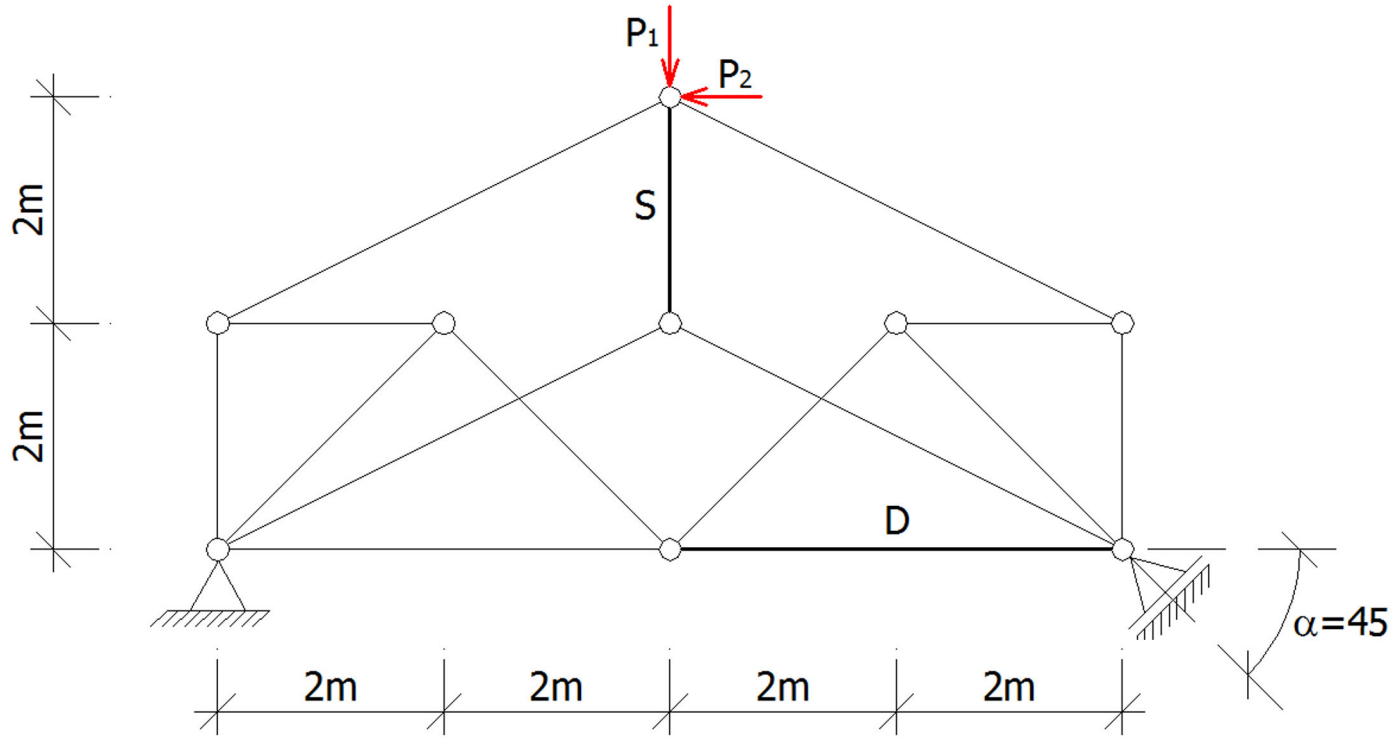
