Statyka Budowli

Laboratorium nr 6

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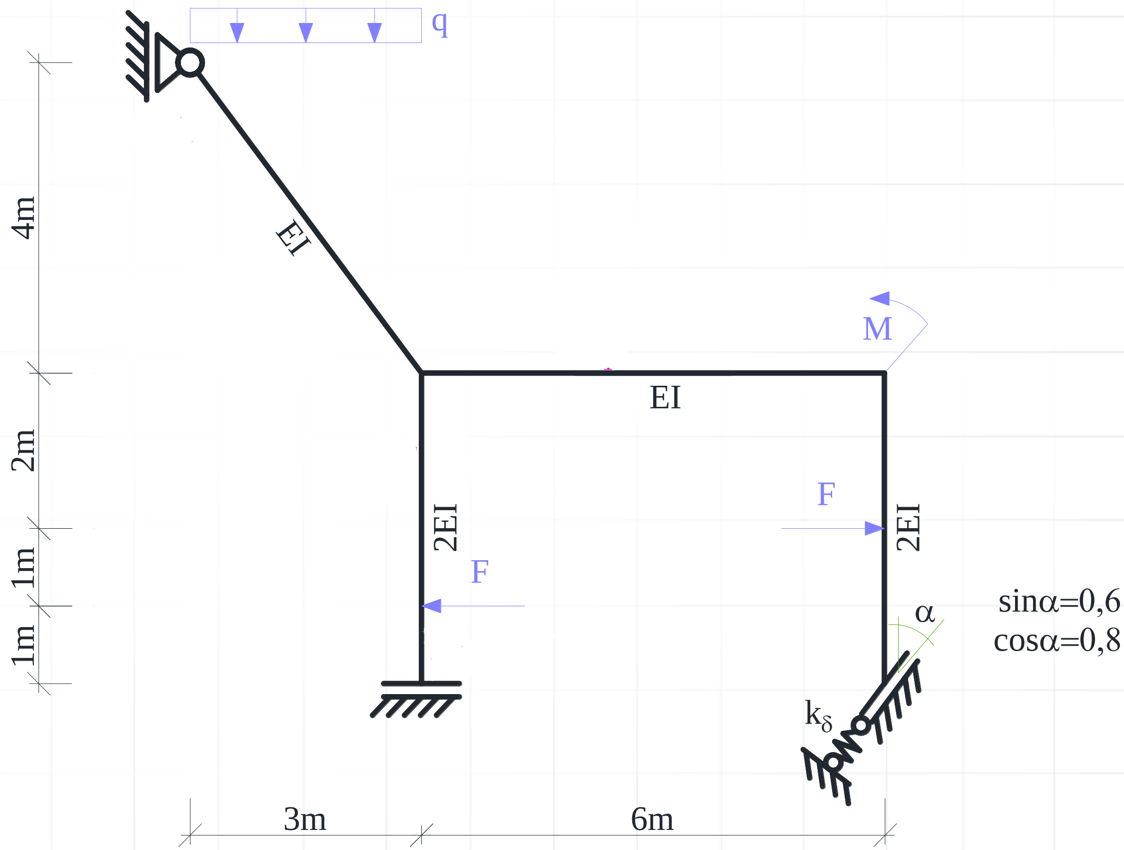
Laboratorium nr 6

METODA PRZEMIESZCZEŃ - RAMA PŁASKA

Dana jest rama płaska o schemacie i obciążeniu mechanicznym jak na rysunku. Należy:

- Sprawdzić warunek ilościowy i jakościowy geometrycznej niezmienności układu.
- Stosując metodę przemieszczeń rozwiązać ramę od podanego obciążenia mechanicznego (obliczyć siły) przekrojowe i sporządzić ich wykresy.
- Przeprowadzić stosowne kontrole rozwiązania.



Rys. 1. Schemat statyczny

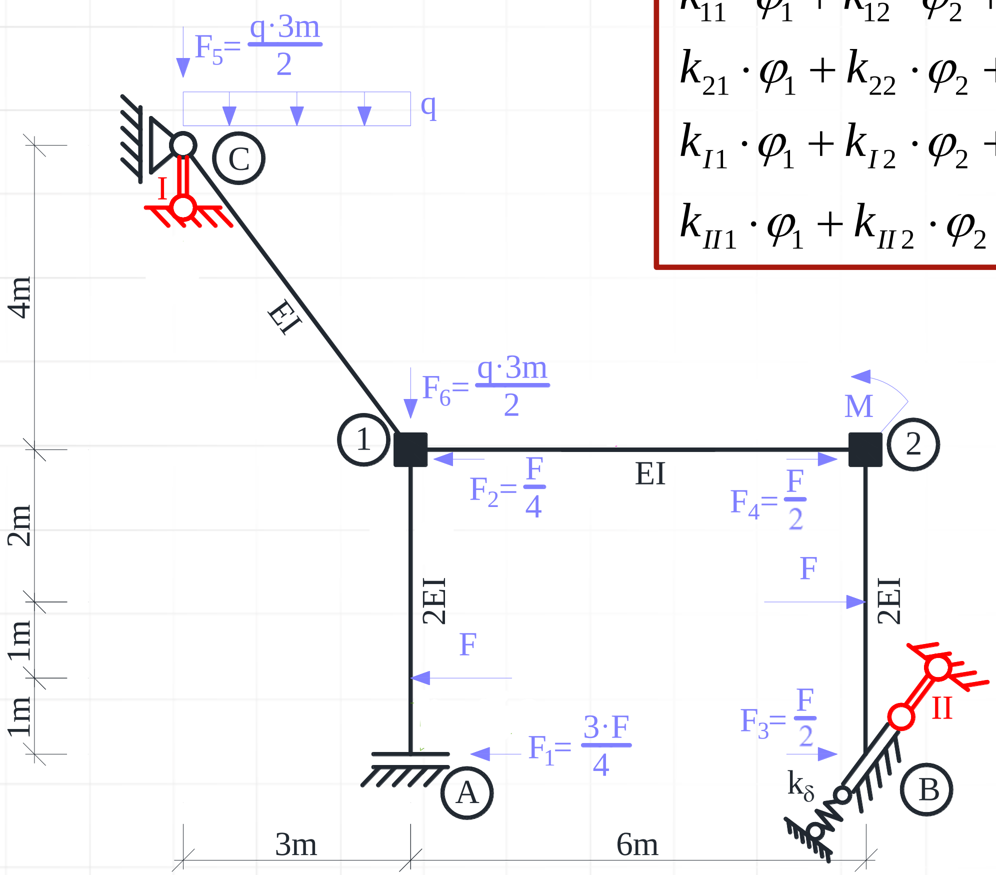
Dane do obliczeń: F = 8 kN;

q = 4 kN/m; M = 20 kN m;

k_δ = 8 EI/m^3

POSTAĆ OGÓLNA UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$\boxed{\begin{aligned}
&k_{11}\cdot\varphi_1 + k_{12}\cdot\varphi_2 + k_{1I}\cdot\delta_I + k_{1II}\cdot\delta_{II} + k_{1o} = 0\\
&k_{21}\cdot\varphi_1 + k_{22}\cdot\varphi_2 + k_{2I}\cdot\delta_I + k_{2II}\cdot\delta_{II} + k_{2o} = 0\\
&k_{I1}\cdot\varphi_1 + k_{I2}\cdot\varphi_2 + k_{I\,I}\cdot\delta_I + k_{I\,II}\cdot\delta_{II} + k_{Io} = 0\\
&k_{II1}\cdot\varphi_1 + k_{II2}\cdot\varphi_2 + k_{II\,I}\cdot\delta_I + k_{II\,II}\cdot\delta_{II} + k_{IIo} = 0
\end{aligned}}$$



Rys. 2. Układ podstawowy metody przemieszczeń

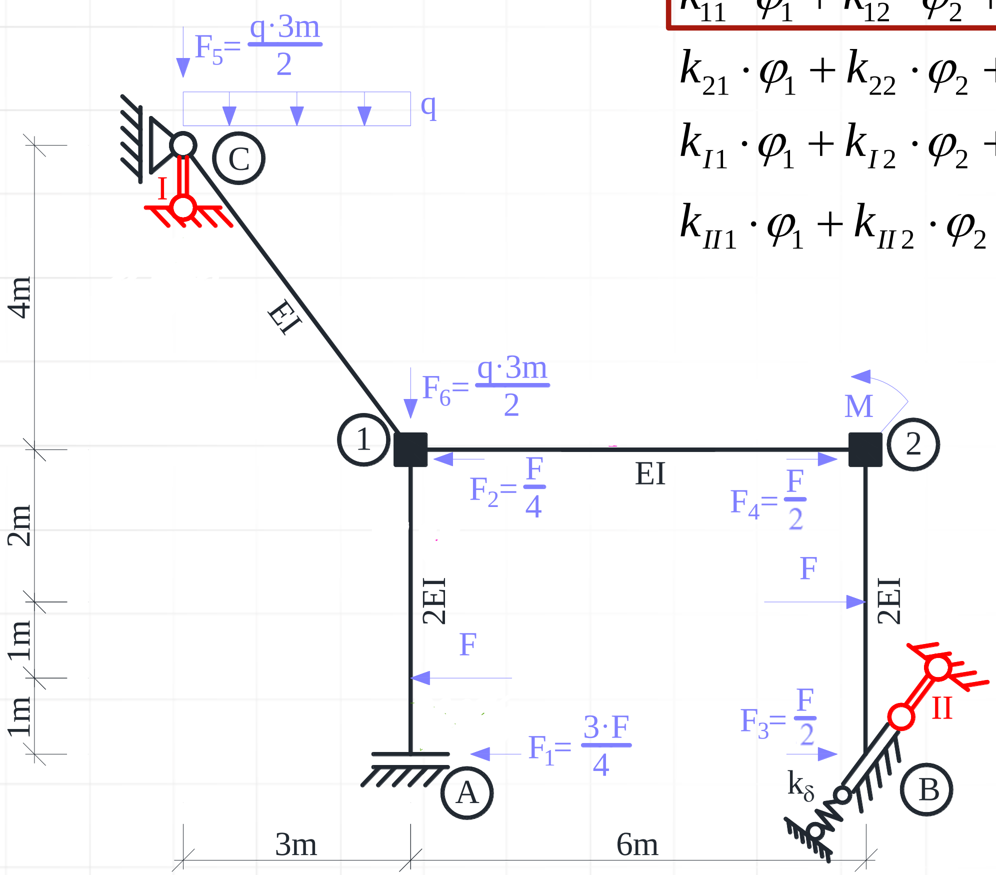
WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$\boxed{k_{11}\cdot\varphi_1 + k_{12}\cdot\varphi_2 + k_{1I}\cdot\delta_I + k_{1II}\cdot\delta_{II} + k_{1o} = 0}$$

$$k_{21}\cdot\varphi_1 + k_{22}\cdot\varphi_2 + k_{2I}\cdot\delta_I + k_{2II}\cdot\delta_{II} + k_{2o} = 0$$

$$k_{I1}\cdot\varphi_1 + k_{I2}\cdot\varphi_2 + k_{I\,I}\cdot\delta_I + k_{I\,II}\cdot\delta_{II} + k_{Io} = 0$$

$$k_{II1}\cdot\varphi_1 + k_{II2}\cdot\varphi_2 + k_{II\,I}\cdot\delta_I + k_{II\,II}\cdot\delta_{II} + k_{IIo} = 0$$



Rys. 2. Układ podstawowy metody przemieszczeń

$\mathbf{K}_{\varphi\varphi}$; $\quad k_{ii} = \sum_j M_{ij}^i + k_i^\varphi$;

$\mathbf{K}_{\varphi\delta}$; $\quad k_{i\beta} = \sum_j M_{ij}^\beta$

$\mathbf{K}_{\varphi o}$; $\quad k_{io} = \begin{cases} k_{iF} = \sum_j M_{ij}^{oF} - M_i^o \\ k_{i\Delta} = \sum_j M_{ij}^{o\Delta} \\ k_{i\Delta} = \sum_j M_{ij}^{oT} \end{cases}$

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

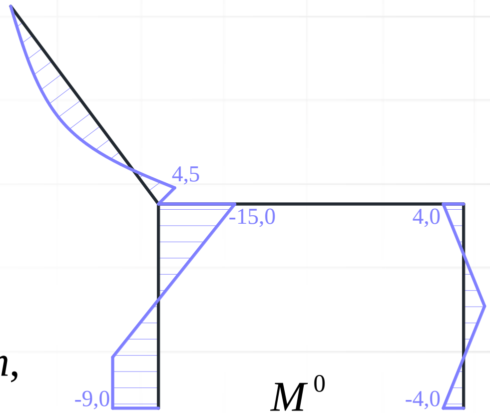
$$k_{11}=\sum_j M_{1j}^1+k_1^{\varphi}=M_{12}^1+M_{1A}^1+M_{1C}^1+k_1^{\varphi}=\left(0{,}6667+0{,}5+0{,}6+0\right)\frac{EI}{m}=\frac{53}{30}\frac{EI}{m}=1{,}7667\frac{EI}{m},$$

$$k_{12}=\sum_j M_{1j}^2=M_{12}^2+M_{1A}^2+M_{1C}^2=\left(0{,}3333+0+0\right)\frac{EI}{m}=0{,}3333\frac{EI}{m},$$

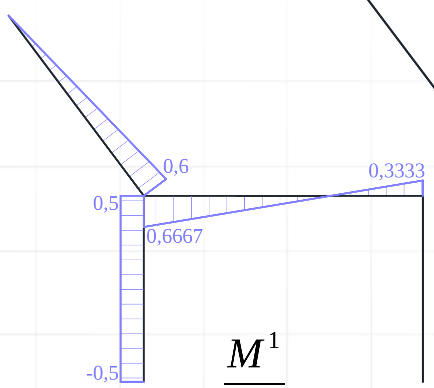
$$k_{1I}=\sum_j M_{1j}^I=M_{12}^I+M_{1A}^I+M_{1C}^I=0+0+\left(-\frac{1}{5}\right)\frac{EI}{m^2}=-0{,}2\frac{EI}{m^2},$$

$$k_{1II}=\sum_j M_{1j}^{II}=M_{12}^{II}+M_{1A}^{II}+M_{1C}^{II}=\frac{2}{15}\frac{EI}{m^2}+0+0=0{,}1333\frac{EI}{m^2},$$

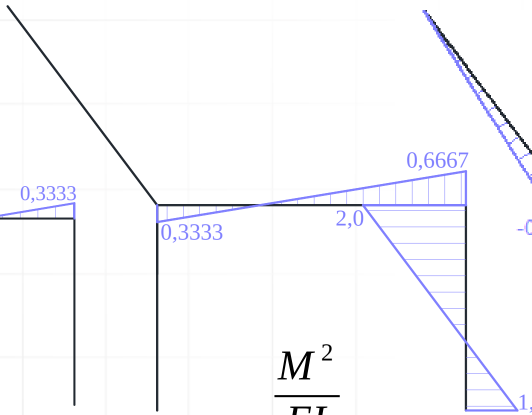
$$k_{1o}=\sum_j M_{1j}^o=M_{12}^o+M_{1A}^o+M_{1C}^o=0-15kN\cdot m+4{,}5kN\cdot m=-10{,}5kN\cdot m,$$



