_g - d$$

Degree of Static Indeterminacy

- The degree of static indeterminacy of a system (number of redundants, or number of hyperstatics) is $n_h = n_M - n_N$ where
  n_M - is the number of unknown member forces, and optionally, reactions in the system;
  n_N - is the number of independent, non-trivial equilibrium equations available for determining these n_M unknown forces.
- In practice, it is more convenient to determine the degree of static indeterminacy of a plane system by using the formula

$$n_h = e - 3t$$

or in space

$$n_h = e - 6b$$

where
e - is the number of constraints in the system,
t - is the number of plane rigid bodies in the 2D system,
b - is the number of rigid bodies in the 3D system