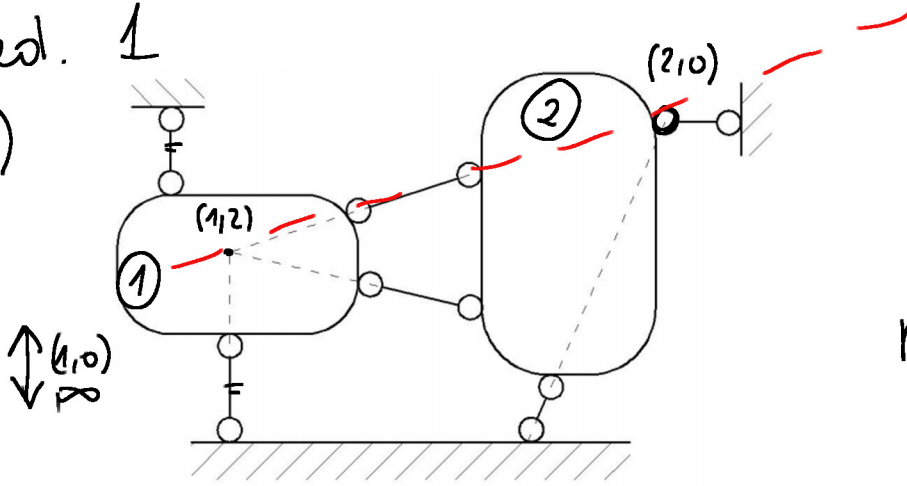


LISTA NR 4

Zad. 1

b)



* war. ilościowy

$$t = 2, e = 6$$

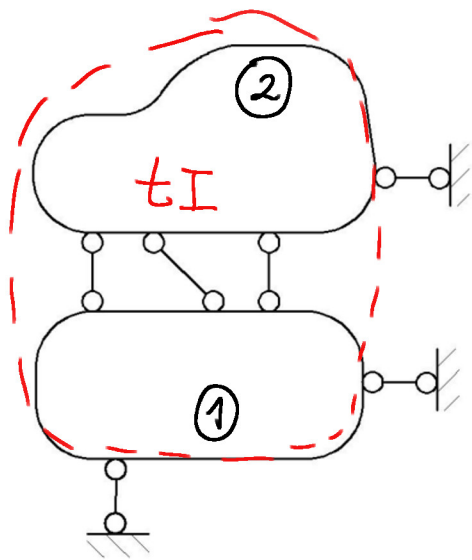
$$n = 6 - 3 \cdot 2 = 0$$

* war. jakościowy

Z tw. o 3. tarwał, układ jest GN

GN i SW

c)



$$t = 2, e = 6$$

$$n = 6 - 3 \cdot 2 = 0$$

* war. jakościowy

1) z tw. o 2. tarwał

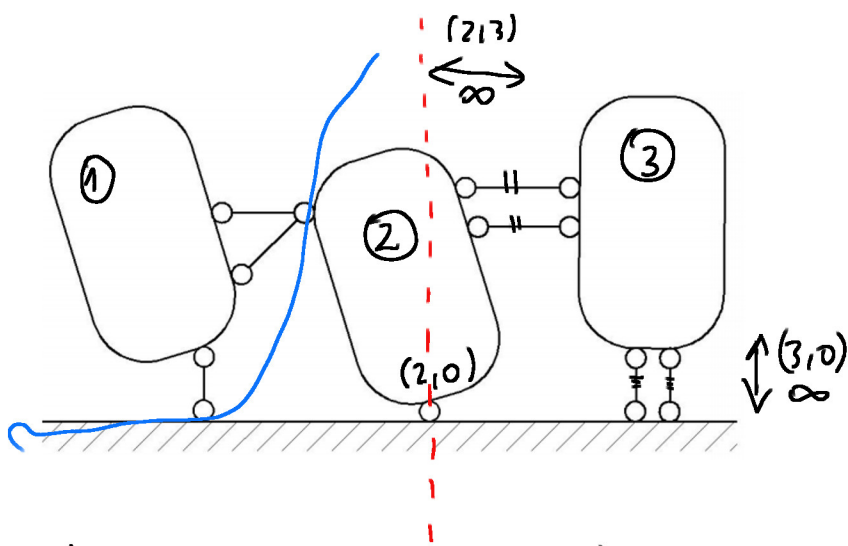
$t_1 - t_2$ tworzą
jedną tarwał tI

2) z tw. o 2. tarwał

$tI - t_0 \rightarrow GN$

GN i SW

d)