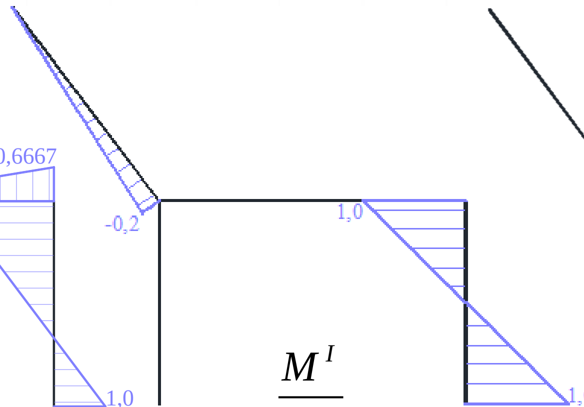
$$\frac{M^0}{kN\cdot m}$$



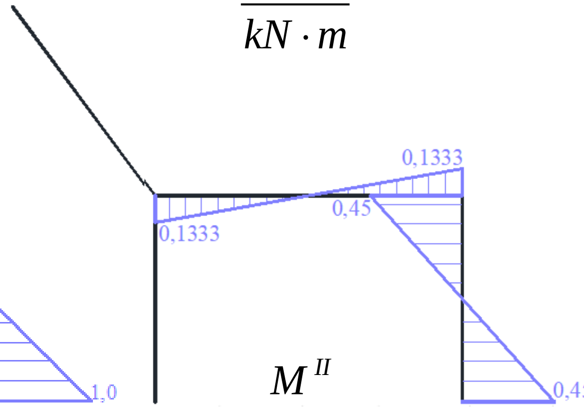
$$\frac{M^1}{\frac{EI}{m}}$$



$$\frac{M^2}{\frac{EI}{m}}$$

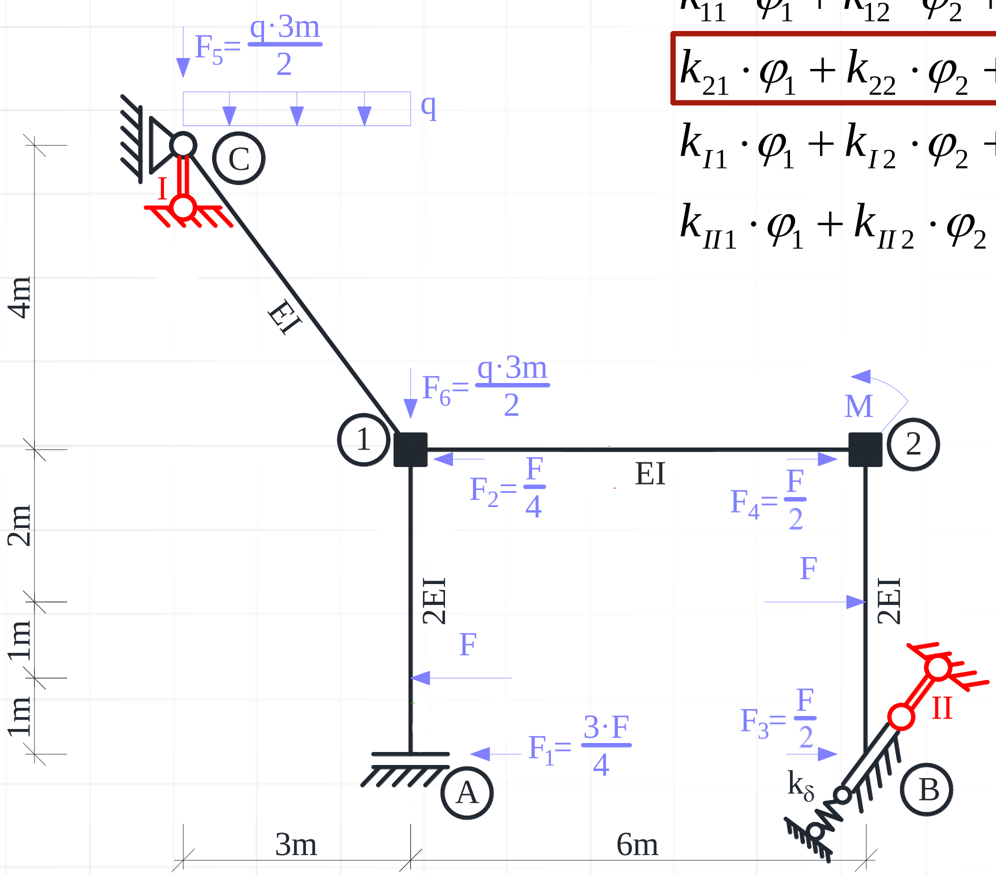


$$\frac{M^I}{\frac{EI}{m^2}}$$



$$\frac{M^{II}}{\frac{EI}{m^2}}$$

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ



Rys. 2. Układ podstawowy metody przemieszczeń

$$k_{11}\cdot\varphi_1 + k_{12}\cdot\varphi_2 + k_{1I}\cdot\delta_I + k_{1II}\cdot\delta_{II} + k_{1o} = 0$$

$$\boxed{k_{21}\cdot\varphi_1 + k_{22}\cdot\varphi_2 + k_{2I}\cdot\delta_I + k_{2II}\cdot\delta_{II} + k_{2o} = 0}$$

$$k_{I1}\cdot\varphi_1 + k_{I2}\cdot\varphi_2 + k_{I\,I}\cdot\delta_I + k_{I\,II}\cdot\delta_{II} + k_{Io} = 0$$

$$k_{II1}\cdot\varphi_1 + k_{II2}\cdot\varphi_2 + k_{II\,I}\cdot\delta_I + k_{II\,II}\cdot\delta_{II} + k_{IIo} = 0$$

$\mathbf{K}_{\varphi\varphi}$; $\quad k_{ii} = \sum_j M_{ij}^{i} + k_i^{\varphi}$:

$\mathbf{K}_{\varphi\delta}$; $\quad k_{i\beta} = \sum_j M_{ij}^{\beta}$

$\mathbf{K}_{\varphi o}$; $\quad k_{io} = \begin{cases} k_{iF} = \sum_j M_{ij}^{oF} - M_i^{o} \\ k_{i\Delta} = \sum_j M_{ij}^{o\Delta} \\ k_{i\Delta} = \sum_j M_{ij}^{oT} \end{cases}$

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

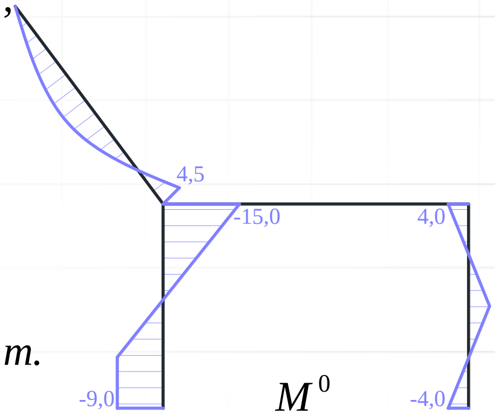
$$k_{21}=\sum_j M^1_{2j}=M^1_{21}+M^1_{2B}=\left(0{,}3333+0\right)\frac{EI}{m}=0{,}3333\frac{EI}{m},$$

$$k_{22}=\sum_j M^2_{2j}+k^{\varphi}_2=M^2_{21}+M^2_{2B}+k^{\varphi}_2=\left(\frac{2}{3}+2+0\right)\frac{EI}{m}=\frac{8}{3}\frac{EI}{m}=2{,}6667\frac{EI}{m},$$

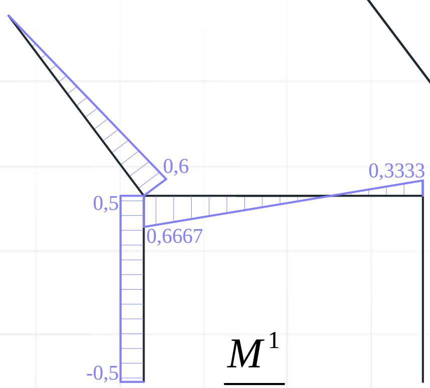
$$k_{2I}=\sum_j M^I_{2j}=M^I_{21}+M^I_{2B}=0+1\frac{EI}{m^2}=1\frac{EI}{m^2},$$

$$k_{2II}=\sum_j M^I_{2j}=M^{II}_{21}+M^{II}_{2B}=\left(\frac{2}{15}\right)\frac{EI}{m^2}+\left(\frac{9}{20}\right)\frac{EI}{m^2}=\frac{7}{12}\frac{EI}{m^2}=0{,}5833\frac{EI}{m^2},$$

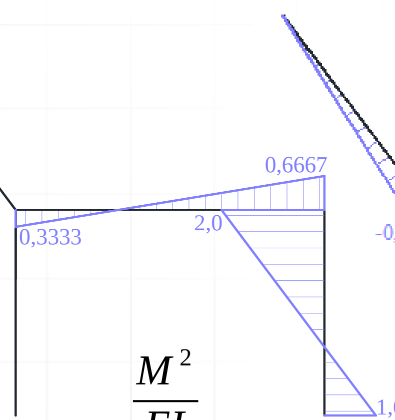
$$k_{2o}=\sum_j M^o_{2j}-M^o_2=M^o_{21}+M^o_{2B}-(-M)=\left[0+4-\left(-20\right)\right]kN\cdot m=24kN\cdot m.$$



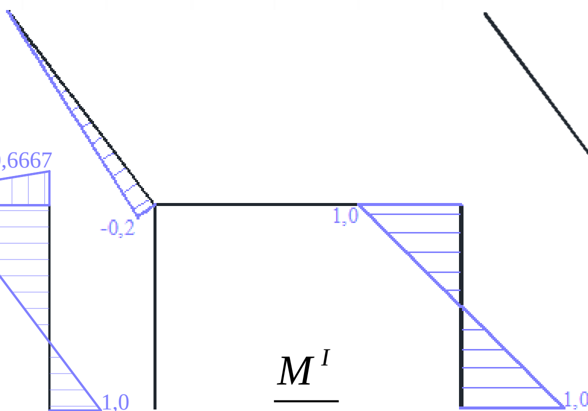
$\dfrac{M^0}{kN\cdot m}$



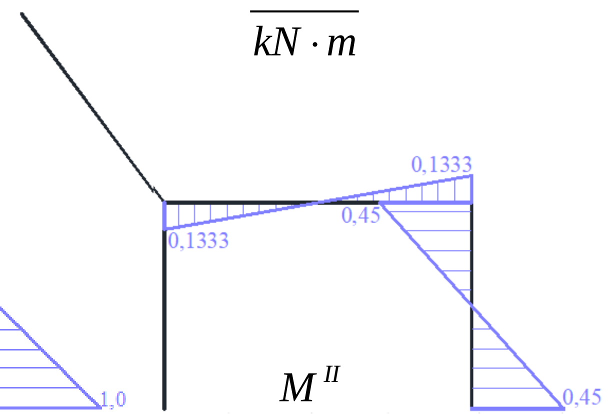
$\dfrac{M^1}{\frac{EI}{m}}$



$\dfrac{M^2}{\frac{EI}{m}}$



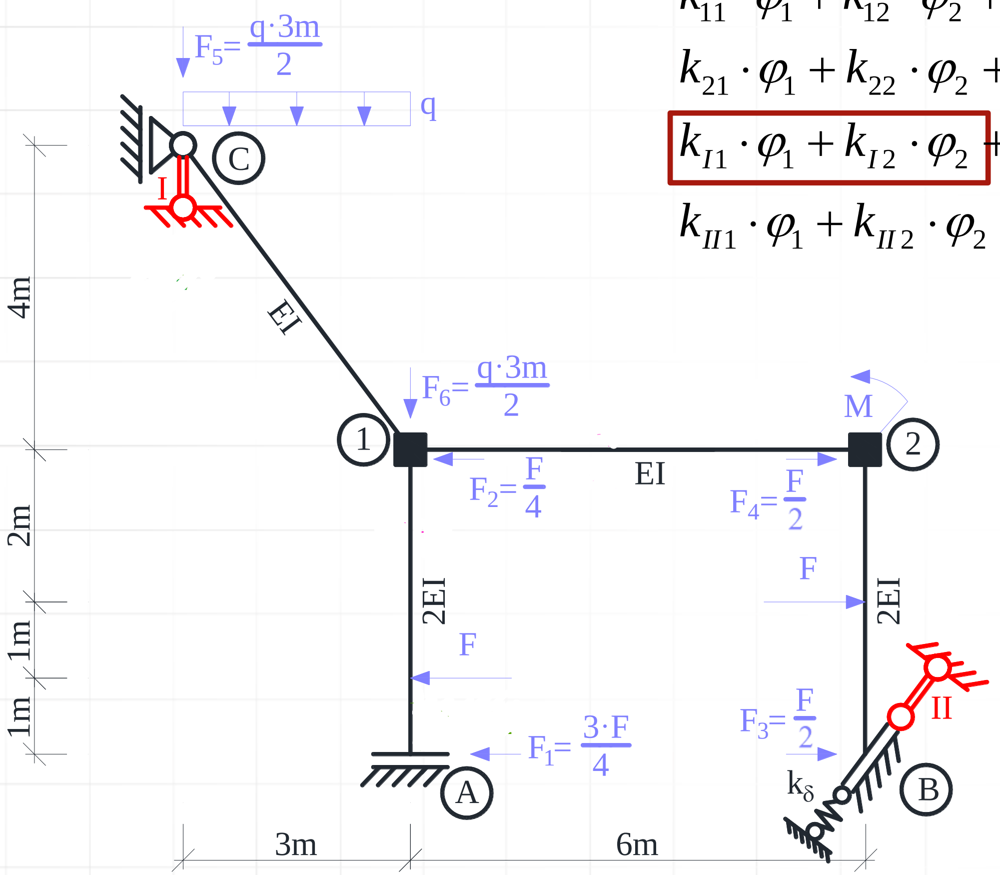
$\dfrac{M^I}{\frac{EI}{m^2}}$



$\dfrac{M^{II}}{\frac{EI}{m^2}}$

Laboratorium nr 6

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ



Rys. 2. Układ podstawowy metody przemieszczeń

$$k_{11} \cdot \varphi_1 + k_{12} \cdot \varphi_2 + k_{1I} \cdot \delta_I + k_{1II} \cdot \delta_{II} + k_{1o} = 0$$

$$k_{21} \cdot \varphi_1 + k_{22} \cdot \varphi_2 + k_{2I} \cdot \delta_I + k_{2II} \cdot \delta_{II} + k_{2o} = 0$$

$$\boxed{k_{I1} \cdot \varphi_1 + k_{I2} \cdot \varphi_2} + k_{I\,I} \cdot \delta_I + k_{I\,II} \cdot \delta_{II} + k_{Io} = 0$$

$$k_{II1} \cdot \varphi_1 + k_{II2} \cdot \varphi_2 + k_{II\,I} \cdot \delta_I + k_{II\,II} \cdot \delta_{II} + k_{IIo} = 0$$

$$\mathbf{K}_{\delta\varphi}\,; \qquad k_{\alpha j} = -\sum (M_{ij}^{j} + M_{ji}^{j}) \cdot \psi_{ij}^{\alpha}$$

Laboratorium nr 6

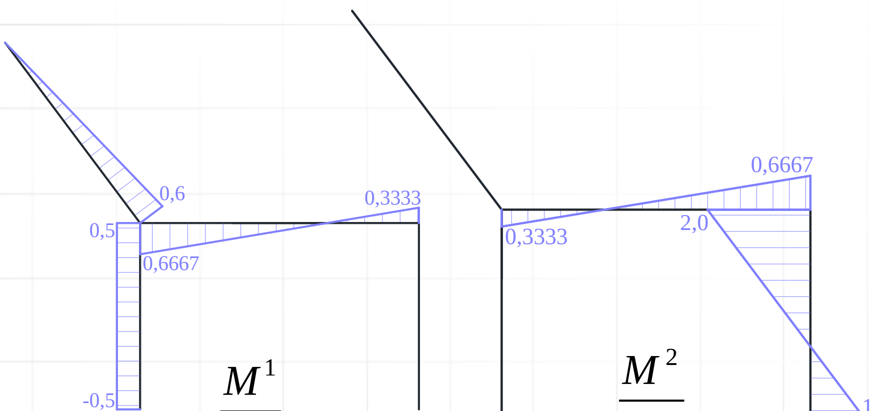
WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$k_{I1}=-\sum_{ij}\left(M_{ij}^{1}+M_{ji}^{1}\right)\psi_{ij}^{I}=-\left(M_{12}^{1}+M_{21}^{1}\right)\psi_{12}^{I}-\left(M_{1A}^{1}+M_{A1}^{1}\right)\psi_{1A}^{I}-\left(M_{1C}^{1}+M_{C1}^{1}\right)\psi_{1C}^{I}-\left(M_{2B}^{1}+M_{B2}^{1}\right)\psi_{2B}^{I}=$$

