

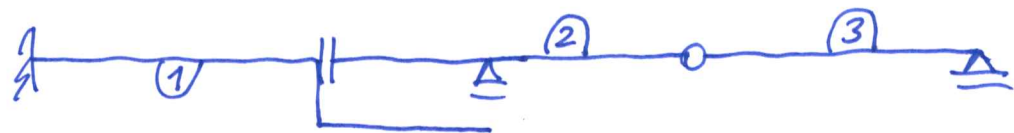
1. Spr GN i SW

* war. ilościowy

$$n = e - 3t, \quad e = 9, \quad t = 3$$

$$n = 9 - 3 \cdot 3 = 0$$

* war. jakościowy

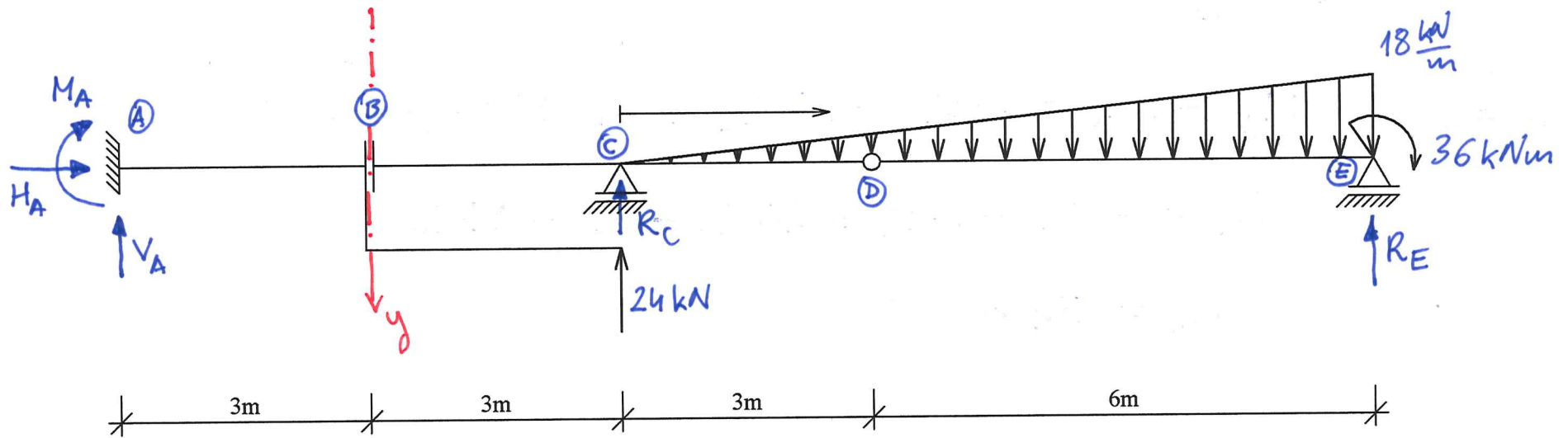


Z tw o 2 tarcele (1) GN

Z tw o 2 tarcele (2) GN

Z tw o 2 tarcele (3) GN

Układ jest geometrycznie niewzruszony i statycznie wyznaczalny ($n=0$)



2. Reakcje

War. konstrukcyjny $\sum y_L^B = 0$ $-V_A - 24 = 0$, $V_A = -24 \text{ kN}$

War. konstrukcyjny $\sum M_L^D = 0$ $-6R_E + 36 + 6 \cdot 6 \cdot 3 + \frac{1}{2} \cdot 6 \cdot 12 \cdot \frac{2}{3} \cdot 6 = 0$

$$R_E = 48 \text{ kN}$$

War. równowagi globalnej $\sum y = 0$ $-V_A - 24 - R_C - R_E + \frac{1}{2} \cdot 9 \cdot 18 = 0$

$$R_C = 24 - 24 - 48 + 81$$

$$R_C = 33 \text{ kN}$$

$$\sum x = 0 \rightarrow H_A = 0$$

$$\sum M_A = 0, \quad M_A - 6R_C - 6 \cdot 24 - 15 \cdot R_E + 36 + \frac{1}{2} \cdot 9 \cdot 18 \cdot 12 = 0$$