$$t = 3$$

$$e = 9$$

$$n = 9 - 3 \cdot 3 = 0$$

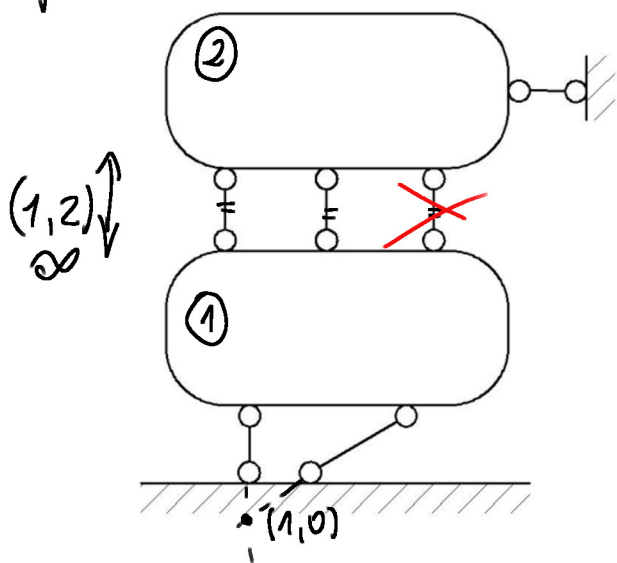
1) $t_2 - t_3 - t_0 \geq t_{\omega} = 3$ warunki \rightarrow GN

2) $t_1 - t_0 \geq t_{\omega} = 2$ warunki \rightarrow GN

GN i SW

f)

(2,0) ?



* war. ilościowy

$$e = 6, t = 2$$

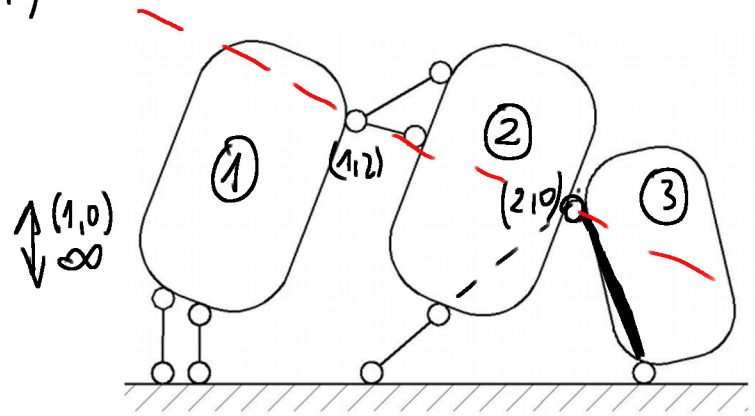
$$n = 6 - 3 \cdot 2 = 0$$

$n < 0$ spełniony

* war. jakościowy

GZ !