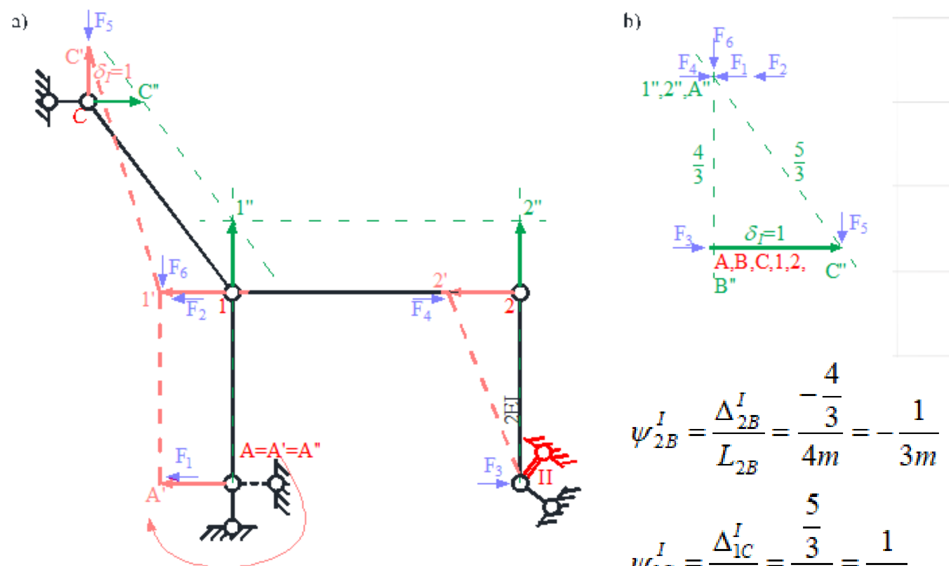
$$=-\left(0,6667\frac{EI}{m}+0,3333\frac{EI}{m}\right)\cdot 0-\left(0,5\frac{EI}{m}-0,5\frac{EI}{m}\right)\cdot 0-\left(0,6\frac{EI}{m}+0\right)\cdot\frac{1}{3m}-\left(0+0\right)\cdot\left(-\frac{1}{3m}\right)=-0,2\frac{EI}{m^{2}},$$

$$k_{I2}=-\sum_{ij}\left(M_{ij}^{2}+M_{ji}^{2}\right)\psi_{ij}^{I}=-\left(M_{12}^{2}+M_{21}^{2}\right)\psi_{12}^{I}-\left(M_{1A}^{2}+M_{A1}^{2}\right)\psi_{1A}^{I}-\left(M_{1C}^{2}+M_{C1}^{2}\right)\psi_{1C}^{I}-\left(M_{2B}^{2}+M_{B2}^{2}\right)\psi_{2B}^{I}=$$

$$=-\left(0,3333\frac{EI}{m}+0,6667\frac{EI}{m}\right)\cdot 0-\left(0-0\right)\cdot 0-\left(0+0\right)\cdot\frac{1}{3m}-\left(1\frac{EI}{m}+2\frac{EI}{m}\right)\cdot\left(-\frac{1}{3m}\right)=1\frac{EI}{m^{2}},$$



$\dfrac{M^{1}}{\frac{EI}{m}}$ $\dfrac{M^{2}}{\frac{EI}{m}}$

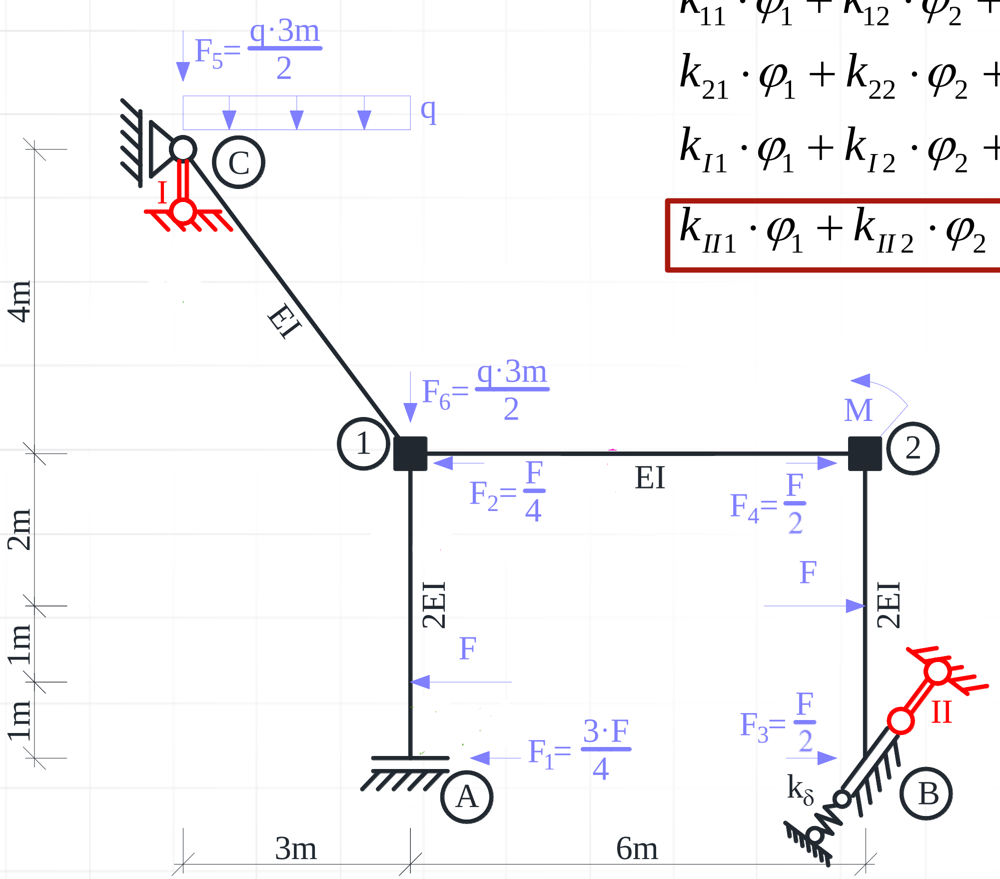


$$\psi_{2B}^{I}=\frac{\Delta_{2B}^{I}}{L_{2B}}=\frac{-\frac{4}{3}}{4m}=-\frac{1}{3m}$$

$$\psi_{1C}^{I}=\frac{\Delta_{1C}^{I}}{L_{1C}}=\frac{\frac{5}{3}}{5m}=\frac{1}{3m}$$

$$\psi_{1A}^{I}=\psi_{12}^{I}=0$$

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ



Rys. 2. Układ podstawowy metody przemieszczeń

$$k_{11}\cdot\varphi_1+k_{12}\cdot\varphi_2+k_{1I}\cdot\delta_I+k_{1II}\cdot\delta_{II}+k_{1o}=0$$

$$k_{21}\cdot\varphi_1+k_{22}\cdot\varphi_2+k_{2I}\cdot\delta_I+k_{2II}\cdot\delta_{II}+k_{2o}=0$$

$$k_{I1}\cdot\varphi_1+k_{I2}\cdot\varphi_2+k_{I\,I}\cdot\delta_I+k_{I\,II}\cdot\delta_{II}+k_{Io}=0$$

$$\boxed{k_{II1}\cdot\varphi_1+k_{II2}\cdot\varphi_2+}k_{II\,I}\cdot\delta_I+k_{II\,II}\cdot\delta_{II}+k_{IIo}=0$$

$$\mathbf{K}_{\delta\varphi};\qquad k_{\alpha j}=-\sum(M_{ij}^{j}+M_{ji}^{j})\cdot\psi_{ij}^{\alpha}$$

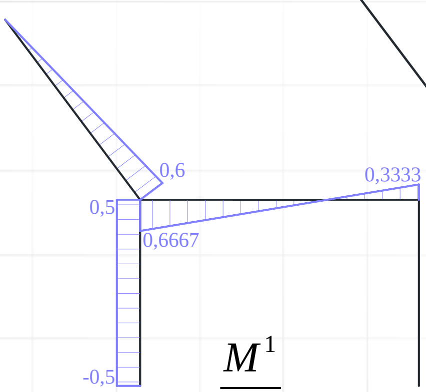
WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$k_{II1}=-\sum_{ij}\left(M_{ij}^{1}+M_{ji}^{1}\right)\psi_{ij}^{II}=-\left(M_{12}^{1}+M_{21}^{1}\right)\psi_{12}^{II}-\left(M_{1A}^{1}+M_{A1}^{1}\right)\psi_{1A}^{II}-\left(M_{1C}^{1}+M_{C1}^{1}\right)\psi_{1C}^{II}-\left(M_{2B}^{1}+M_{B2}^{1}\right)\psi_{2B}^{II}=$$

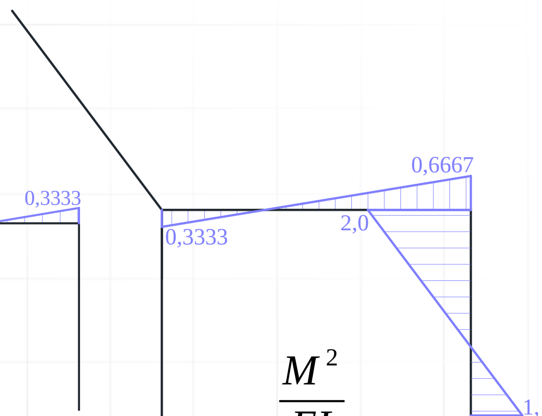
$$=-\left(0,6667\frac{EI}{m}+0,3333\frac{EI}{m}\right)\cdot\left(-\frac{4}{30m}\right)-\left(0,5\frac{EI}{m}-0,5\frac{EI}{m}\right)\cdot 0-\left(0,6\frac{EI}{m}+0\right)\cdot 0-\left(0+0\right)\cdot\left(-\frac{3}{20m}\right)=\frac{2}{15}\frac{EI}{m^{2}},$$

$$k_{II2}=-\sum_{ij}\left(M_{ij}^{2}+M_{ji}^{2}\right)\psi_{ij}^{II}=-\left(M_{12}^{2}+M_{21}^{2}\right)\psi_{12}^{II}-\left(M_{1A}^{2}+M_{A1}^{2}\right)\psi_{1A}^{II}-\left(M_{1C}^{2}+M_{C1}^{2}\right)\psi_{1C}^{II}-\left(M_{2B}^{2}+M_{B2}^{2}\right)\psi_{2B}^{II}=$$

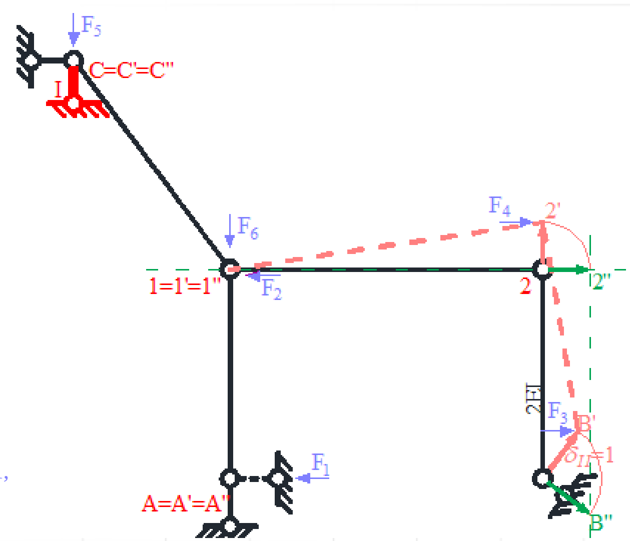
$$=-\left(0,3333\frac{EI}{m}+0,6667\frac{EI}{m}\right)\cdot\left(-\frac{4}{30m}\right)-\left(0-0\right)\cdot 0-\left(0+0\right)\cdot 0-\left(1\frac{EI}{m}+2\frac{EI}{m}\right)\cdot\left(-\frac{3}{20m}\right)=0,5833\frac{EI}{m^{2}},$$

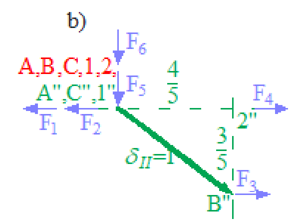


$$\frac{M^{1}}{\frac{EI}{m}}$$



$$\frac{M^{2}}{\frac{EI}{m}}$$





$$\psi_{12}^{II}=\frac{\Delta_{12}^{II}}{L_{12}}=\frac{-\frac{4}{5}}{6m}=-\frac{2}{15m}$$

$$\psi_{2B}^{II}=\frac{\Delta_{2B}^{II}}{L_{2B}}=\frac{-\frac{3}{5}}{4m}=-\frac{3}{20m}$$

$$\psi_{1A}^{II}=\psi_{1C}^{II}=0$$

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

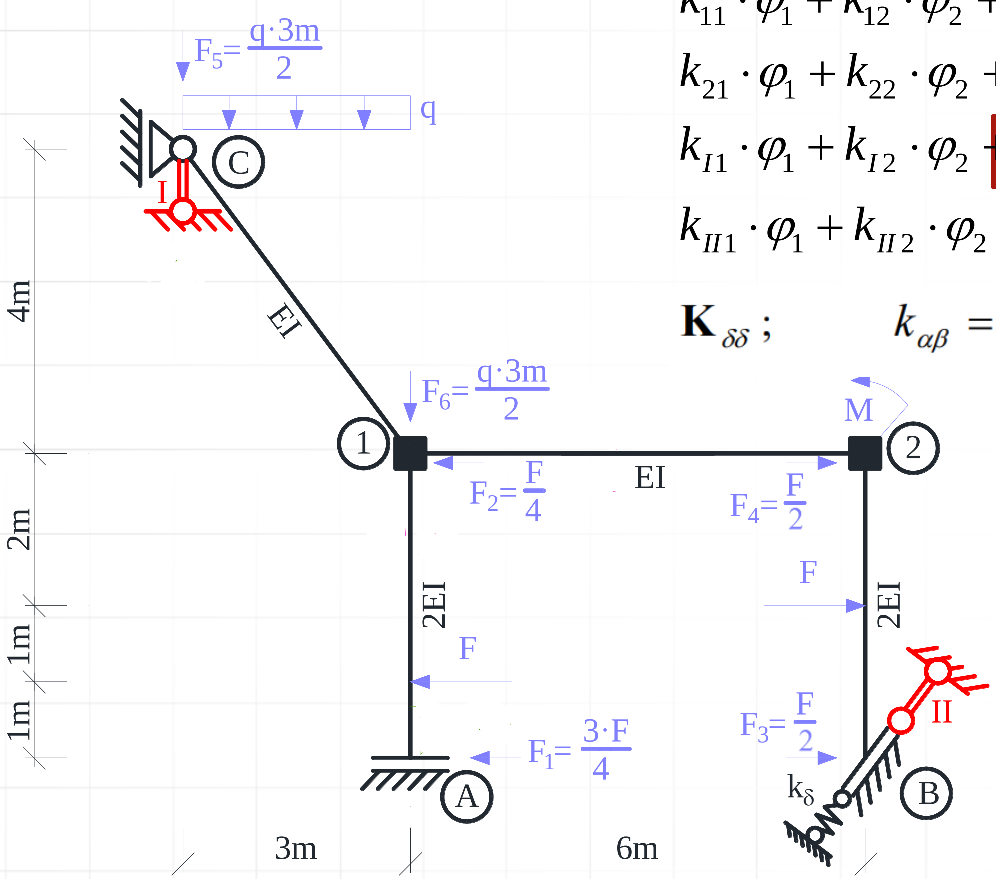
$$k_{11}\cdot\varphi_1 + k_{12}\cdot\varphi_2 + k_{1I}\cdot\delta_I + k_{1II}\cdot\delta_{II} + k_{1o} = 0$$

$$k_{21}\cdot\varphi_1 + k_{22}\cdot\varphi_2 + k_{2I}\cdot\delta_I + k_{2II}\cdot\delta_{II} + k_{2o} = 0$$

$$k_{I1}\cdot\varphi_1 + k_{I2}\cdot\varphi_2 + \boxed{k_{I\,I}\cdot\delta_I + k_{I\,II}\cdot\delta_{II}} + k_{Io} = 0$$

$$k_{II1}\cdot\varphi_1 + k_{II2}\cdot\varphi_2 + k_{II\,I}\cdot\delta_I + k_{II\,II}\cdot\delta_{II} + k_{IIo} = 0$$

$$\mathbf{K}_{\delta\delta}\;; \qquad k_{\alpha\beta} = -\sum_{ii}(M_{ij}^{\beta} + M_{ji}^{\beta})\cdot\psi_{ij}^{\alpha} + \sum_{s} k_s^{\delta}\cdot\Delta L_s^{\alpha}\cdot\Delta L_s^{\beta}$$