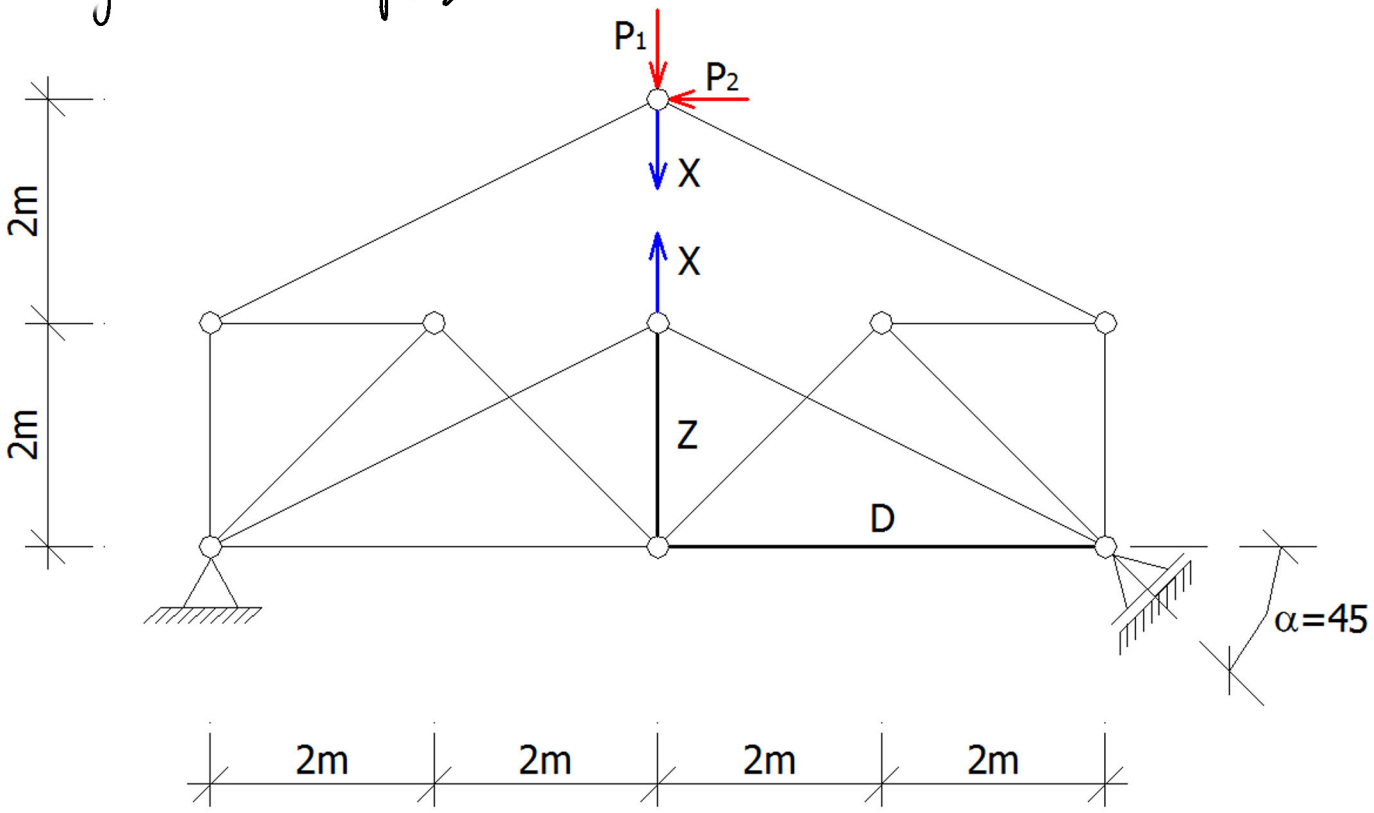
METODA WYMIANY PRĘTÓW (HENNEBERGA)