$$M_A = 6 \cdot 33 + 6 \cdot 24 + 15 \cdot 48 - 36 - 81 \cdot 12$$

$$M_A = 54 \text{ kN}$$

Sperrdrehung

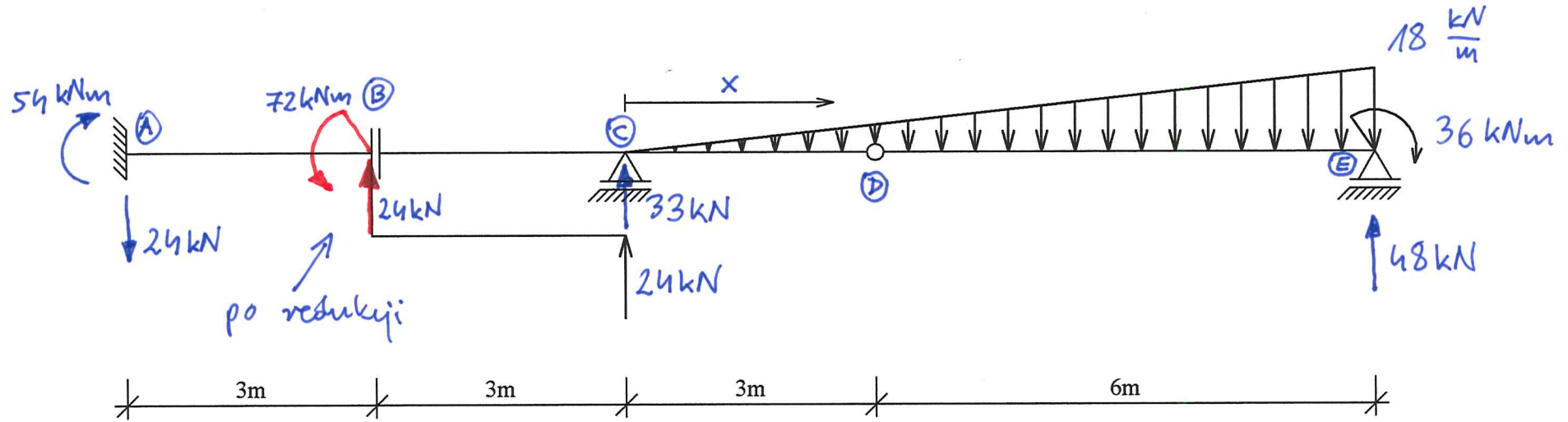
$$\sum M_D = 0$$

$$M_A + 9V_A + 3R_C + 3 \cdot 24 - 6 \cdot R_E + 36 + \frac{1}{2} \cdot 9 \cdot 18 \cdot 3 =$$

$$= 54 + 9 \cdot (-24) + 3 \cdot 33 + 3 \cdot 24 - 6 \cdot 48 + 36 + 81 \cdot 3 =$$