Rys. 2. Układ podstawowy metody przemieszczeń

Laboratorium nr 6

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$k_{I,I}=-\sum_{ij}\left(M_{ij}^{I}+M_{ji}^{I}\right)\psi_{ij}^{I}+\sum_{ij}k_{s}^{\delta}\cdot\delta_{s}^{I}\cdot\delta_{s}^{I}=-\left(M_{12}^{I}+M_{21}^{I}\right)\psi_{12}^{I}-\left(M_{1A}^{I}+M_{A1}^{I}\right)\psi_{1A}^{I}-\left(M_{1C}^{I}+M_{C1}^{I}\right)\psi_{1C}^{I}-$$

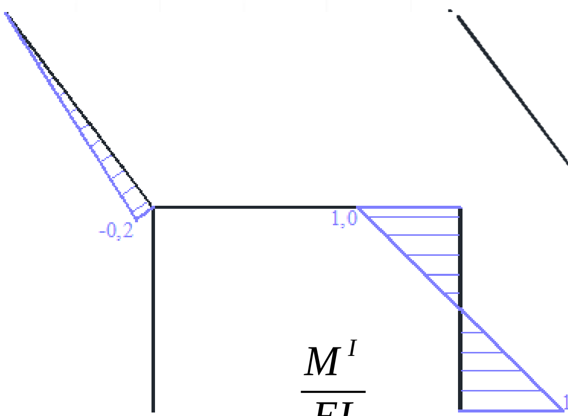
$$-\left(M_{2B}^{I}+M_{B2}^{I}\right)\psi_{2B}^{I}+k^{\delta}\cdot\delta^{I}\cdot\delta^{I}=-(0+0)\cdot 0-(0-0)\cdot 0-\left(-\frac{1}{5}\frac{EI}{m^{2}}+0\right)\cdot\frac{1}{3m}-\left(1\frac{EI}{m^{2}}+1\frac{EI}{m^{2}}\right)\cdot\left(-\frac{1}{3m}\right)+$$

$$+8\frac{EI}{m^{3}}\cdot 0\cdot 0=0,7333\frac{EI}{m^{3}},$$

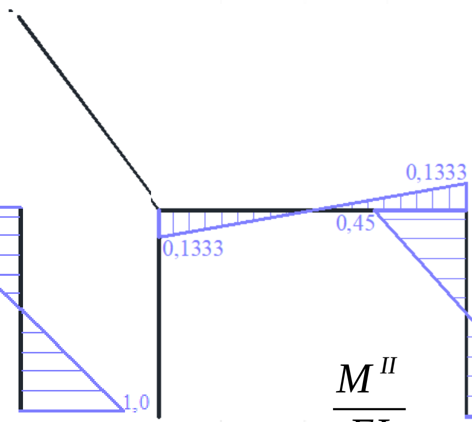
$$k_{I,II}=-\sum_{ij}\left(M_{ij}^{II}+M_{ji}^{II}\right)\psi_{ij}^{I}+\sum_{ij}k_{s}^{\delta}\cdot\delta_{s}^{I}\cdot\delta_{s}^{II}=-\left(M_{12}^{II}+M_{21}^{II}\right)\psi_{12}^{I}-\left(M_{1A}^{II}+M_{A1}^{II}\right)\psi_{1A}^{I}-\left(M_{1C}^{II}+M_{C1}^{II}\right)\psi_{1C}^{I}-$$

$$-\left(M_{2B}^{II}+M_{B2}^{II}\right)\psi_{2B}^{I}+k^{\delta}\cdot\delta^{I}\cdot\delta^{II}=-\left(\frac{2}{15}\frac{EI}{m^{2}}+\frac{2}{15}\frac{EI}{m^{2}}\right)\cdot 0-(0-0)\cdot 0-(0+0)\cdot 0-\left(\frac{9}{20}\frac{EI}{m^{2}}+\frac{9}{20}\frac{EI}{m^{2}}\right)\cdot\left(-\frac{1}{3m}\right)+$$

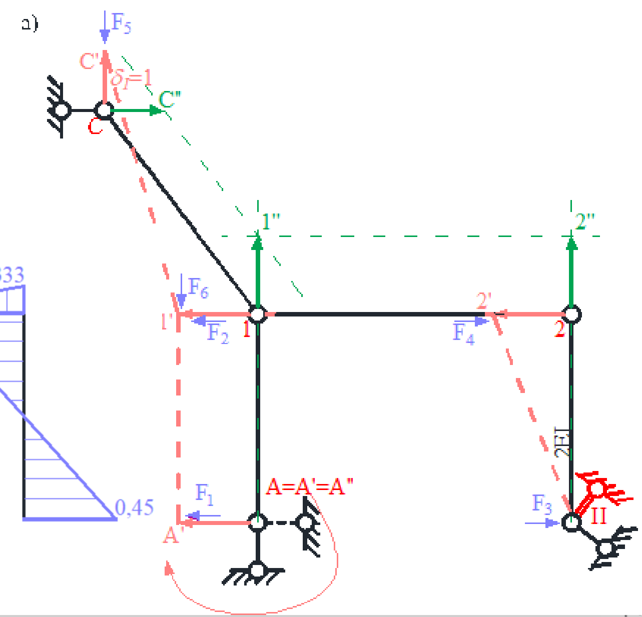
$$+8\frac{EI}{m^{3}}\cdot 0\cdot 1=0,3\frac{EI}{m^{3}},$$

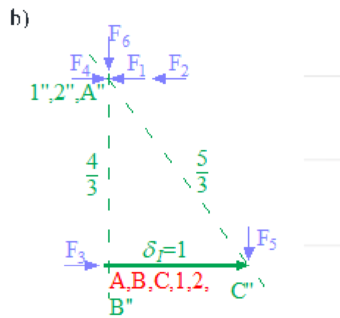


$$\frac{M^{I}}{\frac{EI}{m^{2}}}$$



$$\frac{M^{II}}{\frac{EI}{m^{2}}}$$





$$\psi_{2B}^{I}=\frac{\Delta_{2B}^{I}}{L_{2B}}=\frac{-\frac{4}{3}}{4m}=-\frac{1}{3m}$$

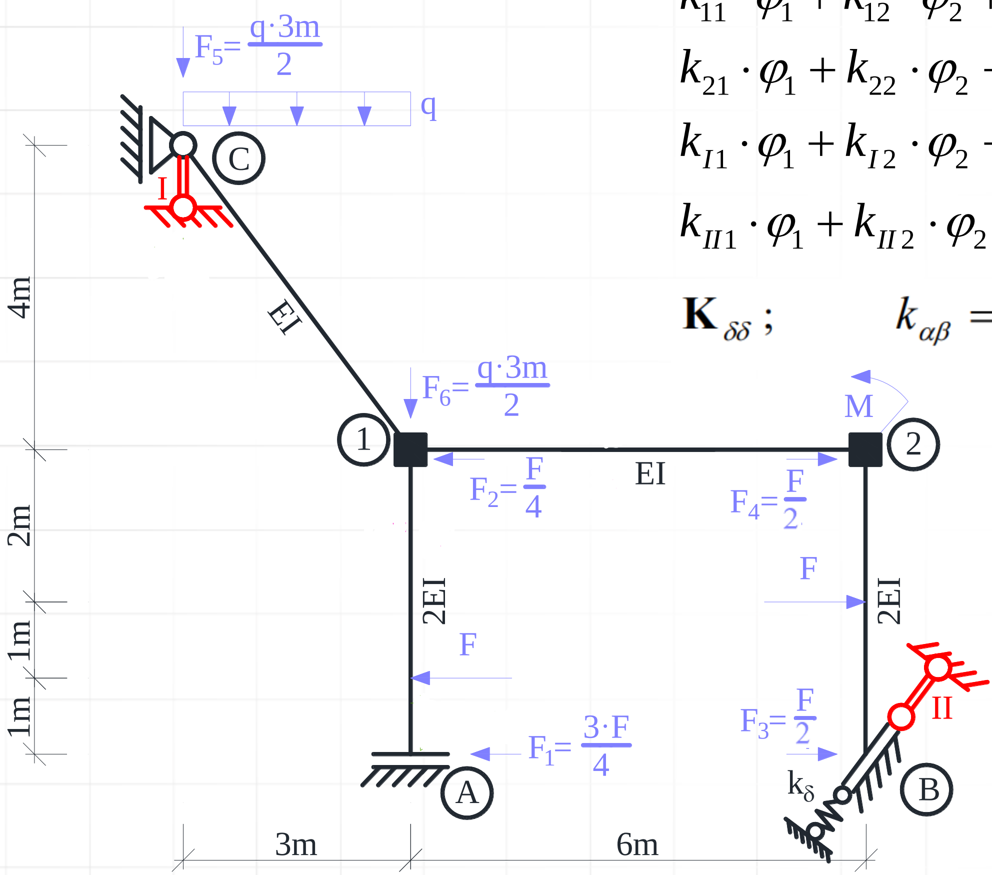
$$\psi_{1C}^{I}=\frac{\Delta_{1C}^{I}}{L_{1C}}=\frac{\frac{5}{3}}{5m}=\frac{1}{3m}$$

$$\psi_{1A}^{I}=\psi_{12}^{I}=0$$

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$k_{11}\cdot\varphi_1 + k_{12}\cdot\varphi_2 + k_{1I}\cdot\delta_I + k_{1II}\cdot\delta_{II} + k_{1o} = 0$$
$$k_{21}\cdot\varphi_1 + k_{22}\cdot\varphi_2 + k_{2I}\cdot\delta_I + k_{2II}\cdot\delta_{II} + k_{2o} = 0$$
$$k_{I1}\cdot\varphi_1 + k_{I2}\cdot\varphi_2 + k_{I\,I}\cdot\delta_I + k_{I\,II}\cdot\delta_{II} + k_{Io} = 0$$
$$k_{II1}\cdot\varphi_1 + k_{II2}\cdot\varphi_2 + \boxed{k_{II\,I}\cdot\delta_I + k_{II\,II}\cdot\delta_{II}} + k_{IIo} = 0$$

$$\mathbf{K}_{\delta\delta}\,; \qquad k_{\alpha\beta} = -\sum_{ij}\left(M_{ij}^{\beta} + M_{ji}^{\beta}\right)\cdot\psi_{ij}^{\alpha} + \sum_{s} k_s^{\delta}\cdot\Delta L_s^{\alpha}\cdot\Delta L_s^{\beta}$$



Rys. 2. Układ podstawowy metody przemieszczeń

Laboratorium nr 6

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$k_{II,I} = -\sum_{ij}\left(M^{I}_{ij}+M^{I}_{ji}\right)\psi^{II}_{ij} + \sum_{ij} k^{\delta}_{s}\cdot\delta^{II}_{s}\cdot\delta^{I}_{s} = -\left(M^{I}_{12}+M^{I}_{21}\right)\psi^{II}_{12} - \left(M^{I}_{1A}+M^{I}_{A1}\right)\psi^{II}_{1A} - \left(M^{I}_{1C}+M^{I}_{C1}\right)\psi^{II}_{1C} -$$

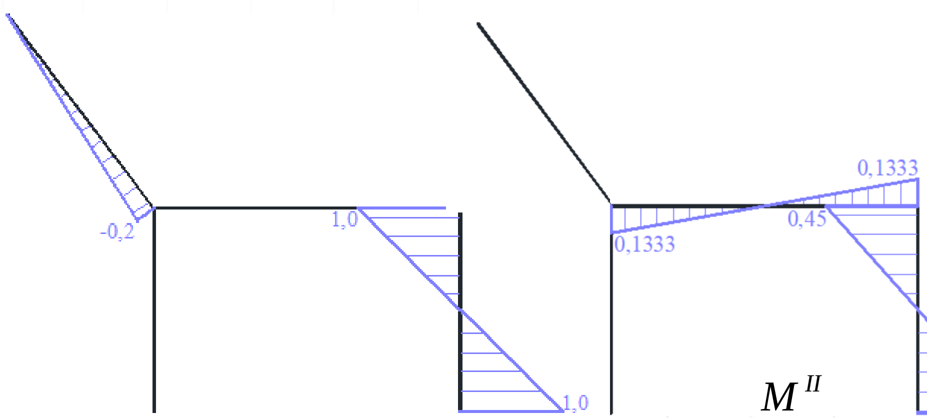
$$-\left(M^{I}_{2B}+M^{I}_{B2}\right)\psi^{II}_{2B} + k^{\delta}\cdot\delta^{II}\cdot\delta^{I} = -\left(0+0\right)\cdot\left(-\frac{4}{30m}\right) - \left(0+0\right)\cdot 0 - \left(-\frac{1}{5}\frac{EI}{m^2}+0\right)\cdot 0 - \left(1\frac{EI}{m^2}+1\frac{EI}{m^2}\right)\cdot\left(-\frac{3}{20m}\right) +$$

$$+8\frac{EI}{m^3}\cdot 1\cdot 0 = 0{,}3\frac{EI}{m^3},$$

$$k_{II,II} = -\sum_{ij}\left(M^{II}_{ij}+M^{II}_{ji}\right)\psi^{II}_{ij} + \sum_{ij} k^{\delta}_{s}\cdot\delta^{II}_{s}\cdot\delta^{II}_{s} = -\left(M^{II}_{12}+M^{II}_{21}\right)\psi^{II}_{12} - \left(M^{II}_{1A}+M^{II}_{A1}\right)\psi^{II}_{1A} - \left(M^{II}_{1C}+M^{II}_{C1}\right)\psi^{II}_{1C} -$$

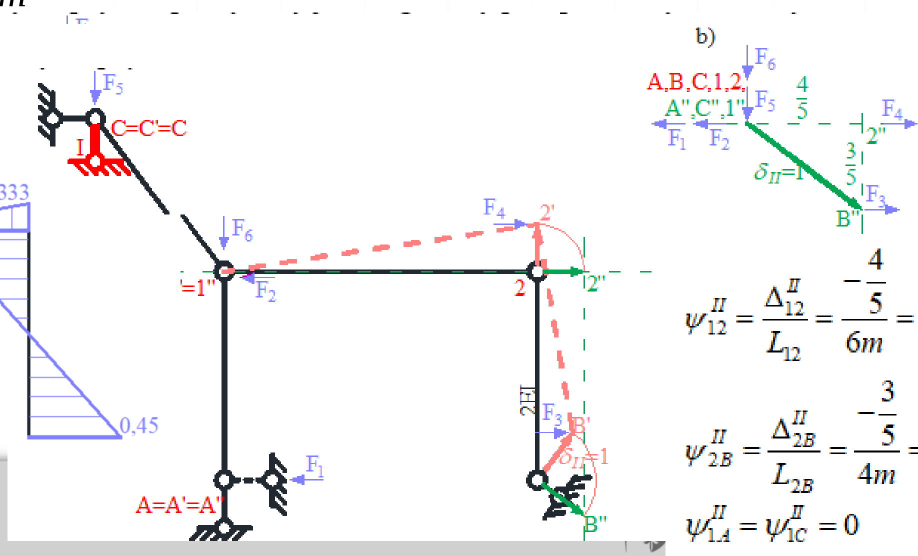
$$-\left(M^{II}_{2B}+M^{II}_{B2}\right)\psi^{II}_{2B} + k^{\delta}\cdot\delta^{II}\cdot\delta^{II} = -\left(\frac{2}{15}\frac{EI}{m^2}+\frac{2}{15}\frac{EI}{m^2}\right)\cdot\left(-\frac{4}{30m}\right) - \left(0-0\right)\cdot 0 - \left(0+0\right)\cdot 0 -$$

$$-\left(\frac{9}{20}\frac{EI}{m^2}+\frac{9}{20}\frac{EI}{m^2}\right)\cdot\left(-\frac{3}{20m}\right) + 8\frac{EI}{m^3}\cdot 1\cdot 1 = 8{,}1705\frac{EI}{m^3},$$