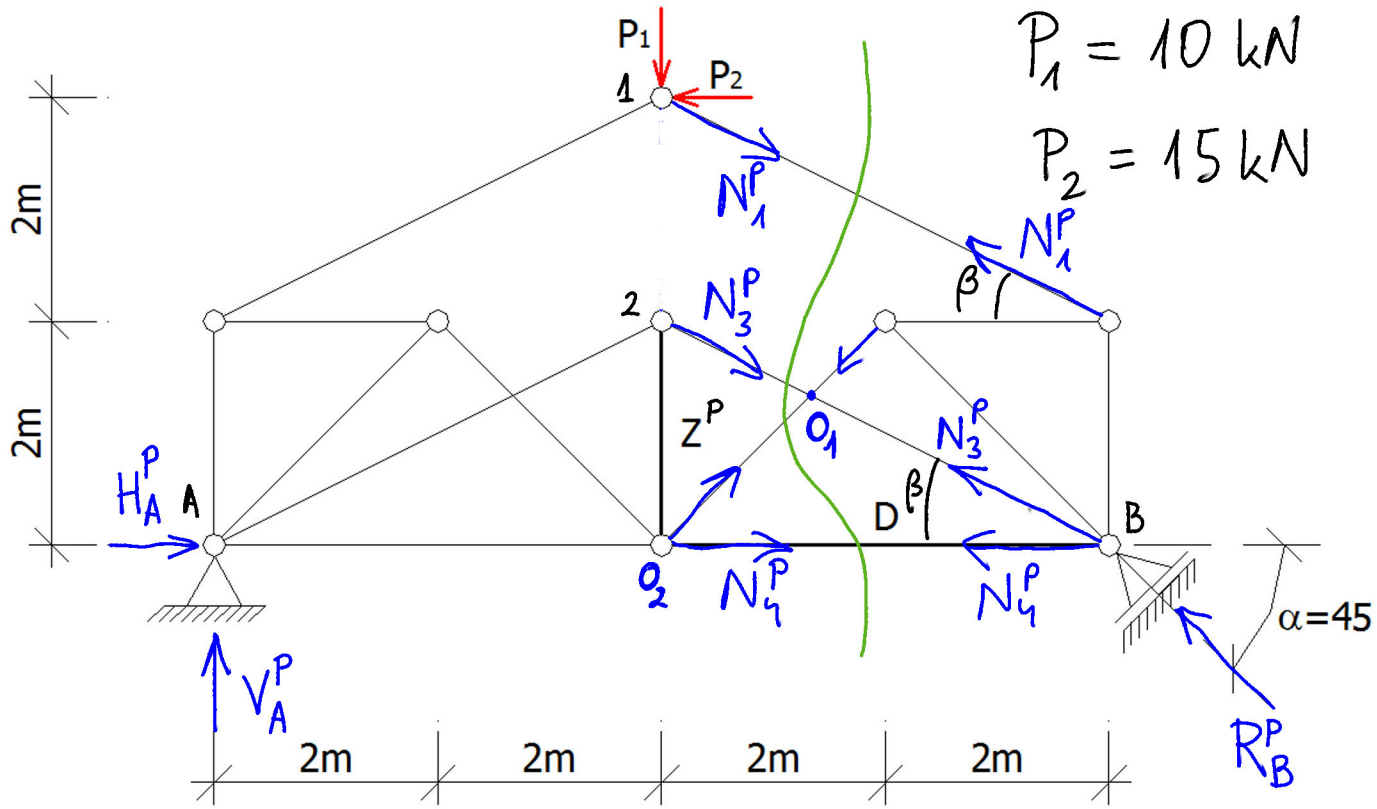
1. Wymiana prętów



WYMOGI

1) Kratownica po wymianie prętów musi być GN i SW
2) Zmodyfikowana kratownica powinna być prosta w dalszych obliczeniach.

2. Rozwiązanie kratownicy od obc. czynnych



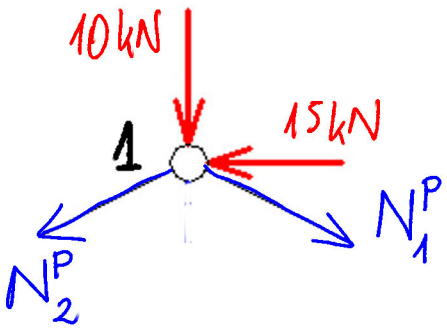
$P_1 = 10\ kN$

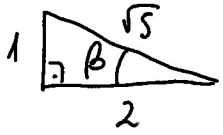
$P_2 = 15\ kN$

$$\Sigma M_A = 0, \quad 10 \cdot 4 - 15 \cdot 4 - 8 \cdot \frac{\sqrt{2}}{2} R_B^P = 0$$