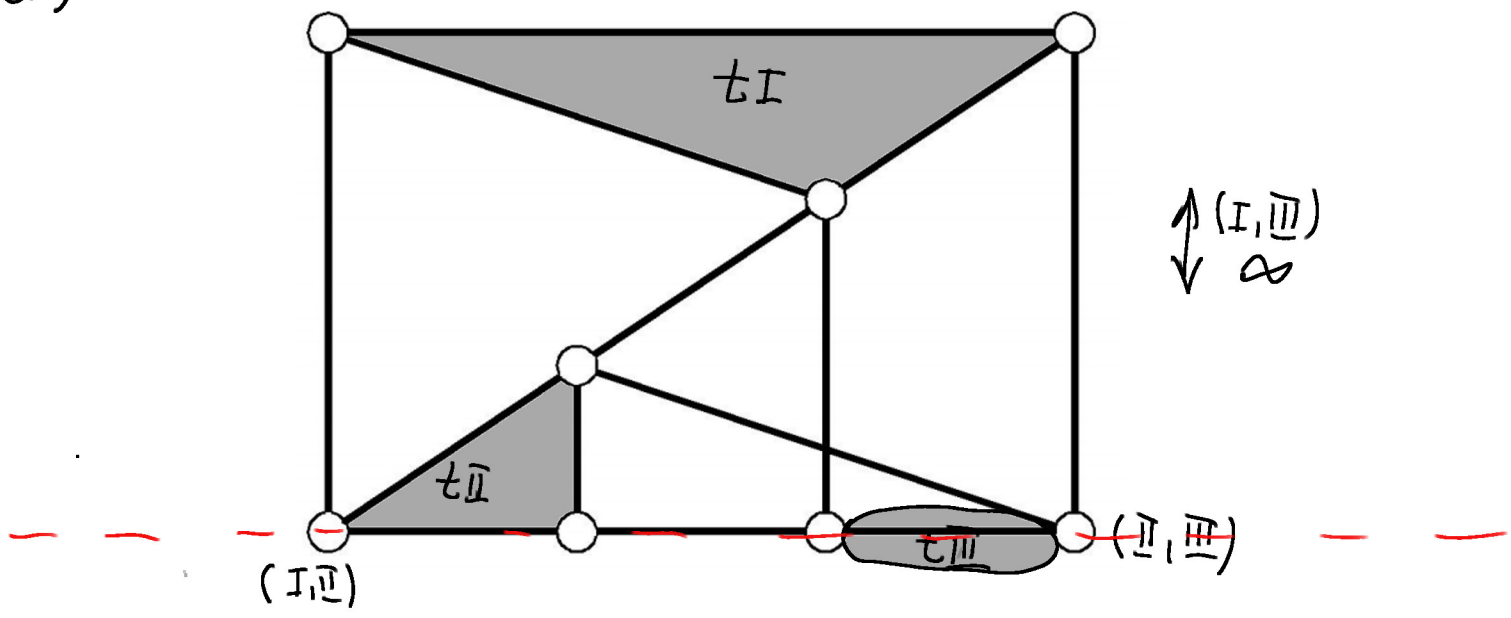
h)