$\frac{EI}{m^2}$

$\frac{M^{II}}{\frac{EI}{m^2}}$



$$\psi^{II}_{12} = \frac{\Delta^{II}_{12}}{L_{12}} = \frac{-\frac{4}{5}}{6m} = -\frac{2}{15m}$$

$$\psi^{II}_{2B} = \frac{\Delta^{II}_{2B}}{L_{2B}} = \frac{-\frac{3}{5}}{4m} = -\frac{3}{20m}$$

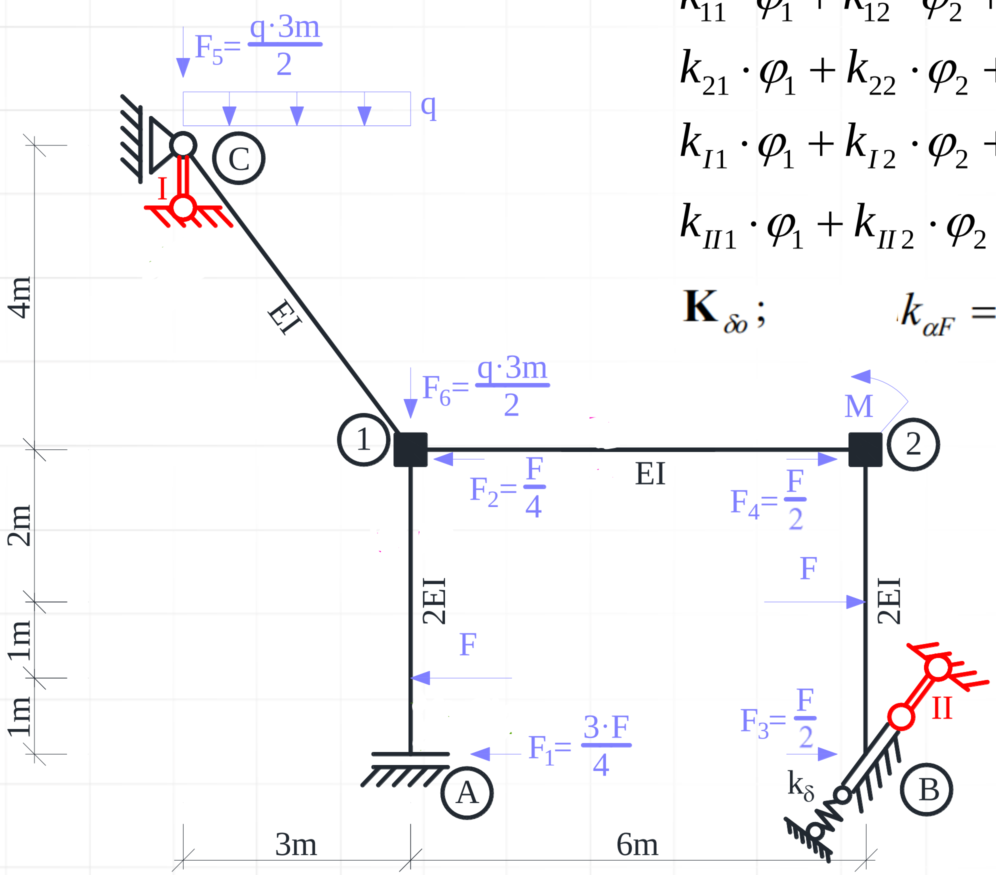
$$\psi^{II}_{1A} = \psi^{II}_{1C} = 0$$

Laboratorium nr 6

WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$k_{11}\cdot\varphi_1 + k_{12}\cdot\varphi_2 + k_{1I}\cdot\delta_I + k_{1II}\cdot\delta_{II} + k_{1o} = 0$$
$$k_{21}\cdot\varphi_1 + k_{22}\cdot\varphi_2 + k_{2I}\cdot\delta_I + k_{2II}\cdot\delta_{II} + k_{2o} = 0$$
$$k_{I1}\cdot\varphi_1 + k_{I2}\cdot\varphi_2 + k_{I\,I}\cdot\delta_I + k_{I\,II}\cdot\delta_{II} + \boxed{k_{Io}} = 0$$
$$k_{II1}\cdot\varphi_1 + k_{II2}\cdot\varphi_2 + k_{II\,I}\cdot\delta_I + k_{II\,II}\cdot\delta_{II} + \boxed{k_{IIo}} = 0$$

$\mathbf{K}_{\delta o}$; $\quad k_{\alpha F} = -\sum (M_{ij}^{oF} + M_{ji}^{oF})\cdot\psi_{ij}^{\alpha} - \sum F_f \cdot \delta_f^{\alpha}$



Rys. 2. Układ podstawowy metody przemieszczeń

Laboratorium nr 6

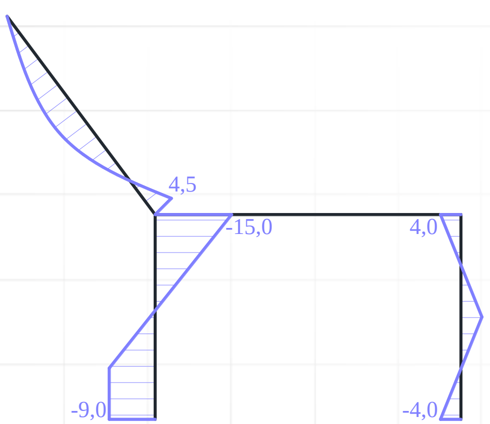
WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$k_{Io} = -\sum_{ij}\left(M^o_{ij}+M^o_{ji}\right)\psi^I_{ij} - \sum_{P} P_P\cdot\delta^I_P = -\left(M^o_{12}+M^o_{21}\right)\psi^I_{12} - \left(M^o_{1A}+M^o_{A1}\right)\psi^I_{1A} - \left(M^o_{1C}+M^o_{C1}\right)\psi^I_{1C} -$$

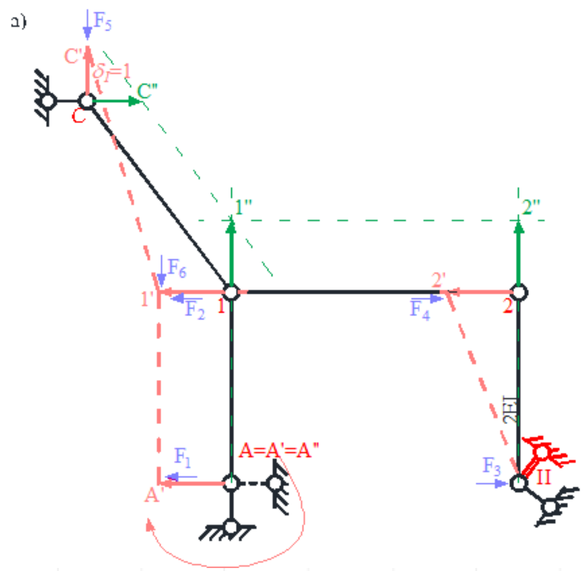
$$-\left(M^o_{2B}+M^o_{B2}\right)\psi^I_{2B} - P_1\cdot\delta^I_1 - P_2\cdot\delta^I_2 - P_3\cdot\delta^I_3 - P_4\cdot\delta^I_4 - P_5\cdot\delta^I_5 - P_6\cdot\delta^I_6 = -(0+0)\cdot 0 -$$

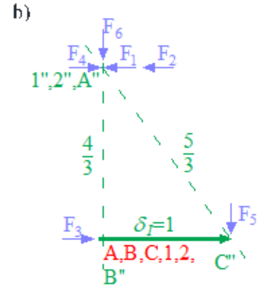
$$-(-15kN\cdot m - 9kN\cdot m)\cdot 0 - (0+4,5kN\cdot m)\cdot\frac{1}{3m} - (4kN\cdot m - 4kN\cdot m)\cdot(-\frac{1}{3m}) - 6kN\cdot\frac{4}{3} - 2kN\cdot\frac{4}{3} -$$

$$-4kN\cdot 0 - 4kN\cdot(-\frac{4}{3}) - 6kN\cdot(-1) - 6kN\cdot 0 = -0,8333kN,$$



$$\frac{M^0}{kN\cdot m}$$





$$\psi^I_{2B} = \frac{\Delta^I_{2B}}{L_{2B}} = \frac{-\frac{4}{3}}{4m} = -\frac{1}{3m}$$

$$\psi^I_{1C} = \frac{\Delta^I_{1C}}{L_{1C}} = \frac{\frac{5}{3}}{5m} = \frac{1}{3m}$$

$$\psi^I_{1A} = \psi^I_{12} = 0$$

Przesunięcia w miejscach sił równoważnych:

$\delta^I_1 = \frac{4}{3}$, $\delta^I_2 = \frac{4}{3}$, $\delta^I_3 = 0$, $\delta^I_4 = -\frac{4}{3}$, ,$\delta^I_5 = -1$, $\delta^I_6 = 0$.

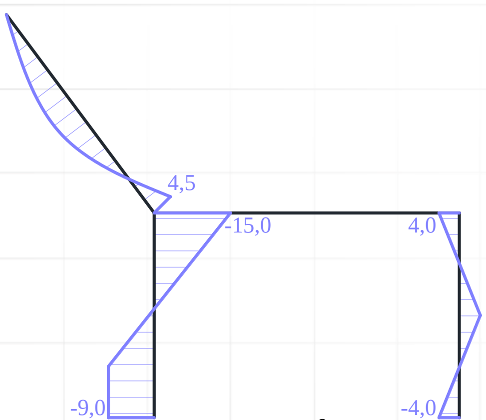
WSPÓŁCZYNNIKI UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ

$$k_{IIo}=-\sum_{ij}\left(M_{ij}^{o}+M_{ji}^{o}\right)\psi_{ij}^{II}-\sum_{P}P_P\cdot\delta_P^{II}=-\left(M_{12}^{o}+M_{21}^{o}\right)\psi_{12}^{II}-\left(M_{1A}^{o}+M_{A1}^{o}\right)\psi_{1A}^{II}-\left(M_{1C}^{o}+M_{C1}^{o}\right)\psi_{1C}^{II}-$$

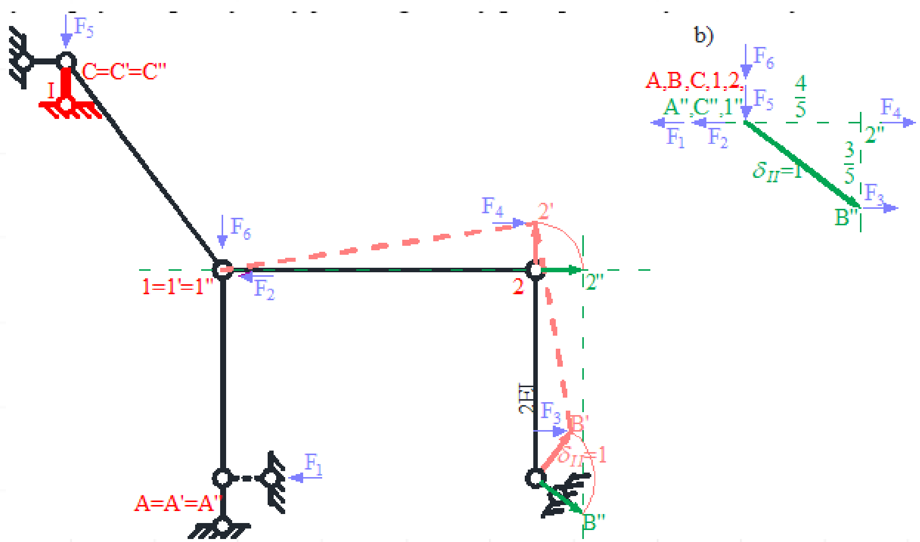
$$-\left(M_{2B}^{o}+M_{B2}^{o}\right)\psi_{2B}^{II}-P_1\cdot\delta_1^{II}-P_2\cdot\delta_2^{II}-P_3\cdot\delta_3^{II}-P_4\cdot\delta_4^{II}-P_5\cdot\delta_5^{II}-P_6\cdot\delta_6^{II}=-(0+0)\cdot(-\frac{4}{30m})-$$

$$-\left(-15kN\cdot m-9kN\cdot m\right)\cdot 0-\left(0+4,5kN\cdot m\right)\cdot 0-\left(4kN\cdot m-4kN\cdot m\right)\cdot(-\frac{3}{20m})-6kN\cdot 0-2kN\cdot 0-$$

$$-4kN\cdot\frac{3}{5}-4kN\cdot 0-6kN\cdot 0-6kN\cdot 0=-2,4kN.$$



$$\frac{M^0}{kN\cdot m}$$



$$\psi_{12}^{II}=\frac{\Delta_{12}^{II}}{L_{12}}=\frac{-\frac{4}{5}}{6m}=-\frac{2}{15m}$$

$$\psi_{2B}^{II}=\frac{\Delta_{2B}^{II}}{L_{2B}}=\frac{-\frac{3}{5}}{4m}=-\frac{3}{20m}$$

$$\psi_{1A}^{II}=\psi_{1C}^{II}=0$$

Przesunięcia w miejscach sił równoważnych:

$$\delta_1^{II}=0,\qquad \delta_2^{II}=0,\qquad \delta_3^{II}=\frac{3}{5},\qquad \delta_4^{II}=0,\qquad \delta_5^{II}=0,\qquad \delta_6^{II}=0$$

Laboratorium nr 6

SZCZEGÓŁOWA POSTAĆ UKŁADU RÓWNAŃ METODY PRZEMIESZCZEŃ I JEGO ROZWIĄZANIE

$$1,7667\frac{EI}{m}\cdot\varphi_1+0,3333\frac{EI}{m}\cdot\varphi_2-0,2\frac{EI}{m^2}\cdot\delta_I+\frac{2}{15}\frac{EI}{m^2}\cdot\delta_{II}-10,5kN\cdot m=0$$

$$0,3333\frac{EI}{m}\cdot\varphi_1+2,66671\frac{EI}{m}\cdot\varphi_2+1\frac{EI}{m^2}\cdot\delta_I+0,5833\frac{EI}{m^2}\cdot\delta_{II}+24kN\cdot m=0$$

$$-0,2\frac{EI}{m^2}\cdot\varphi_1+1\frac{EI}{m^2}\cdot\varphi_2+0,7333\frac{EI}{m^3}\cdot\delta_I+0,3\frac{EI}{m^3}\cdot\delta_{II}-0,8333kN=0$$

$$\frac{2}{15}\frac{EI}{m^2}\cdot\varphi_1+0,5833\frac{EI}{m^2}\cdot\varphi_2+0,3\frac{EI}{m^3}\cdot\delta_I+8,1705\frac{EI}{m^3}\cdot\delta_{II}-2,4kN=0$$

$$\varphi_1=15,6360\frac{kN\cdot m^2}{EI},\qquad \varphi_2=-26,6186\frac{kN\cdot m^2}{EI},$$

$$\delta_I=41,5294\frac{kN\cdot m^3}{EI},\qquad \delta_{II}=0,4142\frac{kN\cdot m^3}{EI}.$$