$$R_B^P = -3{,}5355\ kN = -2{,}5\sqrt{2}\ kN$$

* Równowaga węzła 1





$$\cos\beta = \frac{2}{\sqrt{5}}$$

$$\sin\beta = \frac{1}{\sqrt{5}}$$

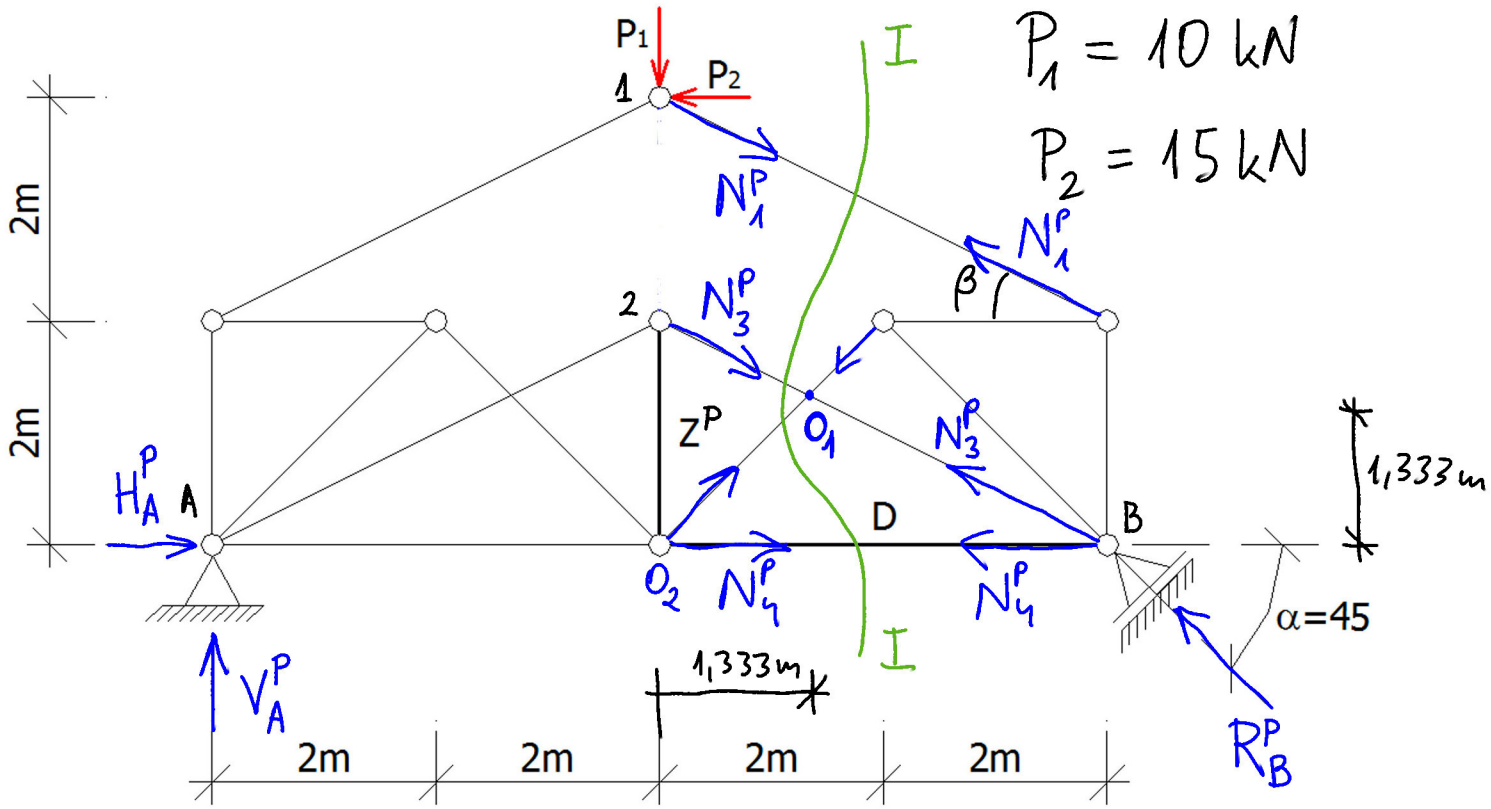
$$\Sigma x = 0$$

$$-\frac{2}{\sqrt{5}} N_2^P - 15 + \frac{2}{\sqrt{5}} N_1^P = 0$$

$$\Sigma y = 0$$

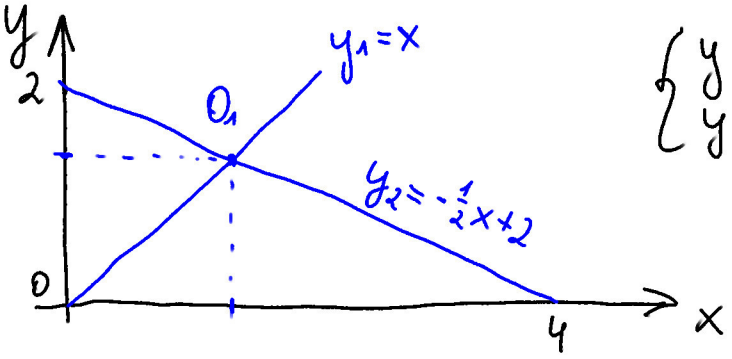
$$\frac{1}{\sqrt{5}} N_2^P + 10 + \frac{1}{\sqrt{5}} N_1^P = 0 \quad / \cdot 2$$

$$5 + \frac{4}{\sqrt{5}} N_1^P = 0 \quad \Rightarrow \quad N_1^P = -2{,}795\ kN$$



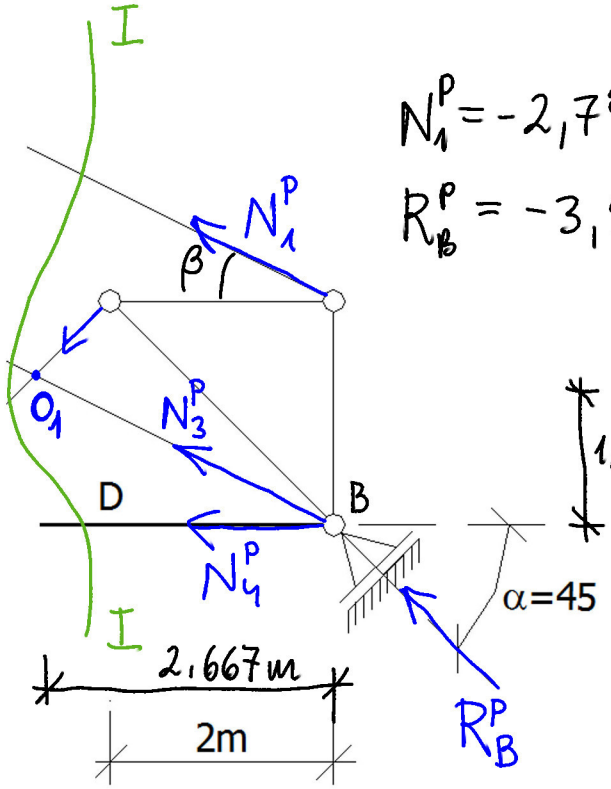
$P_1 = 10\ kN$

$P_2 = 15\ kN$



$$\begin{cases} y = x \\ y = -\frac{1}{2}x + 2 \end{cases} \qquad \frac{3}{2}x = 2 \Rightarrow \begin{cases} x = \frac{4}{3} \\ y = \frac{4}{3} \end{cases}$$