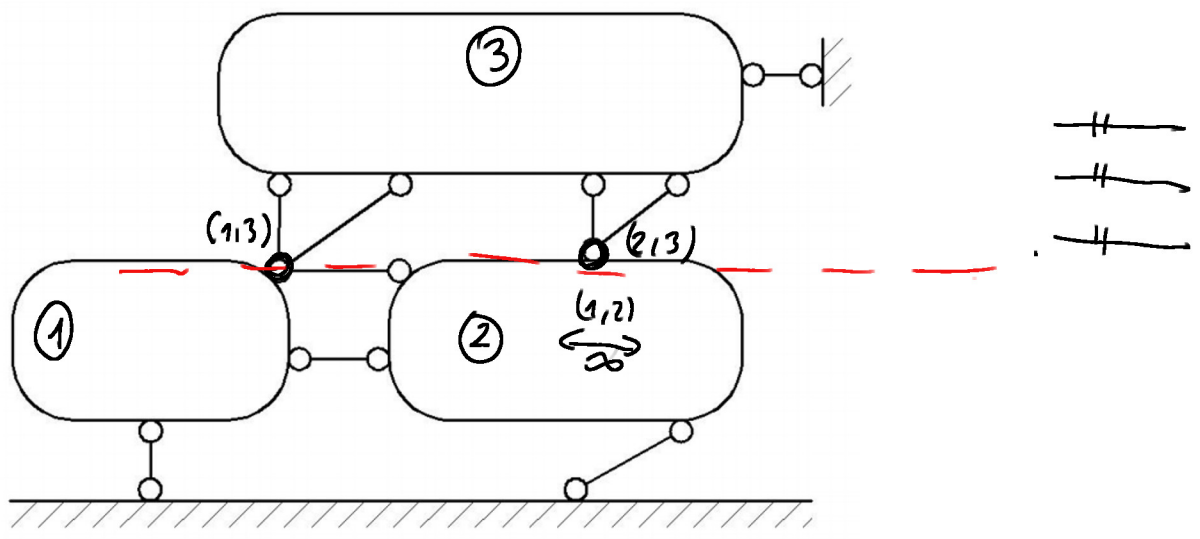
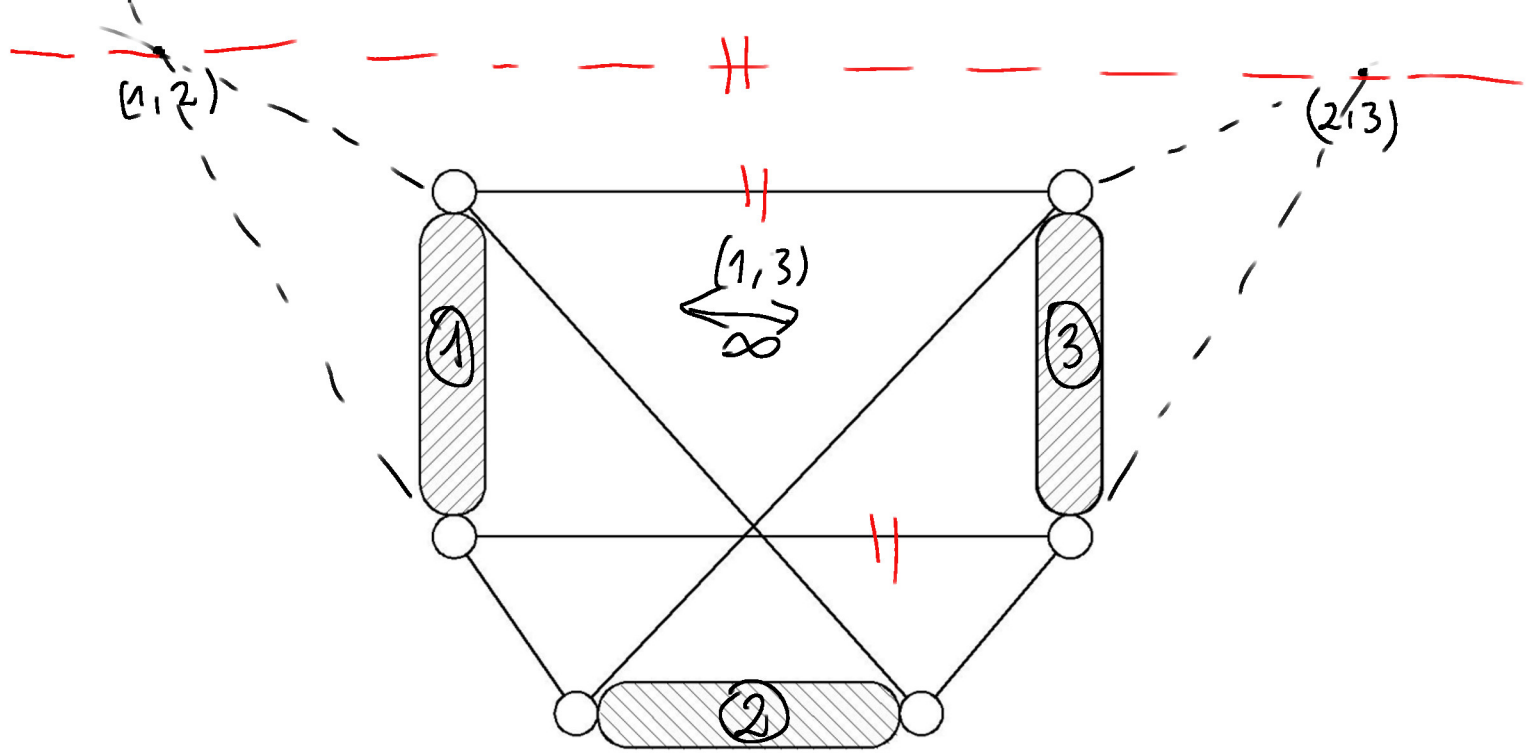
GN i' SW

Zad. 2.

a)



A well knowny whated tower we
 may to. 0 3. towerd towerd
 jeking towerd!