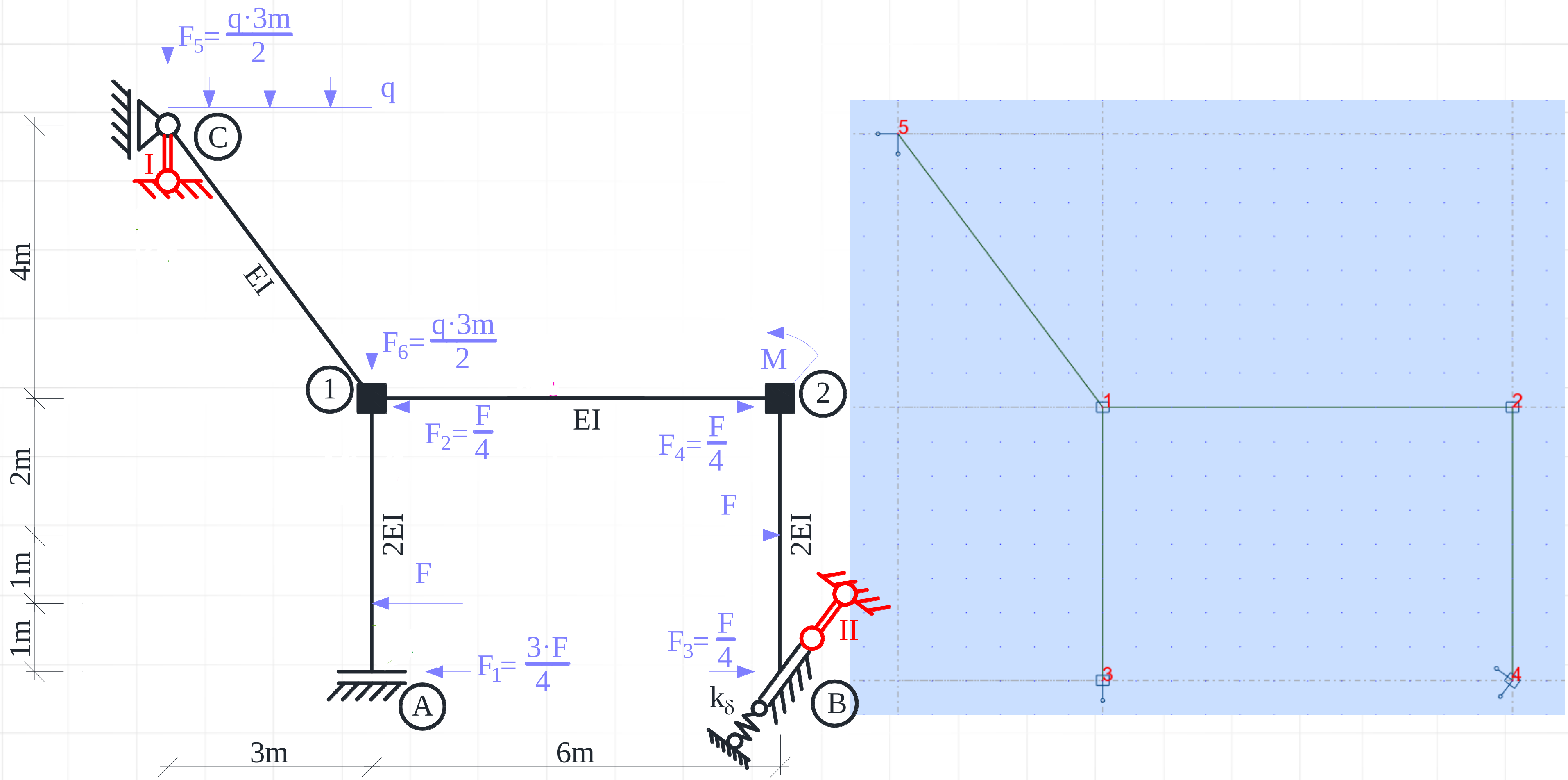
Zadanie domowe (projekt nr 2)

1. Sprawdzenie SW i GN układu
2. Dobranie układu podstawowego metody przemieszczeń
3. Rozwiązanie układu podstawowego metody przemieszczeń od składowych stanów obciążeń
4. Obliczenie współczynników układu równań metody przemieszczeń.

Laboratorium nr 6

UKŁAD WPROWADZANY DO PROGRAMU ROBOT-UKŁAD PODSTAWOWY

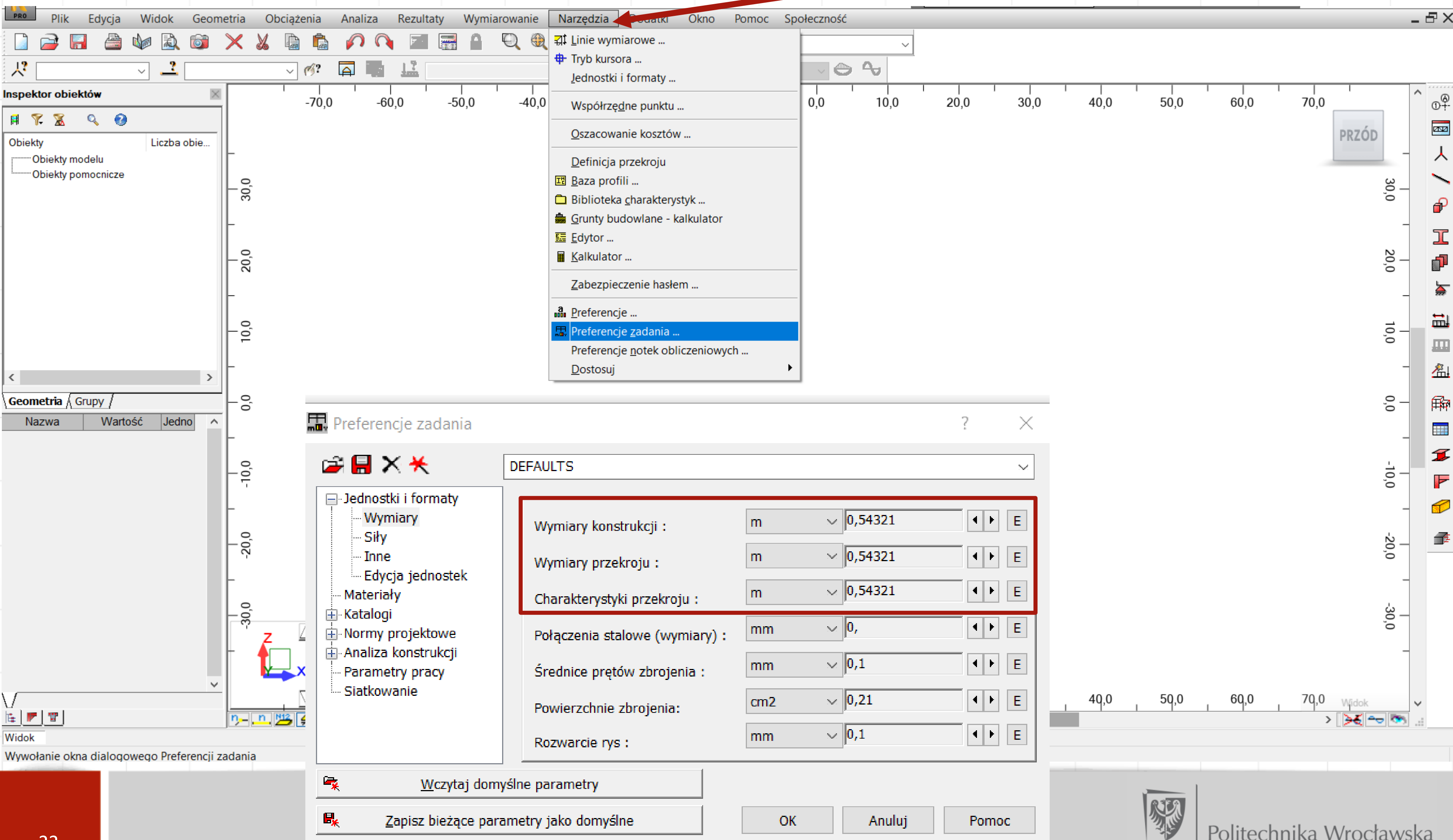
Do programu ROBOT wprowadzamy układ podstawowy metody przemieszczeń.



Laboratorium nr 6

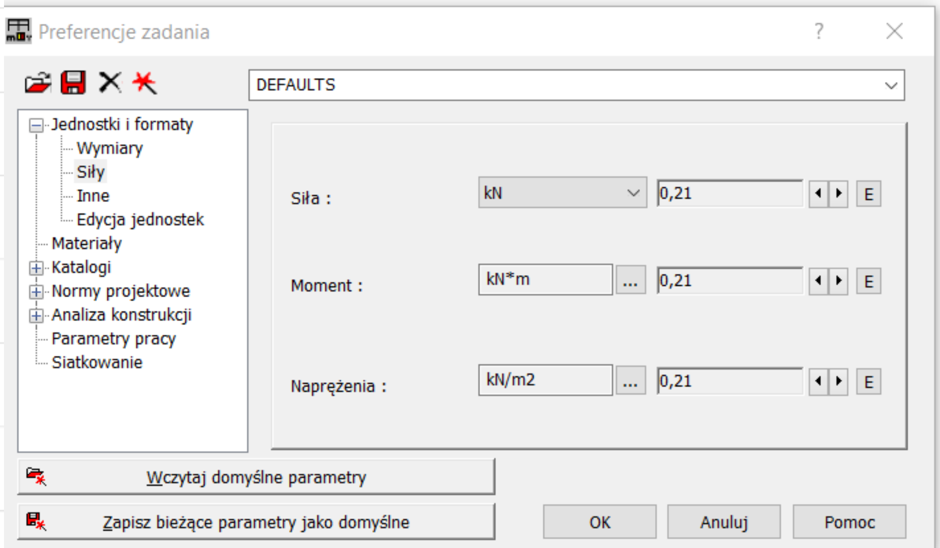
1. USTAWIENIE PREFERENCJI ZADANIA

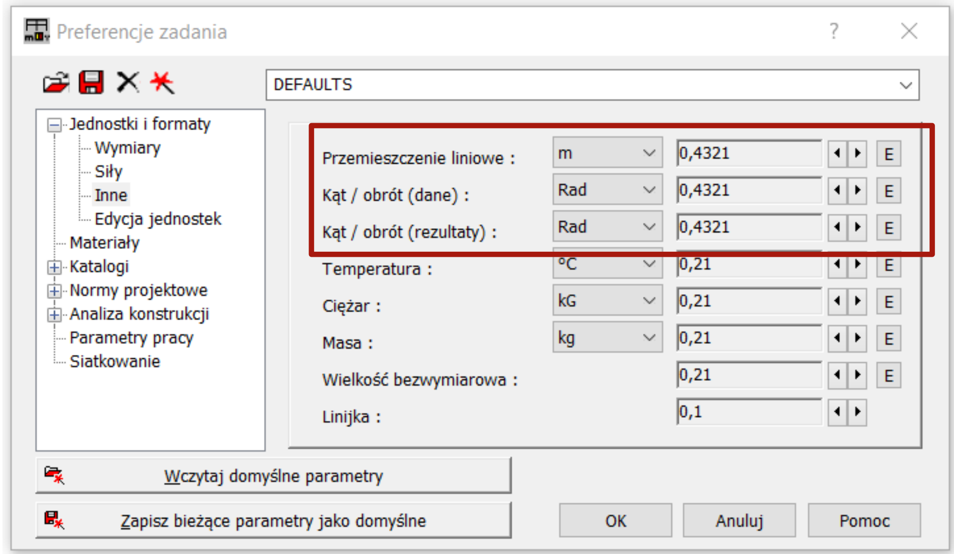
1. Wybieramy Narzędzia->Preferencje zadania

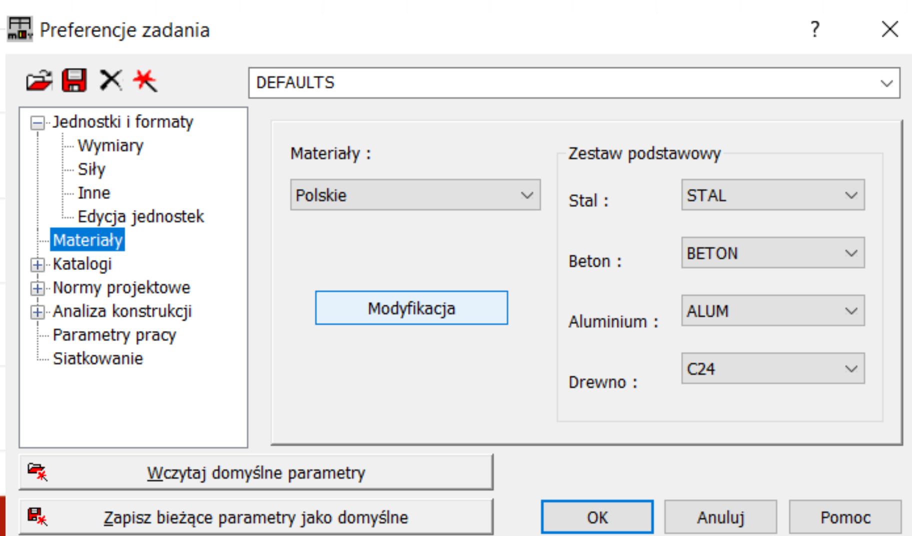


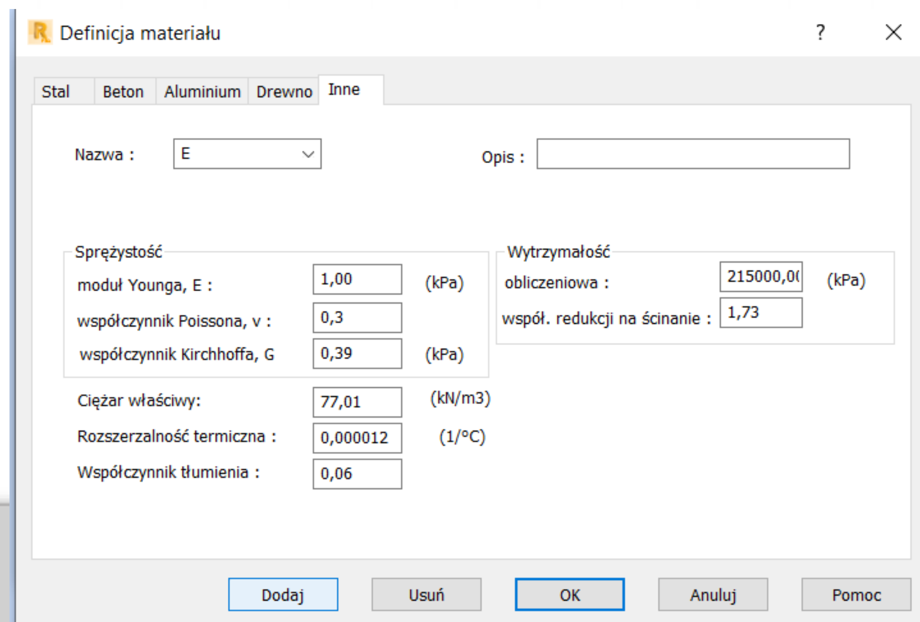
Laboratorium nr 6

1. USTAWIENIE PREFERENCJI ZADANIA









UKŁAD WPROWADZANY DO PROGRAMU ROBOT-UKŁAD PODSTAWOWY

Zdefiniowanie przekroju:

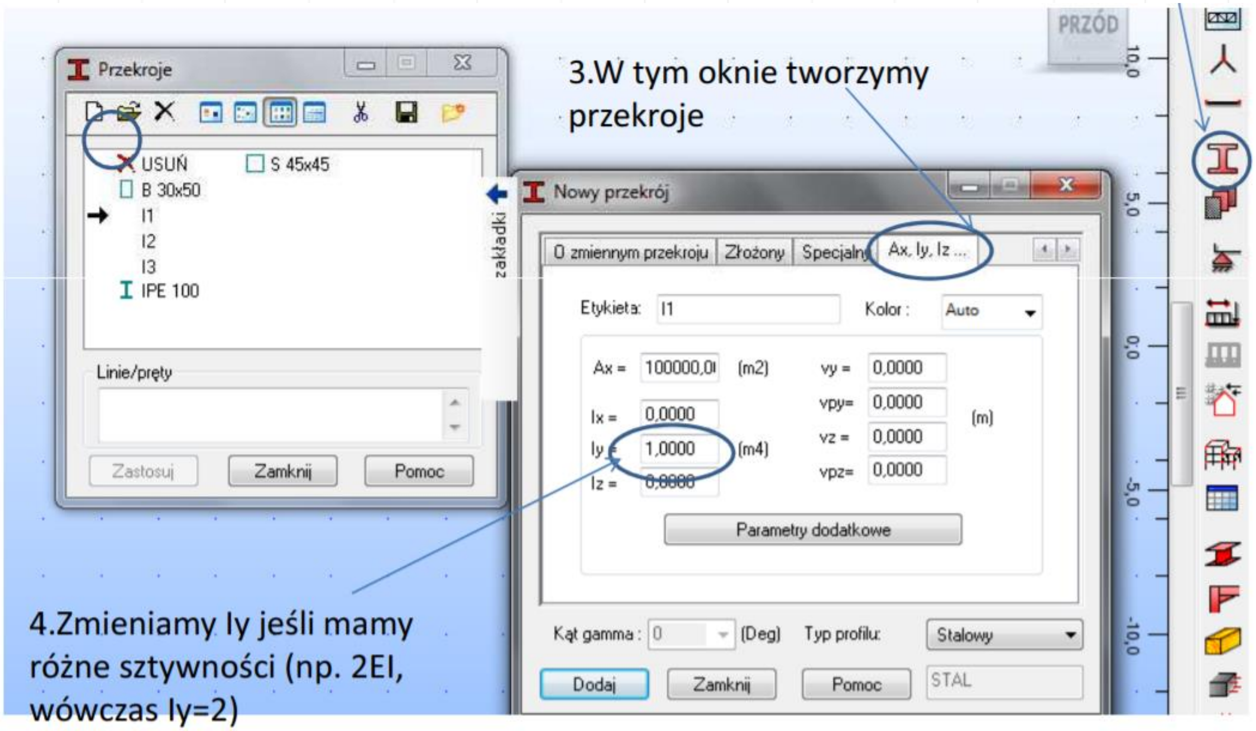
A_x=1 000 000 m^2

I_y=1m^4 (dla 1 EI)