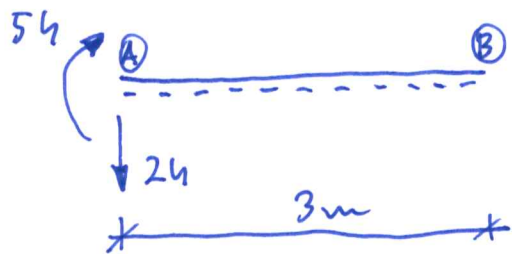
$$= 504 - 504 = 0$$

SCHEMAT OBLICZENIOWY



3. Siły przekrojowe

3.1. Przedział A-B



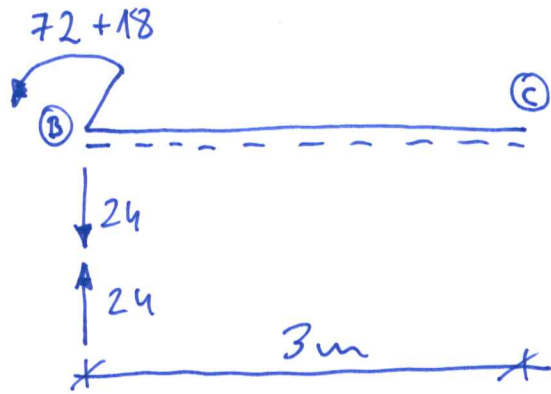
$$M_{AB} = 54 \text{ kNm}$$

$$M_{BA} = 54 - 3 \cdot 24 = -18 \text{ kNm}$$

$$T_{BA} = T_{AB} = -24 \text{ kN}$$

$$N_{BA} = N_{AB} = 0$$

3.2. Prędykt B-C



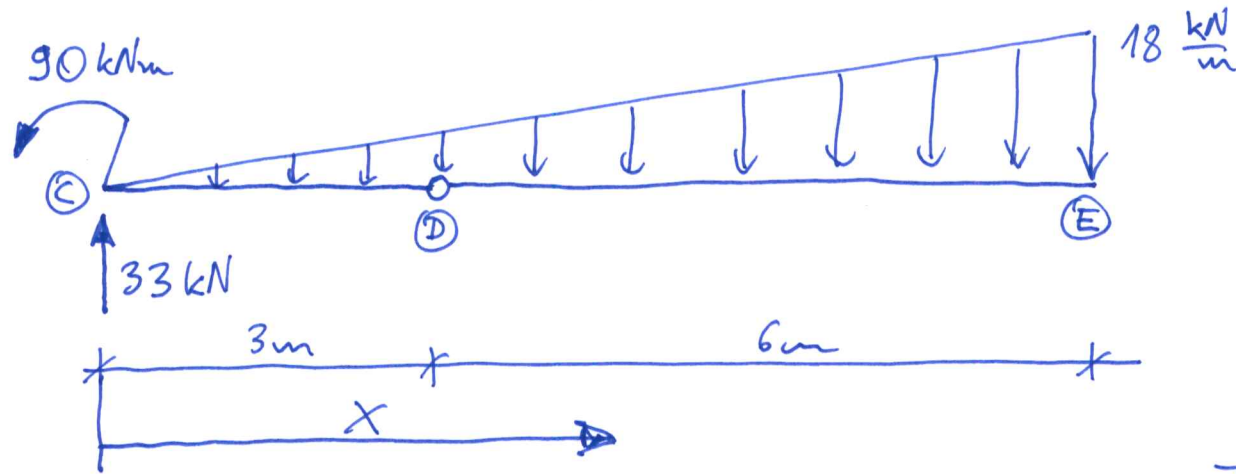
$$M_{BC} = -72 - 18 = -90 \text{ kNm}$$

$$M_{CB} = -72 - 18 = -90 \text{ kNm}$$

$$T_{BC} = T_{CB} = 24 - 24 = 0$$

$$N_{BC} = N_{CB} = 0$$

3.3. Prędykt C-D-E ($x \in <0; 9>$)



$$M_{CE}(x) = -90 + 33x - \frac{1}{2}x \cdot q_x \cdot \frac{1}{3}x$$

$$= -90 + 33x - \frac{1}{2}x \cdot \frac{1}{3}x \cdot 2x =$$

$$= -\frac{1}{3}x^3 + 33x - 90$$

$$T_{CE}(x) = 33 - \frac{1}{2}x \cdot 2x = -x^2 + 33$$

$$N_{CE}(x) \equiv 0$$

$$T_{CE}(x_0) = 0$$

$$-x^2 + 33 = 0$$

$$x_0 = \underline{\underline{5,745 \text{ m}}} \quad - \text{niejsze ekstremalnego momentu}$$