$$t = 3, \quad e = 9, \quad n = 9 - 3 \cdot 3 = 0$$

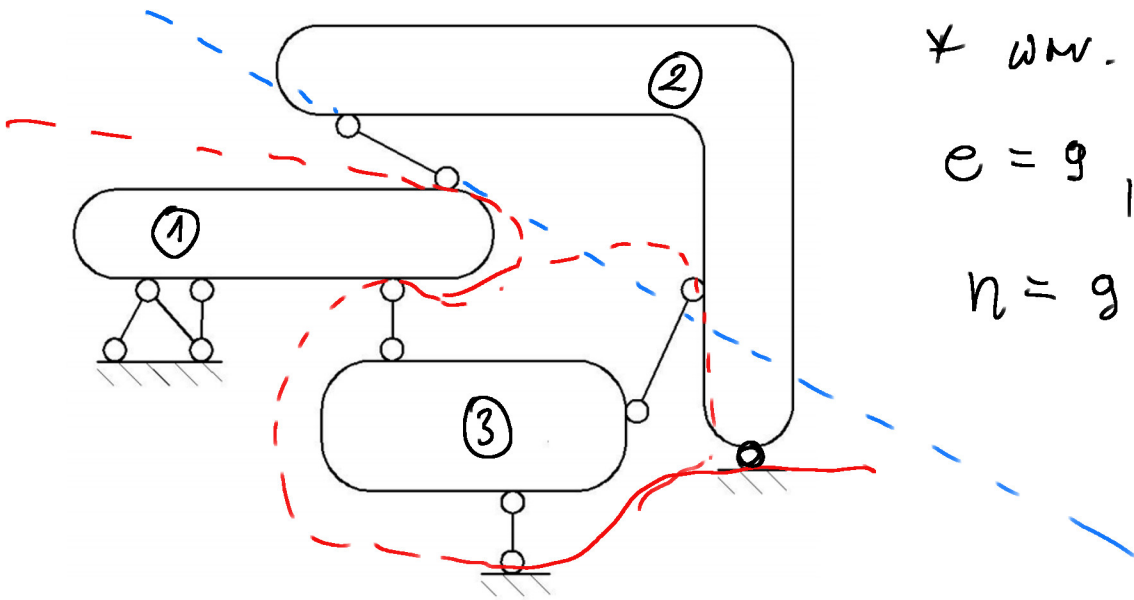
* war. jakościowy niespełniony

GZ

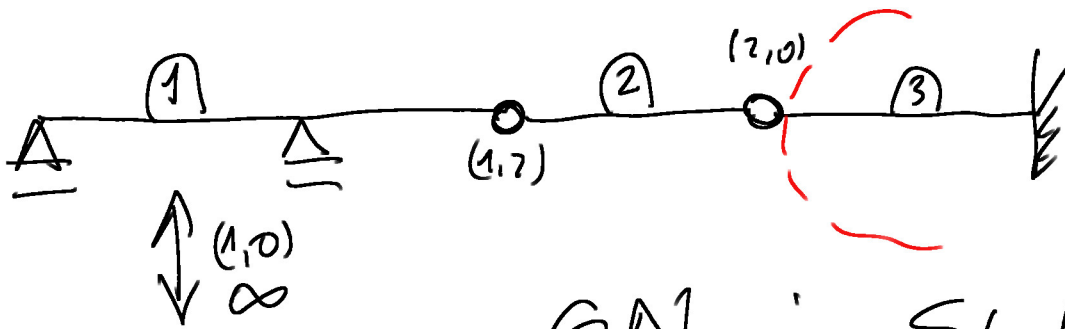
* wmv. ilos'ciomy

$$e = 9, \quad t = 3$$

$$n = 9 - 3 \cdot 3 = 0$$



GN i SW



GN i SW

Przykład. Układ prętlemy

Dane:

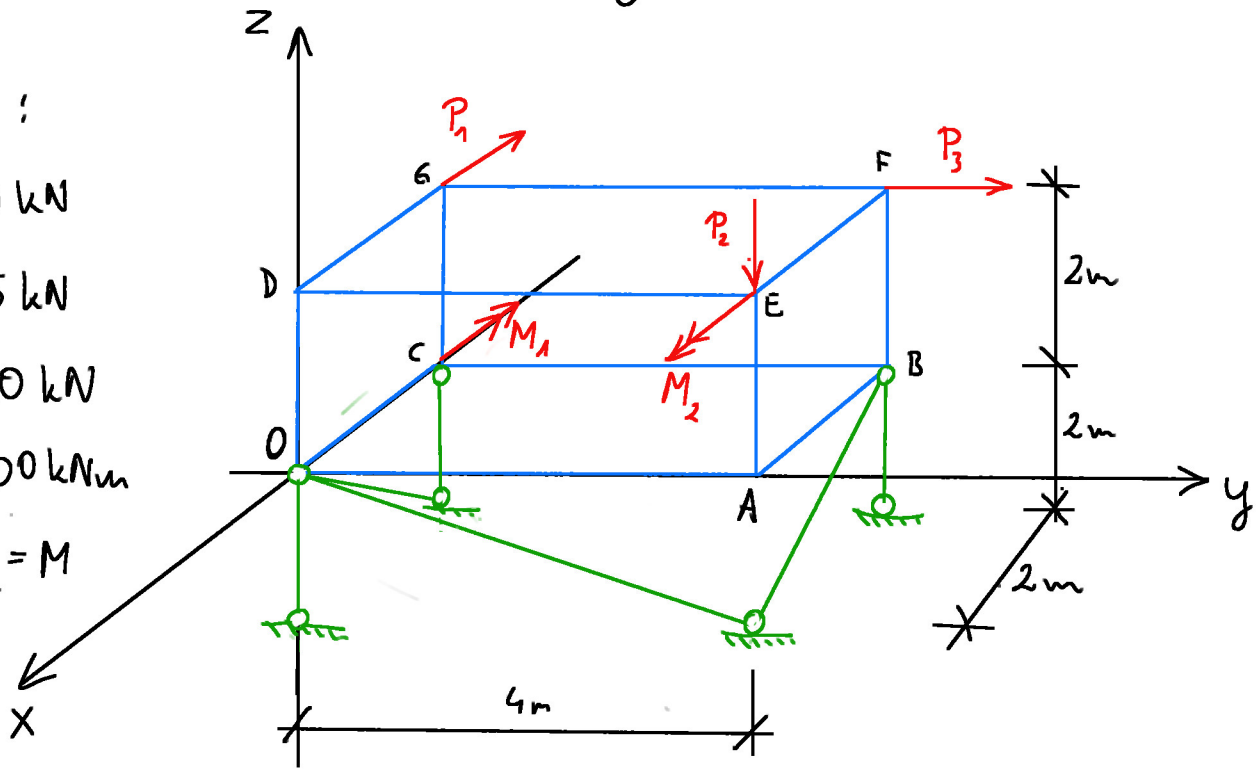
$$P_1 = 10 \text{ kN}$$

$$P_2 = 15 \text{ kN}$$

$$P_3 = 20 \text{ kN}$$

$$M_1 = 100 \text{ kNm}$$

$$M_1 = M_2 = M$$



1. Redukcja sił czynnych do biegunu O

$$\bar{S} = \sum_i \bar{P}_i, \quad S_x = \sum_i P_{ix}, \quad S_y = \sum_i P_{iy}, \quad S_z = \sum_i P_{iz}$$

$$\bar{P}_1 = -10i, \quad \bar{P}_2 = -15k, \quad \bar{P}_3 = +20j$$

$$\bar{M}_1 = -100i, \quad \bar{M}_2 = +100i$$

$$S_x = -10 \text{ kN}$$

$$S_y = 20 \text{ kN}$$

$$S_z = -15 \text{ kN}$$

$$\bar{S} = -10i + 20j - 15k \quad [\text{kN}]$$

$$\bar{M}_0 = \sum_i (\bar{r}_i \times \bar{P}_i + \bar{M}_i) = \begin{vmatrix} i & j & k \\ 0 & 0 & 2 \\ -10 & 0 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 4 & 0 \\ 0 & 0 & -15 \end{vmatrix} +$$

$$+ \begin{vmatrix} i & j & k \\ -2 & 0 & 2 \\ 0 & 20 & 0 \end{vmatrix} + (-100i) + 100i = -20j - 60i - 40k - 40i =$$

$$= -100i - 20j - 40k \quad [\text{kNm}]$$

$$|\vec{S}| = \sqrt{(-10)^2 + 20^2 + (-15)^2} = 26,926 \text{ kN}$$

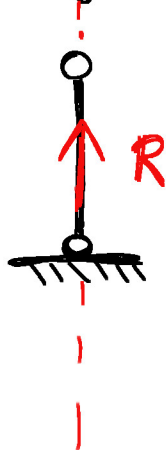
$$|\vec{M}_0| = \sqrt{(-100)^2 + (-20)^2 + (-40)^2} = 109,545 \text{ kNm}$$

2. Sprawdzić istnienie wypadkowej

$$w = \vec{S} \circ \vec{M}_0 = -10 \cdot (-100) + 20 \cdot (-20) + (-15) \cdot (-40) = 1200 \neq 0$$

Wypadkowa nie istnieje!

