Wrocław University of Technology

DYNAMICS LECTURE 6

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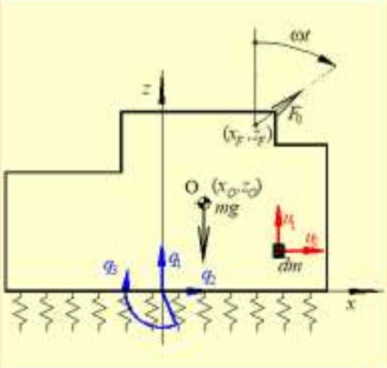
BLOCK FOUNDATION

- Special cases of excitation in an one-degree-of-freedom system: inertial excitation and kinematic excitation.
- Use of the modal transformation method for analysing harmonic vibration of a block foundation.

BLOCK FOUNDATION

One of the most important problems in structural dynamics is the analysis of vibrations generated by a machine attached to a block foundation. The foundation is placed on the surface of elastic ground, Fig.



It is assumed that:

- The block foundation is a rigid body.
- The base contact area is placed on the horizontal plane *xy*.
- The structure is symmetric with respect to the plane *xz*.
- The axes *x*, *y* are the principal axes of the area of the foundation contact surface.
- The axes x,y,z pass through the centroid of the area of the foundation contact surface.
- The elastic ground is a non-inertial one described by three parameters:
  - the stiffness coefficients in the horizontal direction are k_x, k_z (in the directions x, z respectively),
  - and the stiffness coefficient in the direction of rotation around the axis y perpendicular to the plane of vibrations is k_φ.
- The mass center of the block foundation (point O), could be located not on the same vertical line as the centroid of the foundation base area.
- The vibrations are harmonically excited by a force rotating in plane *xz*.
- The force may be located not in the mass center of the block foundation.

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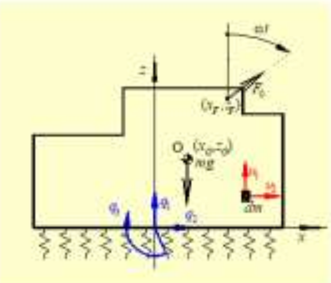
COORDINATES

- Three kinds of coordinates are sufficient to describe the dynamic properties of the structure: generalized, local and block mass center coordinates.



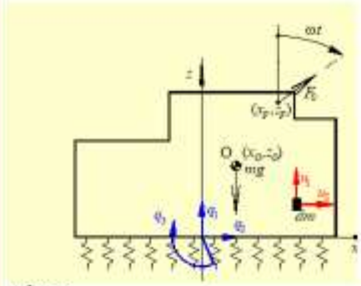
- Taking into account the above assumptions, the space vibration of the structure can be reduced to the problem of the plane vibrations in the symmetry plane *x z*.
- Three generalized coordinates are sufficient to describe the dynamic properties of the structure.
- These generalized, local and block mass center coordinates are shown in Fig.
- In the vector notation they have a form

$$\mathbf{u} = [u_1 \quad u_2]^T \qquad \mathbf{q} = [q_1 \quad q_2 \quad q_3]^T$$

- The transformation of generalized to local coordinates can be written down in form

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & z \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

INERTIA MATRIX



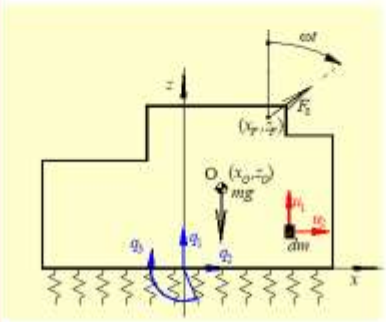
Finally, the inertia matrix has the form

$$\mathbf{B} = \begin{bmatrix} m & 0 & -S_{yz} \\ 0 & m & S_{xy} \\ -S_{yz} & S_{xy} & J_A \end{bmatrix}$$

where

$S_{xy} = m z_O$	The static moment of mass with respect to the plane *xy* .
$S_{yz} = m x_O$	The static moment of mass with respect to the plane *yz*.
$J_A = m(x_O^2 + z_O^2) + J_O$	The polar mass moment of inertia about the axis through the dynamic center - point A, i.e. about the axis through the centroid of the contact area and perpendicular to the plane of vibrations.
J_o	The moment of mass inertia of the machine-foundation structure with respect to the axis passing through the mass center of the machine-foundation structure.
m	The mass of machine-foundation structure