$$T_{CE}(0) = 33 \text{ kN}$$

$$T_{CE}(3\text{m}) = 24 \text{ kN}$$

$$T_{CE}(5,745\text{m}) = 0$$

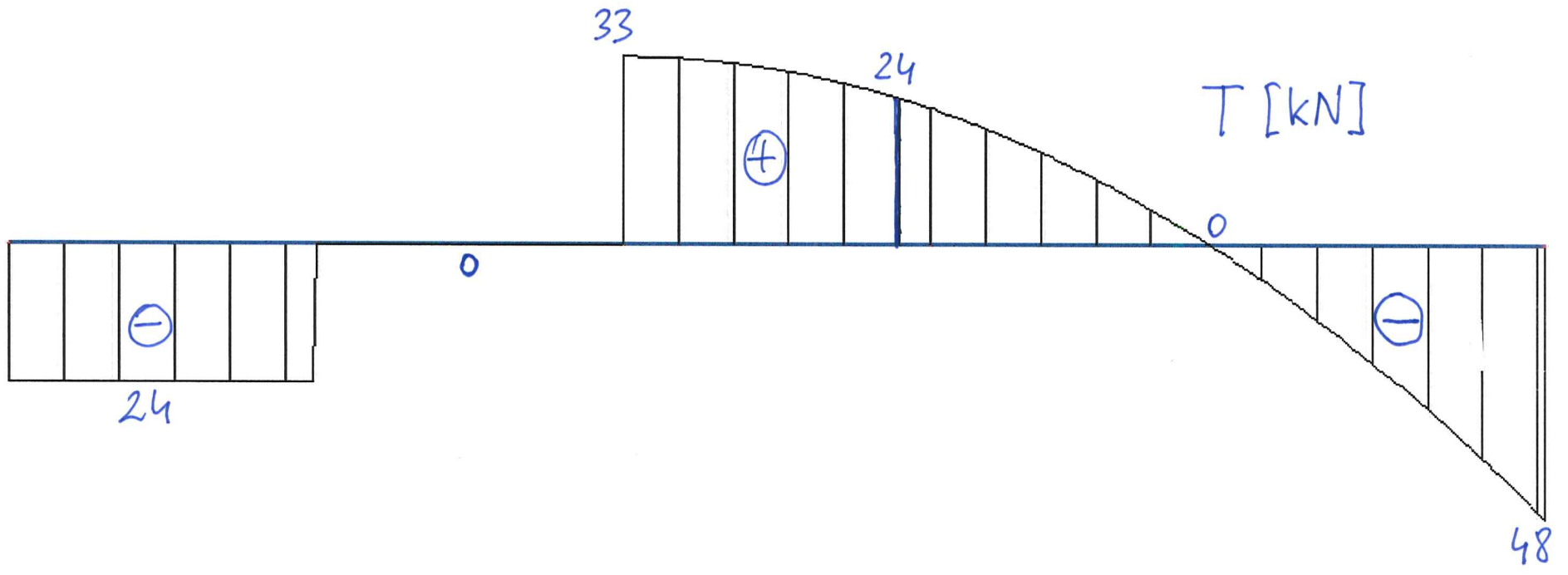
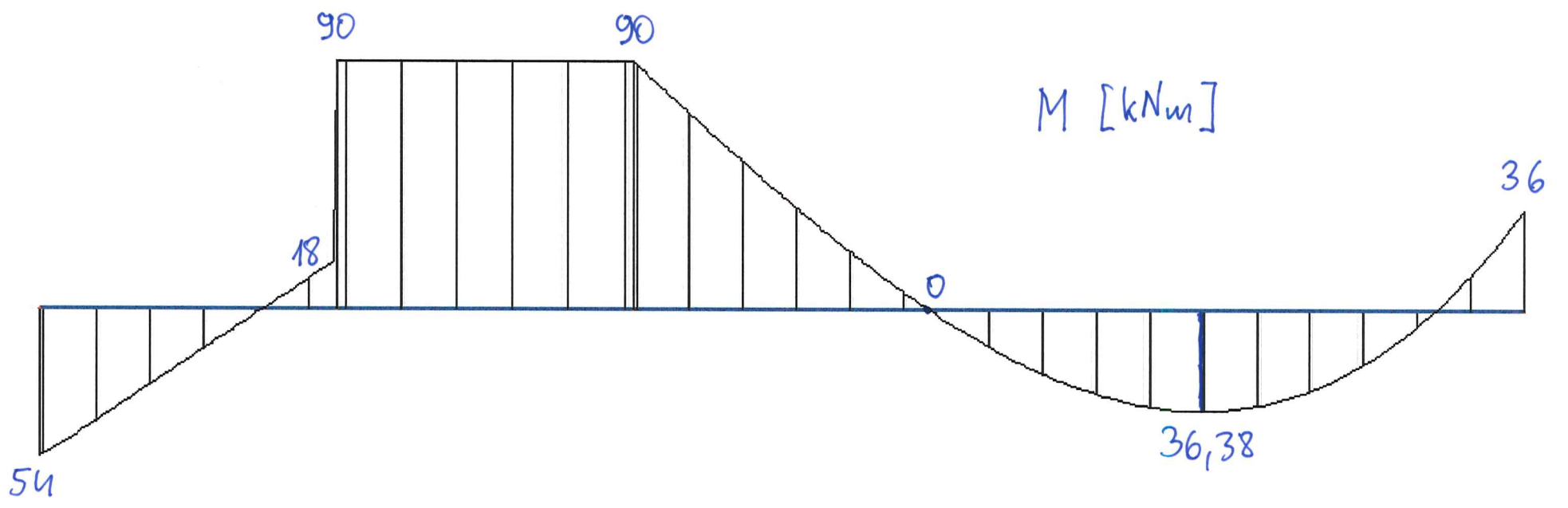
$$T_{CE}(9\text{m}) = -48 \text{ kN}$$