3. Wyznaczyć reakcje



$$\vec{R}_1 = R_1 \cdot \vec{k}$$

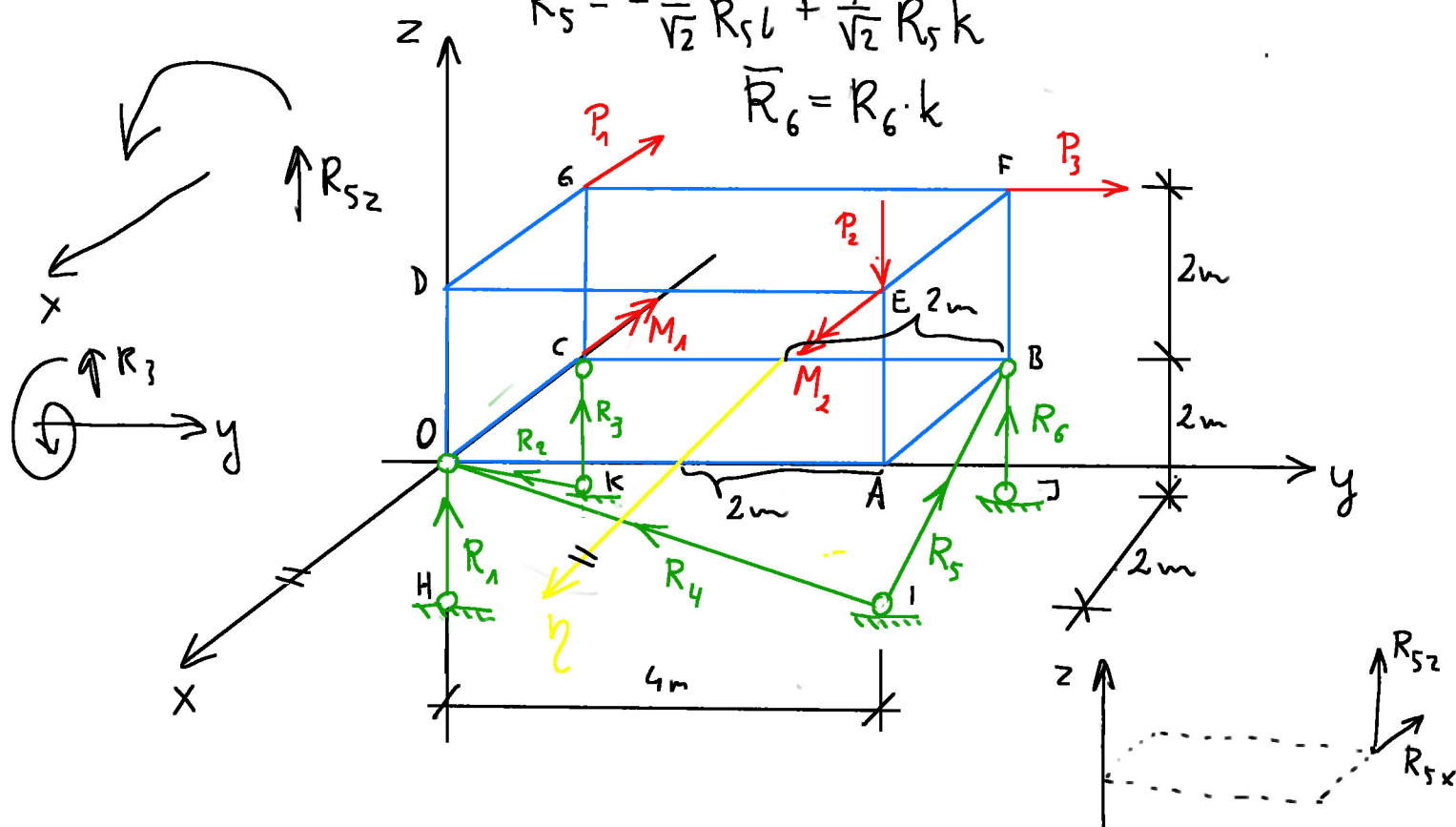
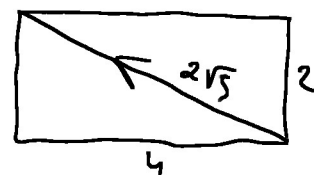
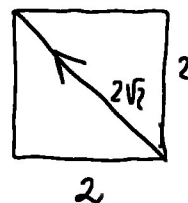
$$\vec{R}_2 = \frac{1}{\sqrt{2}} R_2 \vec{i} + \frac{1}{\sqrt{2}} R_2 \vec{k}$$