STIFFNESS and DAMPING MATRIX



- The potential energy of the ground elasticity is

$$E_p = \frac{1}{2}(k_z A q_1^2 + k_x A q_2^2 + k_\phi J_A q_3^2)$$

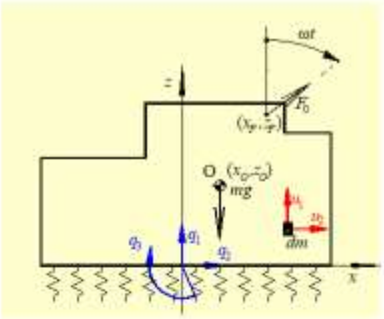
- The stiffness matrix

$$\mathbf{K} = \mathrm{diag}\left(k_z A \quad k_x A \quad k_\varphi J_A\right)$$

- where *A* is the area of surface of contact between structure and elastic ground.

- The damping matrix can be assumed on the basis of one of the hypotheses of damping, even though it is most frequently assumed that $\mathbf{C} = \kappa \mathbf{K}$, i.e. structural damping.

GENERALIZED FORCES



After reducing the rotating force from a given point of its localization to the point of localization of generalized coordinates (this point is the centroid *A* of the area of the foundation contact surface), the generalized forces vector can be obtained in the following form

$$\mathbf{F}(t)=\begin{bmatrix}0\\1\\z_F\end{bmatrix}\mathbf{F}_o\sin\omega t+\begin{bmatrix}1\\0\\-x_F\end{bmatrix}\mathbf{F}_o\cos\omega t$$

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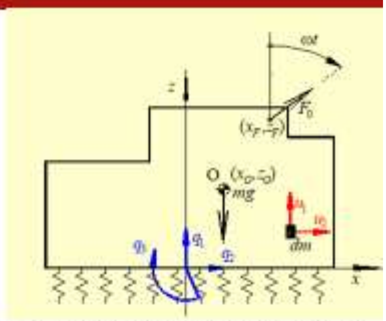
GENERALIZED FORCES

Since the weight of the block foundation is usually significant, and because of the assumption that the mass center, point O, of the block foundation is not located on the same vertical line as the centroid of the foundation base area, it seems to be advisable to take into account an amendment resulting from the second order theory. This second order theory amendment is, in this situation, the moment of the gravity force (weight of the block foundation) about the axis *y* through the centroid of the foundation base area. This axis is perpendicular to the plane of vibrations. Since the mass center, point O, of the block foundation is moving during the vibrations, this additional moment of force depends on time. This additional moment of force can be written down in the form



$$\Delta\mathbf{F}(t)=\begin{bmatrix}0\\0\\mg\,z_O q_3\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&gS_{xy}\end{bmatrix}\cdot\begin{bmatrix}q_1\\q_2\\q_3\end{bmatrix}$$

This vector should be transferred onto the left side of the matrix equation of motion.

$$\mathbf{B}\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{F}(t)$$

After this operation the matrix equation of motion has the standard form

$$\mathbf{K}=\mathrm{diag}(k_z A \quad k_x A \quad k_\varphi J_A-gS_{xy})$$

GENERALIZED FORCES TRANSMITTED TO THE GROUND

$$\mathbf{Q} = \mathbf{F}_T = \mathbf{F} + \omega^2 \mathbf{B} \mathbf{q}$$

- There are three elements in this vector.
  - the first element $Q_1 = F_{Tz}$ is the vertical transmitted force;
  - the second element $Q_2 = F_{Tx}$ is the horizontal transmitted force;
  - the third element $Q_3 = M_{Ty}$ is the moment of transmitted force around the axis y.
- These transmitted forces make it possible to find the dynamic stresses in the foundation contact surface.
- These stresses ought to be calculated separately for sinusoidal and cosinusoidal components.
- The final amplitudal values of stresses should be calculated with the use of formula

$$\mathrm{am}\,\sigma = \sqrt{\sigma_S^2 + \sigma_C^2}$$

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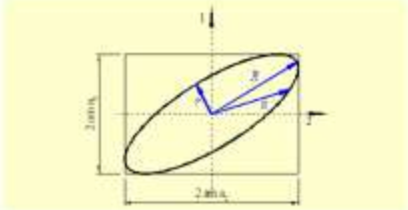
AMPLITUDE OF THE LOCAL CARTESIAN DISPLACEMENTS