* Przejście Rittera I-I z prawej strony



$N_1^P = -2{,}795\ kN$

$R_B^P = -3{,}5355\ kN$

$$\sum M_{O_1}^P = 0$$

$$1{,}333 \cdot N_4^P - 0{,}167 \cdot \frac{2}{\sqrt{5}} N_1^P + -\frac{1}{\sqrt{5}} N_1^P \cdot 2{,}667 - 2{,}667 \cdot \frac{\sqrt{2}}{2} R_B^P + 1{,}333 \cdot \frac{\sqrt{2}}{2} R_B^P = 0$$

$$1{,}333 N_4^P + 8{,}333 = 0$$

$$N_4^P = -6{,}25\ kN$$

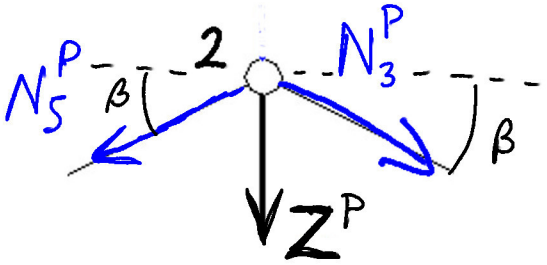
$$\Sigma M^P_{O_2} = 0$$

$$-\frac{1}{\sqrt{5}} N_3^P \cdot 4 - \frac{\sqrt{2}}{2} R_B^P \cdot 4 - \frac{2}{\sqrt{5}} N_1^P \cdot 2 - \frac{1}{\sqrt{5}} N_1^P \cdot 4 = 0$$