$$\vec{R}_3 = R_3 \cdot \vec{k}$$

$$\vec{R}_4 = -\frac{2}{\sqrt{5}} R_4 \vec{j} + \frac{1}{\sqrt{5}} R_4 \vec{k}$$

$$\vec{R}_5 = -\frac{1}{\sqrt{2}} R_5 \vec{i} + \frac{1}{\sqrt{2}} R_5 \vec{k}$$

$$\vec{R}_6 = R_6 \cdot \vec{k}$$



$$\bar{S}^P \neq 0, \quad \bar{S}^R$$

$$\bar{M}_0^P \neq 0, \quad \bar{M}_0^R$$

$$\bar{S} = 0 \rightarrow \bar{S}^P + \bar{S}^R = 0$$

$$\bar{M}_0 = 0 \rightarrow \bar{M}_0^P + \bar{M}_0^R = 0$$

6 równań równowagi

$$1) \sum x = 0, \quad S_x^P + S_x^R = 0$$

$$2) \sum y = 0, \quad S_y^P + S_y^R = 0$$

$$3) \sum z = 0, \quad S_z^P + S_z^R = 0$$

$$4) \sum M_{ix} = 0, \quad M_x^P + M_x^R = 0$$

$$5) \sum M_{iy} = 0, \quad M_y^P + M_y^R = 0$$

$$6) \sum M_{iz} = 0, \quad M_z^P + M_z^R = 0$$

$$1) -10 + \frac{1}{\sqrt{2}}R_2 - \frac{1}{\sqrt{2}}R_5 = 0$$

$$2) 20 - \frac{2}{\sqrt{3}}R_4 = 0$$

$$3) -15 + R_1 + \frac{1}{\sqrt{2}}R_2 + R_3 + \frac{1}{\sqrt{5}}R_4 + \frac{1}{\sqrt{2}}R_5 + R_6 = 0$$

$$4) -100 + 4 \cdot \frac{1}{\sqrt{2}}R_5 + 4 \cdot R_6 = 0$$

$$5) -20 + 2R_3 + 2 \cdot \frac{1}{\sqrt{2}}R_5 + 2 \cdot R_6 = 0$$

$$6) -40 + 4 \cdot \frac{1}{\sqrt{2}}R_5 = 0$$

$$\begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{5}} & 0 & 0 \\ 1 & \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{2}} & 1 \\ 0 & 0 & 0 & 0 & \frac{4}{\sqrt{2}} & 4 \\ 0 & 0 & 2 & 0 & \frac{2}{\sqrt{2}} & 2 \\ 0 & 0 & 0 & 0 & \frac{4}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \end{bmatrix} = \begin{bmatrix} 10 \\ -20 \\ 15 \\ 100 \\ 20 \\ 40 \end{bmatrix}$$

A

\bar{R}

\bar{P}

$$A \cdot \bar{R} = \bar{P}$$

$$\bar{R} = A^{-1} \cdot \bar{P} \iff$$

$$\boxed{\det A \neq 0}$$

\Uparrow

WARUNEK
GEOMETRYCZNEJ
NIEZMIENNOŚCI
UKŁADU

Tutaj $A \neq 0$ zatem

$$R_1 = -25 \text{ kN}$$

$$R_2 = 28,284 \text{ kN}$$

$$R_3 = -15 \text{ kN}$$

$$R_4 = 22,361 \text{ kN}$$

$$R_5 = 14,142 \text{ kN}$$

$$R_6 = 15 \text{ kN}$$

SPRAWDZENIE

$$\sum M_y = 0$$

$$\begin{aligned}\sum M_b = 0, & \quad -2 \cdot 15 - 2 \cdot 20 - 100 + 100 - 2 \cdot (-25) + \\ & \quad - 2 \cdot \left(\frac{1}{\sqrt{2}} \cdot 20\sqrt{2} \right) - 2 \cdot (-15) - 2 \cdot \left(\frac{1}{\sqrt{5}} \cdot 22,361 \right) + \\ & \quad + 2 \cdot \left(\frac{1}{\sqrt{2}} \cdot 10\sqrt{2} \right) + 2 \cdot 15 = 230 - 230 = 0\end{aligned}$$