- The generalized coordinates vector is also useful to obtain local displacements of chosen points of foundation.
- The amplitude of the local cartesian displacements of any point may be obtained from expressions

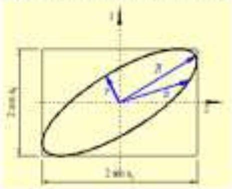
$$\mathrm{am}\,u_1 = \sqrt{u_{1S}^2 + u_{1C}^2} \qquad \mathrm{am}\,u_2 = \sqrt{u_{2S}^2 + u_{2C}^2}$$

- These amplitudes describe the "frame of trajectory" of the vibrating point.
- The trajectory is an ellipse, see Fig. (Lissajous figures - periodic vibration).
- This ellipse is inscribed into a rectangular frame with dimensions .



TRAJECTORY OF VIBRATION OF A GIVEN POINT *A* OF BLOCK FOUNDATION

- It is important to notice that the dimensions of the "frame of trajectory" change depending on the directions of the local cartesian coordinates, while the trajectory itself is an invariant figure, i.e. it describes the motion of the point in an objective way.
- For this reason it is important to determine the trajectory.
- It can be found that



$$u^2(t) = \mathbf{u}^T\mathbf{u} = (\mathbf{u}_S^T \sin\omega t + \mathbf{u}_C^T \cos\omega t)\cdot(\mathbf{u}_S \sin\omega t + \mathbf{u}_C \cos\omega t) =$$
$$= \frac{1}{2}(\mathbf{u}_C^T\mathbf{u}_C + \mathbf{u}_S^T\mathbf{u}_S) + \sqrt{\frac{1}{4}(\mathbf{u}_C^T\mathbf{u}_C - \mathbf{u}_S^T\mathbf{u}_S)^2 + (\mathbf{u}_S^T\mathbf{u}_C)^2}\cos(2\omega t - 2\theta)$$

where

$$2\theta = \arctan\frac{2\mathbf{u}_S^T\mathbf{u}_C}{\mathbf{u}_C^T\mathbf{u}_C - \mathbf{u}_S^T\mathbf{u}_S}$$

$$\left.\begin{matrix}\widetilde{R} = \max u(t) \\ \widetilde{r} = \min u(t)\end{matrix}\right\} = \sqrt{\frac{1}{2}(\mathbf{u}_C^T\mathbf{u}_C + \mathbf{u}_S^T\mathbf{u}_S) \pm \sqrt{\frac{1}{4}(\mathbf{u}_C^T\mathbf{u}_C - \mathbf{u}_S^T\mathbf{u}_S)^2 + (\mathbf{u}_S^T\mathbf{u}_C)^2}}$$

- The maximum value of the principle amplitude occurs when $\cos(2\omega t - 2\theta) = 1$, which means that $\omega t = \theta$. Then

$$\widetilde{\mathbf{R}} = \begin{bmatrix}\widetilde{R}_1 \\ \widetilde{R}_2\end{bmatrix} = \mathbf{u}_S \sin\theta + \mathbf{u}_C \cos\theta \qquad \widetilde{R} = \sqrt{\widetilde{\mathbf{R}}^T\widetilde{\mathbf{R}}}$$

- The minimum value of the principle amplitude occurs when $\cos(2\omega t - 2\theta) = -1$, which means that $\omega t = \theta + \pi/2$. Then

$$\widetilde{\mathbf{r}} = \begin{bmatrix}\widetilde{r}_1 \\ \widetilde{r}_2\end{bmatrix} = \mathbf{u}_S \sin\theta + \mathbf{u}_C \cos\theta \qquad \widetilde{r} = \sqrt{\widetilde{\mathbf{r}}^T\widetilde{\mathbf{r}}}$$