$$-\frac{4}{\sqrt{5}} N_3^P - 2\sqrt{2} \cdot (-2{,}5\sqrt{2}) - \frac{4}{\sqrt{5}} (-2{,}795) - \frac{4}{\sqrt{5}} (-2{,}795) = 0$$

$$N_3^P = 11{,}18 \text{ kN}$$

* Równowaga węzła 2



$$\Sigma x = 0$$

$$-\frac{2}{\sqrt{5}} N_5^P + \frac{2}{\sqrt{5}} N_3^P = 0$$

$$N_5^P = N_3^P = 11{,}18 \text{ kN}$$

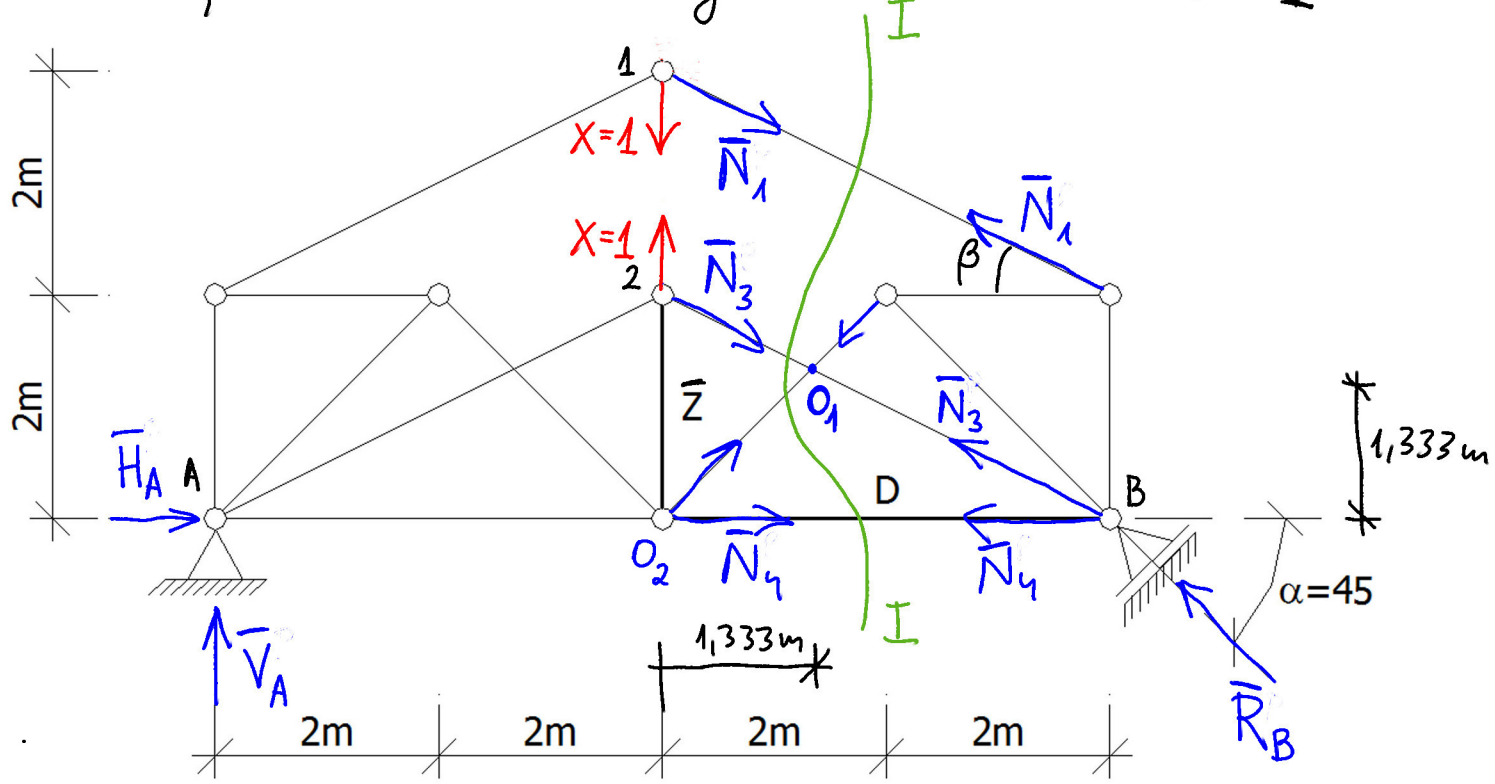
$$\Sigma y = 0$$

$$\frac{1}{\sqrt{5}} N_5^P + \frac{1}{\sqrt{5}} N_3^P + Z^P = 0$$

$$\frac{2}{\sqrt{5}} (11{,}18) + Z^P = 0$$

$$Z^P = -10 \text{ kN}$$

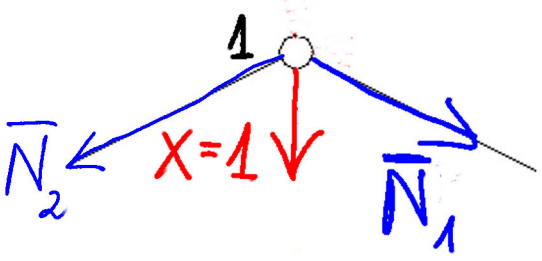
3. Rozwiązanie kratownicy w stanie $X = 1$



* Reakcja $\bar{R}_B$

$$\Sigma M_A = 0 \;,\; -8 \cdot \frac{\sqrt{2}}{2} \bar{R}_B = 0 \Rightarrow \underline{\bar{R}_B = 0}$$