1m^4 (dla 1 EI)

I_x=0

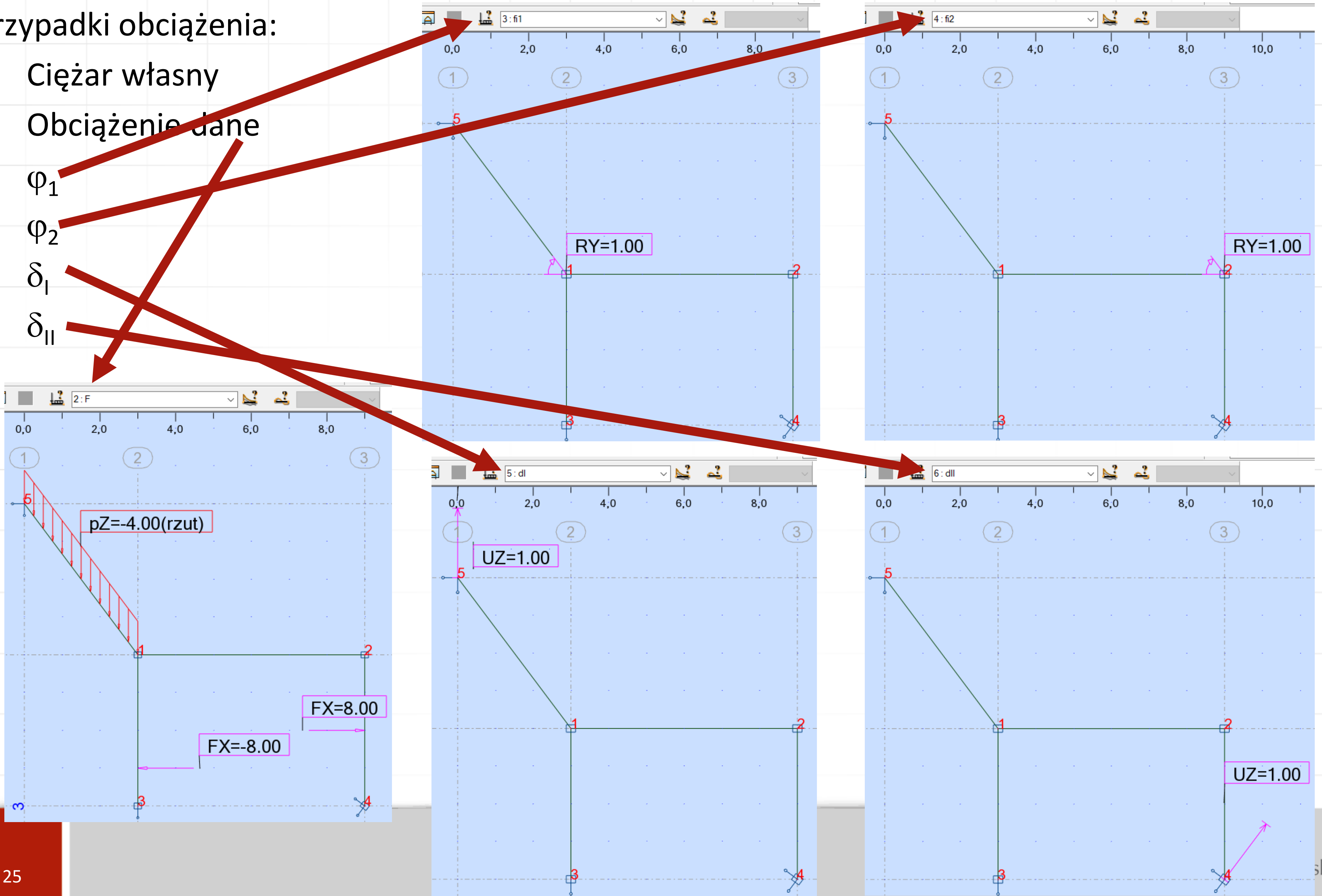
I_z=0



UKŁAD WPROWADZANY DO PROGRAMU ROBOT-UKŁAD PODSTAWOWY

Przypadki obciążenia:

1. Ciężar własny
2. Obciążenie dane
3. φ_1
4. φ_2
5. δ_I
6. δ_{II}



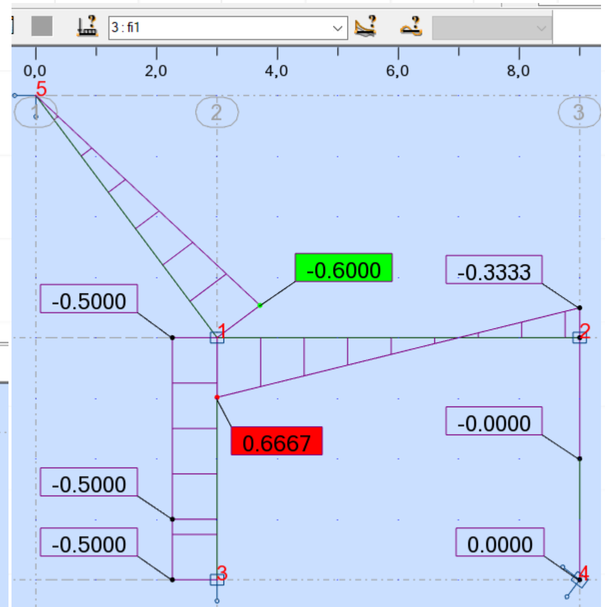
UKŁAD WPROWADZANY DO PROGRAMU ROBOT-UKŁAD PODSTAWOWY

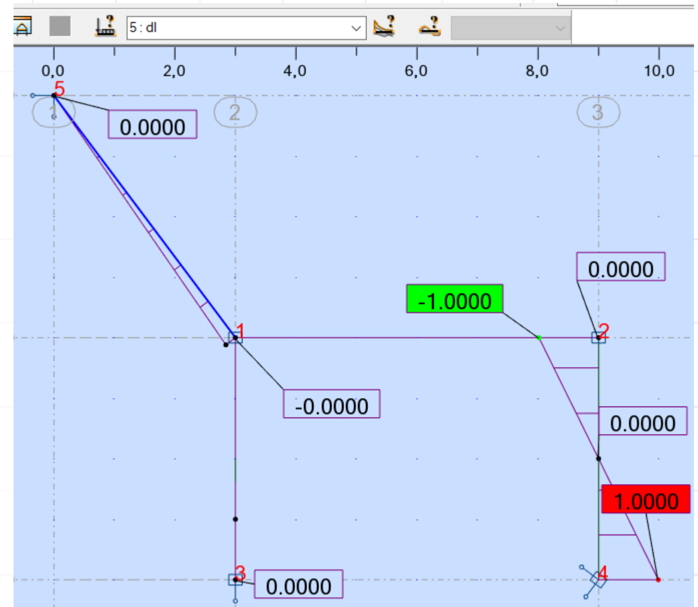
Wykresy momentów

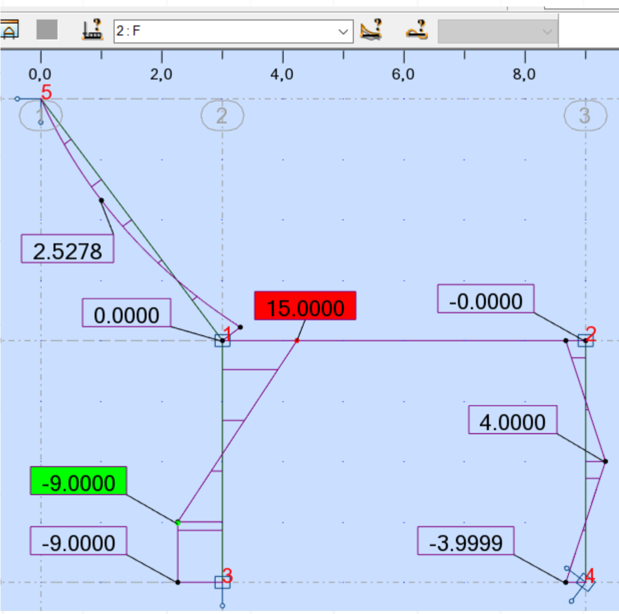
Zginających w układzie

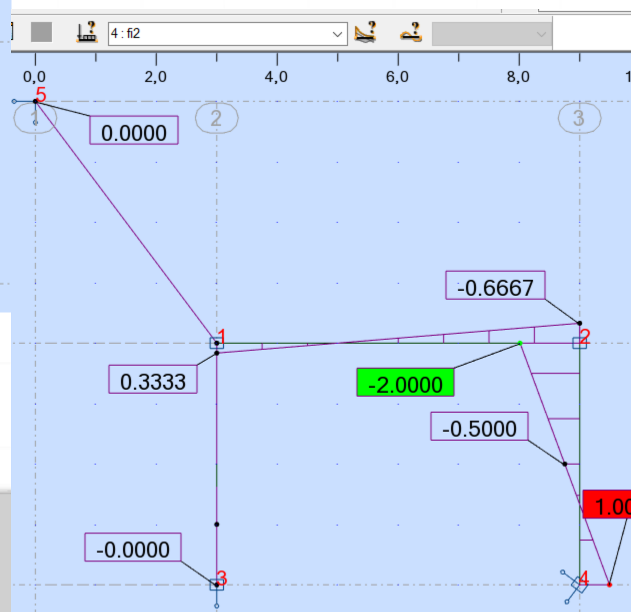
podstawowym metody

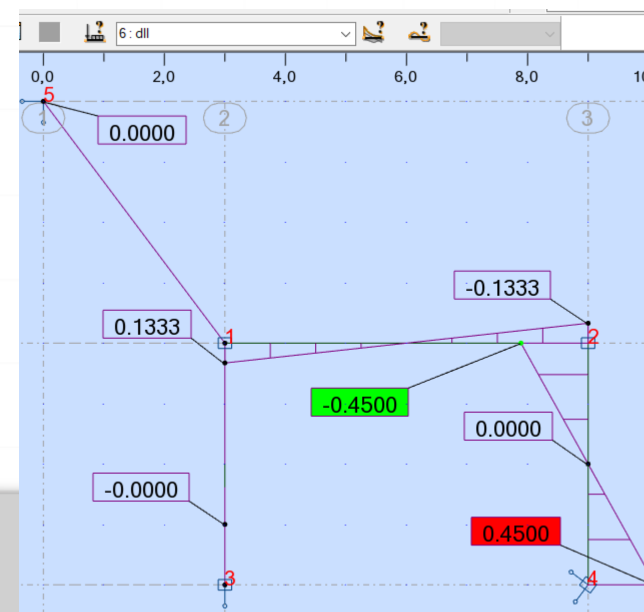
przemieszczeń







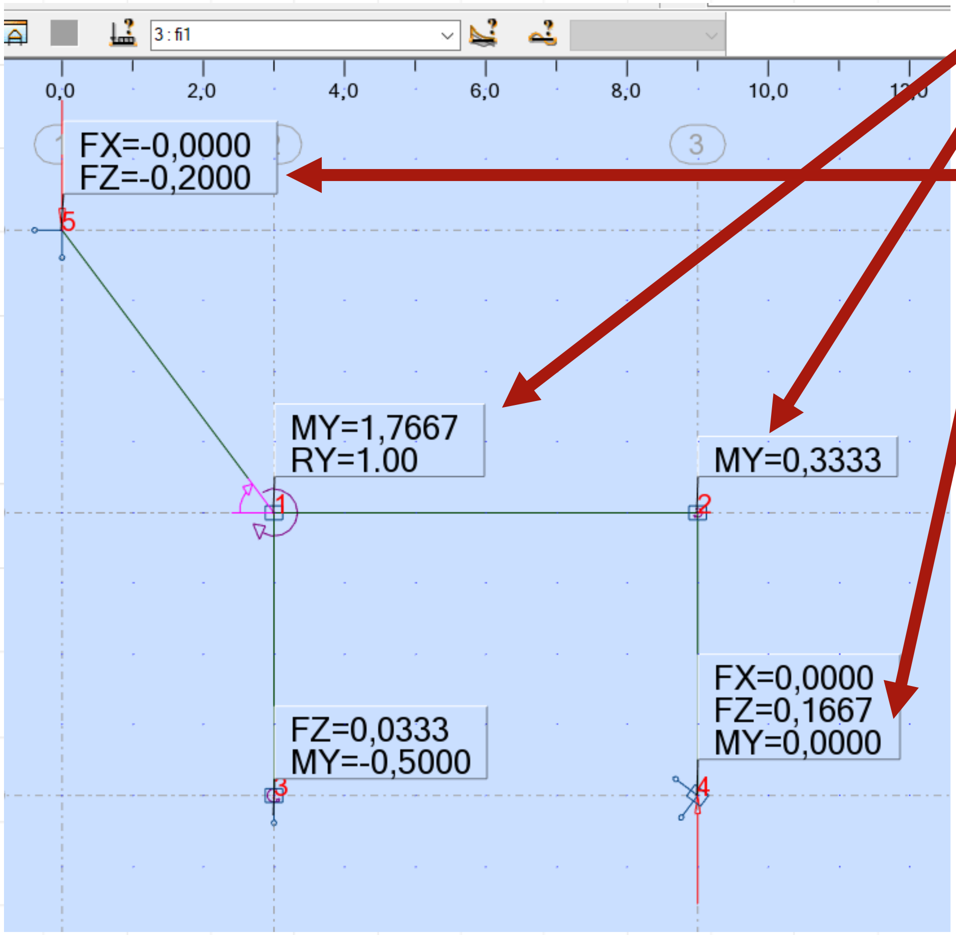




Laboratorium nr 6

UKŁAD WPROWADZANY DO PROGRAMU ROBOT-UKŁAD PODSTAWOWY

Odczytywanie współczynników: reakcje w dodanych więziach.



$$\boxed{k_{11}} \cdot \varphi_1 + k_{12} \cdot \varphi_2 + k_{1I} \cdot \delta_I + k_{1II} \cdot \delta_{II} + k_{1o} = 0$$

$$\boxed{k_{21}} \cdot \varphi_1 + k_{22} \cdot \varphi_2 + k_{2I} \cdot \delta_I + k_{2II} \cdot \delta_{II} + k_{2o} = 0$$