* Równowaga węzła 1



$$\Sigma x = 0$$

$$-\frac{2}{\sqrt{5}} \bar{N}_2 + \frac{2}{\sqrt{5}} \bar{N}_1 = 0$$

$$\bar{N}_2 = \bar{N}_1$$

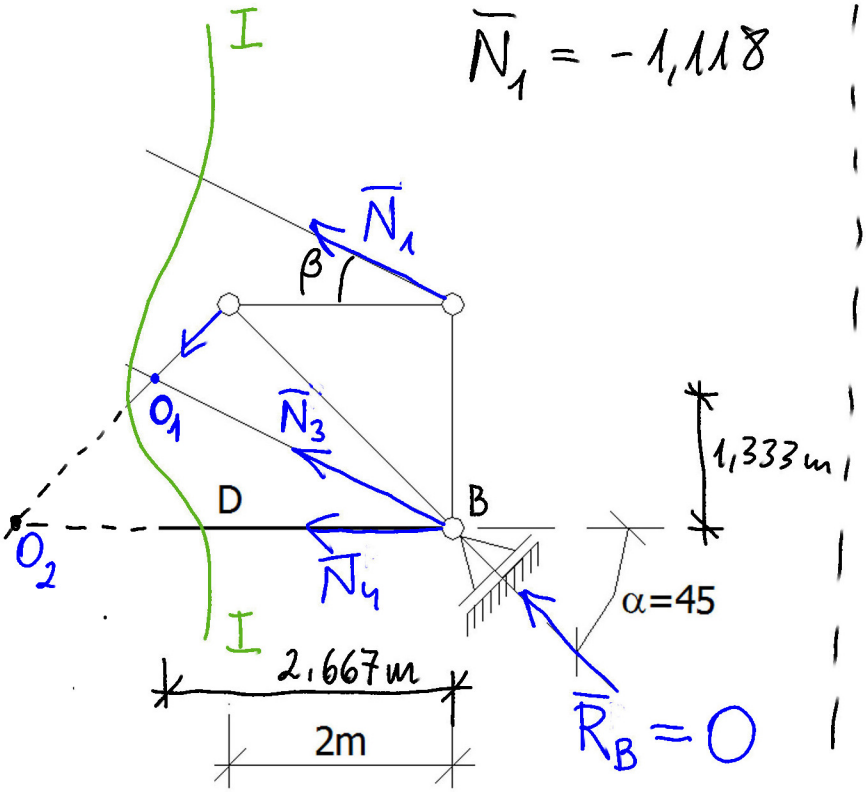
$$\Sigma y = 0$$

$$\frac{1}{\sqrt{5}} \bar{N}_2 + \frac{1}{\sqrt{5}} \bar{N}_1 + 1 = 0$$

$$\frac{2}{\sqrt{5}} \bar{N}_1 = -1$$

$$\underline{\bar{N}_1 = -\frac{\sqrt{5}}{2} = -1{,}118}$$

* Przecięcie Rittera I - I



$\bar{N}_1 = -1{,}118$

$\sum M^P_{O_1} = 0$

$1{,}333 \cdot \bar{N}_4 - 0{,}667 \cdot \frac{2}{\sqrt{5}} \bar{N}_1 +$

$- \frac{1}{\sqrt{5}} \bar{N}_1 \cdot 2{,}667 - 2{,}667 \cdot \frac{\sqrt{2}}{2} \bar{R}_B +$

$+ 1{,}333 \cdot \frac{\sqrt{2}}{2} \bar{R}_B = 0$

$1{,}333 \bar{N}_4 + 2 = 0$

$\bar{N}_4 = -1{,}5$

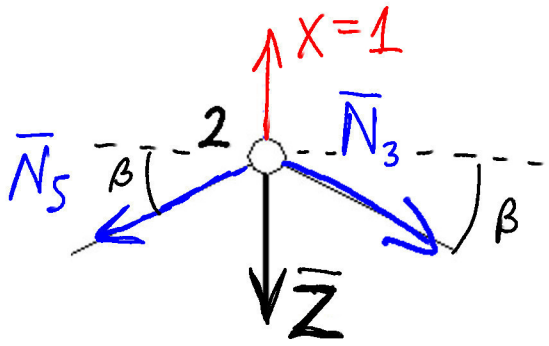
$\sum M^P_{O_2} = 0$

$- \frac{1}{\sqrt{5}} \bar{N}_3 \cdot 4 - \frac{\sqrt{2}}{2} \bar{R}_B \cdot 4 - \frac{2}{\sqrt{5}} \bar{N}_1 \cdot 2 - \frac{1}{\sqrt{5}} \bar{N}_1 \cdot 4 = 0$