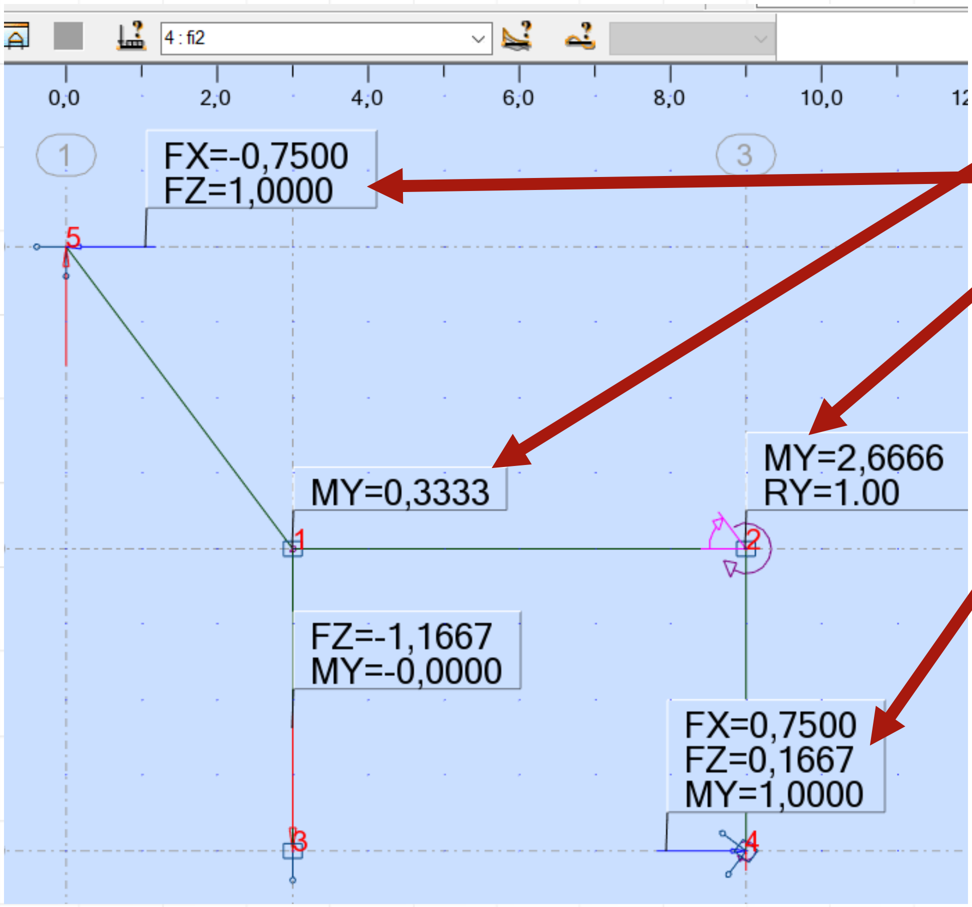
$$\boxed{k_{I1}} \cdot \varphi_1 + k_{I2} \cdot \varphi_2 + k_{I\,I} \cdot \delta_I + k_{I\,II} \cdot \delta_{II} + k_{Io} = 0$$

$$\boxed{k_{II1}} \cdot \varphi_1 + k_{II2} \cdot \varphi_2 + k_{II\,I} \cdot \delta_I + k_{II\,II} \cdot \delta_{II} + k_{IIo} = 0$$

$$k_{I1} = FX \cdot \sin\alpha + FZ \cdot \cos\alpha =$$

$$0 \cdot 0{,}6 + 0{,}1667m \cdot 0{,}8 = 0{,}1334m$$

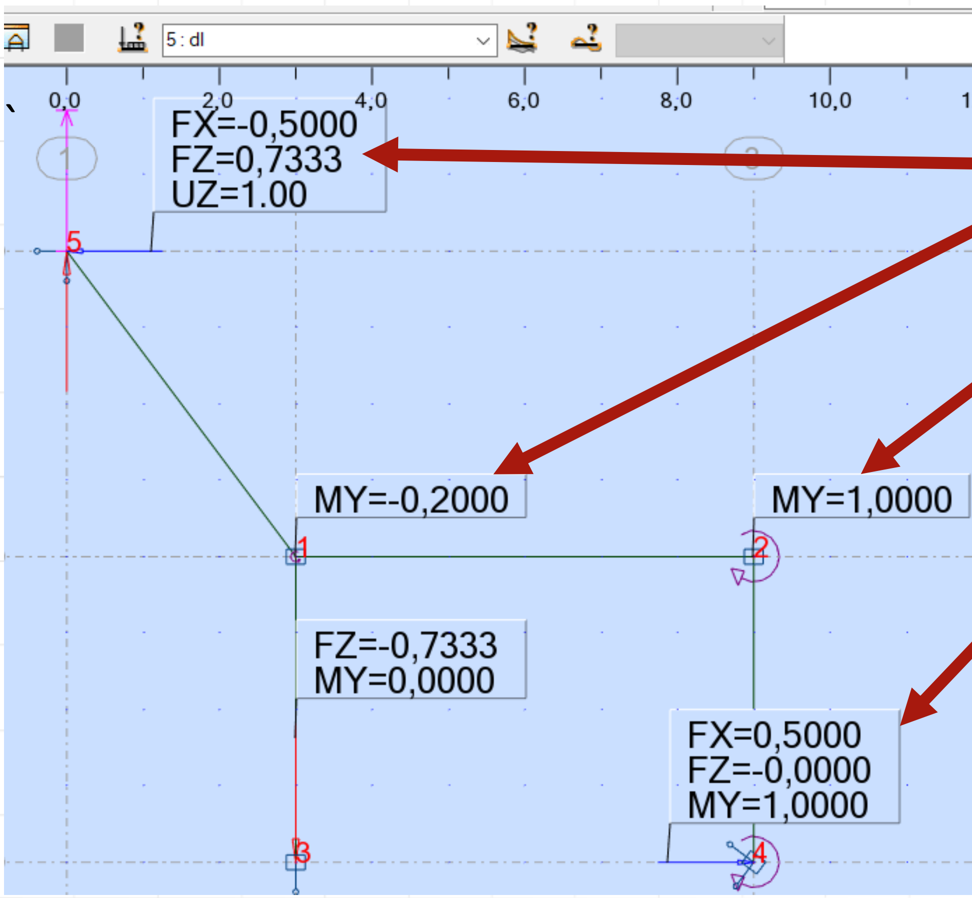
KONTROLA KINEMATYCZNA ROZWIĄZANIA



$$k_{11}\cdot\varphi_1+k_{12}\cdot\varphi_2+k_{1I}\cdot\delta_I+k_{1II}\cdot\delta_{II}+k_{1o}=0$$
$$k_{21}\cdot\varphi_1+k_{22}\cdot\varphi_2+k_{2I}\cdot\delta_I+k_{2II}\cdot\delta_{II}+k_{2o}=0$$
$$k_{I1}\cdot\varphi_1+k_{I2}\cdot\varphi_2+k_{I\,I}\cdot\delta_I+k_{I\,II}\cdot\delta_{II}+k_{Io}=0$$
$$k_{II1}\cdot\varphi_1+k_{II2}\cdot\varphi_2+k_{II\,I}\cdot\delta_I+k_{II\,II}\cdot\delta_{II}+k_{IIo}=0$$

$$k_{II2}=FX\cdot\sin\alpha+FZ\cdot\cos\alpha=$$
$$0,75m\cdot0,6+0,1667m\cdot0,8=0,5834m$$

KONTROLA KINEMATYCZNA ROZWIĄZANIA



$$k_{11}\cdot\varphi_1 + k_{12}\cdot\varphi_2 + k_{1I}\cdot\delta_I + k_{1II}\cdot\delta_{II} + k_{1o} = 0$$

$$k_{21}\cdot\varphi_1 + k_{22}\cdot\varphi_2 + k_{2I}\cdot\delta_I + k_{2II}\cdot\delta_{II} + k_{2o} = 0$$

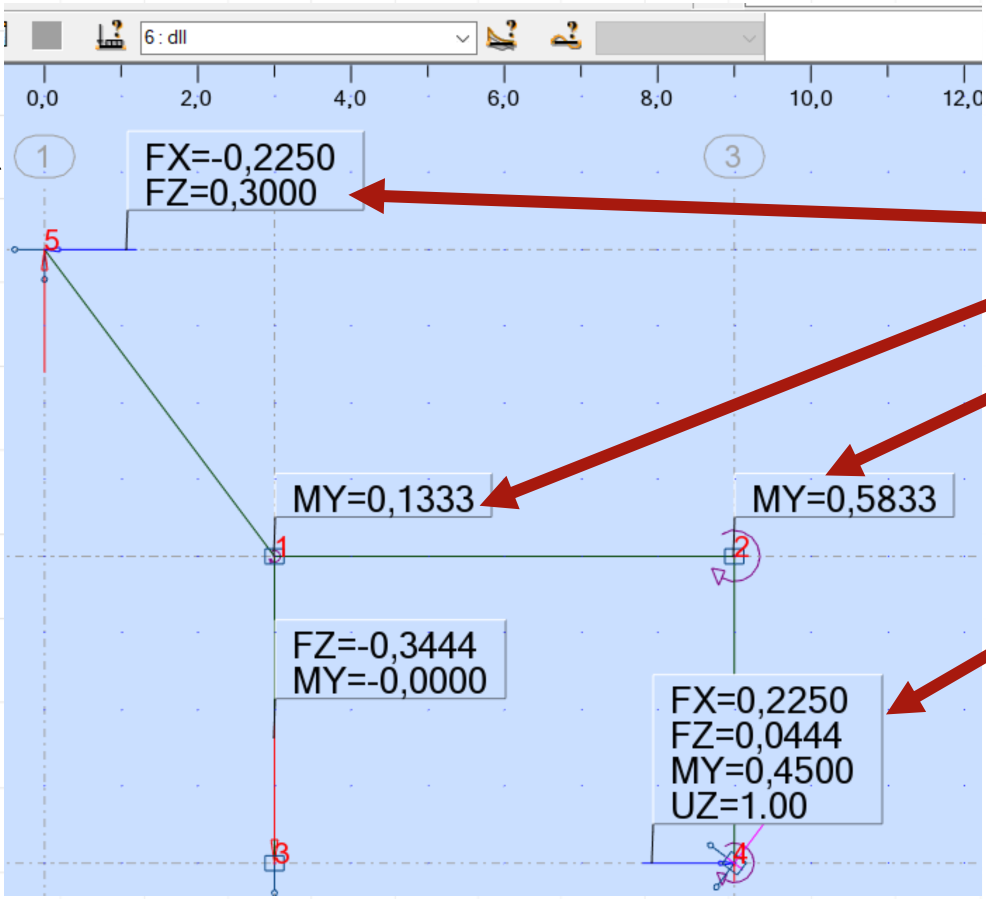
$$k_{I1}\cdot\varphi_1 + k_{I2}\cdot\varphi_2 + k_{I\,I}\cdot\delta_I + k_{I\,II}\cdot\delta_{II} + k_{Io} = 0$$

$$k_{II1}\cdot\varphi_1 + k_{II2}\cdot\varphi_2 + k_{II\,I}\cdot\delta_I + k_{II\,II}\cdot\delta_{II} + k_{IIo} = 0$$

$$k_{II\,I} = FX\cdot\sin\alpha + FZ\cdot\cos\alpha =$$

$$0,5m\cdot 0,6 + 0\cdot 0,8 = 0,3m$$

KONTROLA KINEMATYCZNA ROZWIĄZANIA



$$k_{11}\cdot\varphi_1 + k_{12}\cdot\varphi_2 + k_{1I}\cdot\delta_I + \boxed{k_{1II}\cdot\delta_{II}} + k_{1o} = 0$$

$$k_{21}\cdot\varphi_1 + k_{22}\cdot\varphi_2 + k_{2I}\cdot\delta_I + \boxed{k_{2II}\cdot\delta_{II}} + k_{2o} = 0$$

$$k_{I1}\cdot\varphi_1 + k_{I2}\cdot\varphi_2 + k_{I\,I}\cdot\delta_I + \boxed{k_{I\,II}\cdot\delta_{II}} + k_{Io} = 0$$

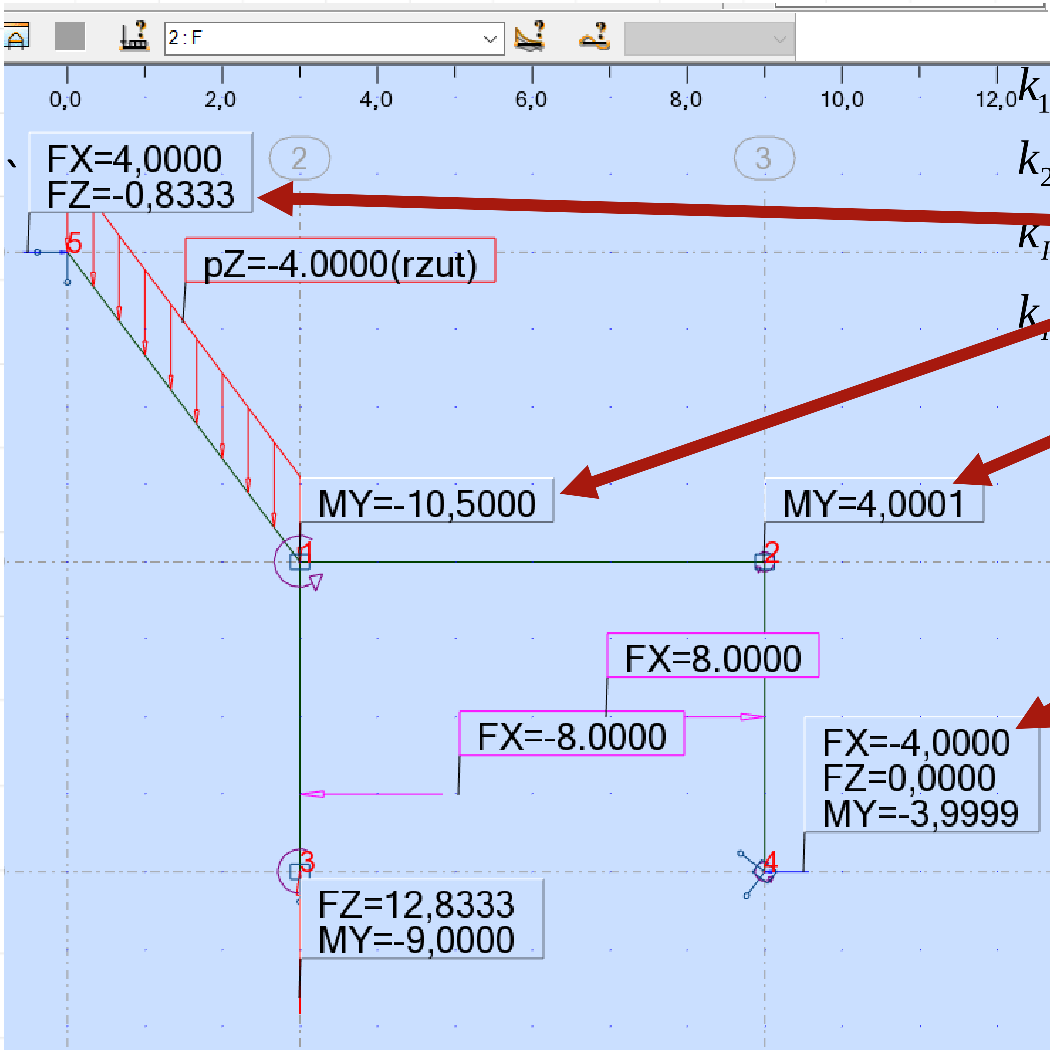
$$k_{II1}\cdot\varphi_1 + k_{II2}\cdot\varphi_2 + k_{II\,I}\cdot\delta_I + \boxed{k_{II\,II}}\cdot\delta_{II} + k_{IIo} = 0$$

$$k_{II\,II} = FX\cdot\sin\alpha + FZ\cdot\cos\alpha =$$

$$0,225m\cdot 0,6 + 0,0444m\cdot 0,8 = 0,1705m$$

Laboratorium nr 6

KONTROLA KINEMATYCZNA ROZWIĄZANIA



$$k_{11}\cdot\varphi_1 + k_{12}\cdot\varphi_2 + k_{1I}\cdot\delta_I + k_{1II}\cdot\delta_{II} + k_{1o} = 0$$

$$k_{21}\cdot\varphi_1 + k_{22}\cdot\varphi_2 + k_{2I}\cdot\delta_I + k_{2II}\cdot\delta_{II} + k_{2o} = 0$$

$$k_{I1}\cdot\varphi_1 + k_{I2}\cdot\varphi_2 + k_{I\,I}\cdot\delta_I + k_{I\,II}\cdot\delta_{II} + k_{Io} = 0$$

$$k_{II1}\cdot\varphi_1 + k_{II2}\cdot\varphi_2 + k_{II\,I}\cdot\delta_I + k_{II\,II}\cdot\delta_{II} + k_{IIo} = 0$$

$$k_{II\,II} = FX\cdot\sin\alpha + FZ\cdot\cos\alpha =$$

$$-4m\cdot 0,6 + 0\cdot 0,8 = -2,4m$$