$- \frac{4}{\sqrt{5}} \bar{N}_3 - \frac{4}{\sqrt{5}} (-1{,}118) - \frac{4}{\sqrt{5}} (-1{,}118) = 0$

$\bar{N}_3 = 2{,}236$

* Równowaga węzła 2



$\sum x = 0$

$- \frac{2}{\sqrt{5}} \bar{N}_5 + \frac{2}{\sqrt{5}} \bar{N}_3 = 0$

$\bar{N}_5 = \bar{N}_3 = 2{,}236$

$$\Sigma y = 0 \qquad \frac{1}{\sqrt{5}}\bar{N}_5 + \frac{1}{\sqrt{5}}\bar{N}_3 + \bar{\bar{Z}} - 1 = 0$$

$$1 + 1 + \bar{\bar{Z}} - 1 = 0$$

$$\boxed{\bar{\bar{Z}} = -1}$$

$$\bar{\bar{Z}} \neq 0 \Rightarrow$$ KRATOWNICA RZECZYWISTA JEST GN.

4. Wyznaczenie sił we wskazanych prętach kratownicy rzeczywistej

$$\boxed{Z = 0} \Rightarrow Z = Z^P + \bar{\bar{Z}} \cdot X = 0$$

$$\boxed{X = -\frac{Z^P}{\bar{\bar{Z}}}}$$

$$\boxed{N_i = N_i^P + \bar{N}_i \cdot X}$$

ZASADA SUPERPOZYCJI STANÓW OBCIĄŻEŃ

* W analizowanej kratownicy mamy

$$X = -\frac{-10\,kN}{-1} = \boxed{-10\ kN = S}$$

$$D = N_4 = N_4^P + \bar{N}_4 \cdot X = -6{,}25\,kN - 1{,}5(-10\,kN) = \boxed{8{,}75\,kN}$$