$$M_{CE}(0) = -90 \text{ kNm}$$

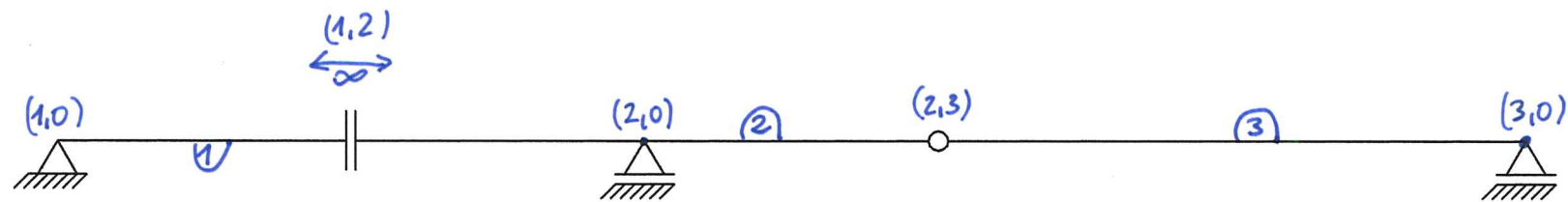
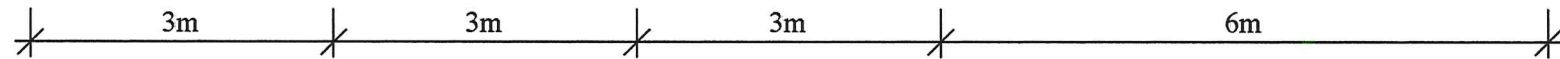
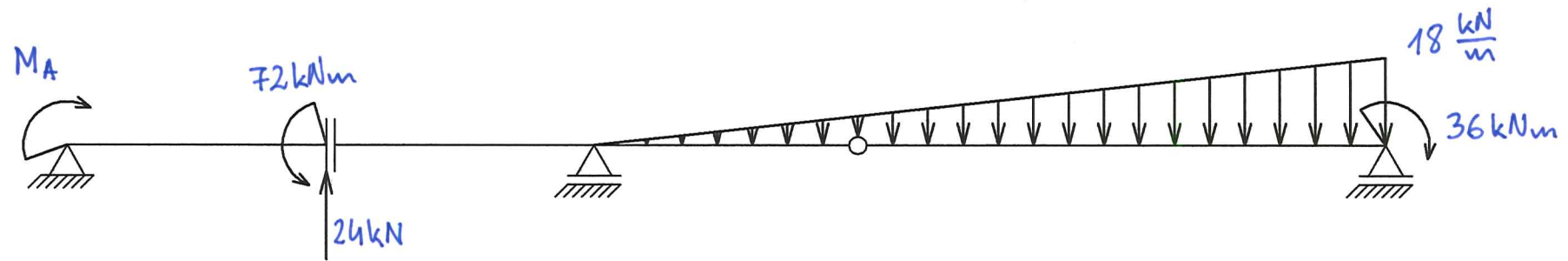
$$M_{CE}(3\text{m}) = 0$$

$$M_{CE}(5,745\text{m}) = M_{CE}^{\max} = 36,38 \text{ kNm}$$

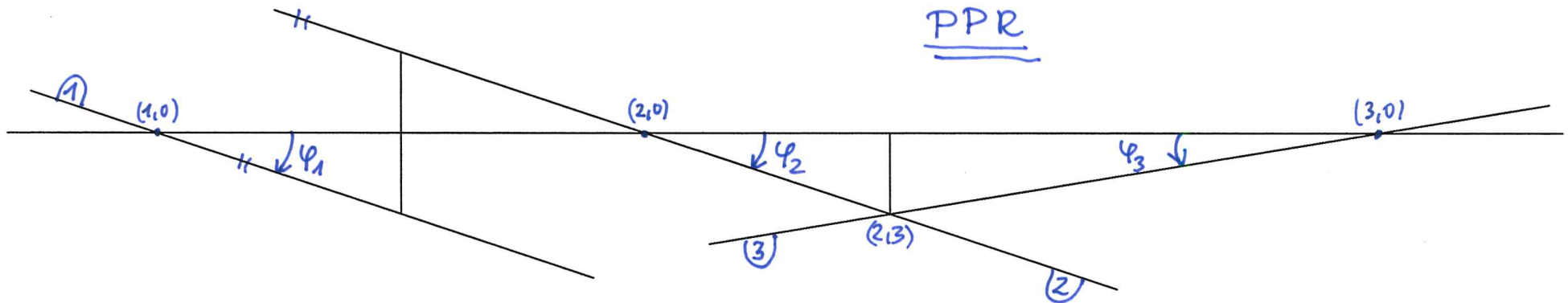
$$M_{CE}(9\text{m}) = -36 \text{ kNm}$$



MECHANIZM „M_A”



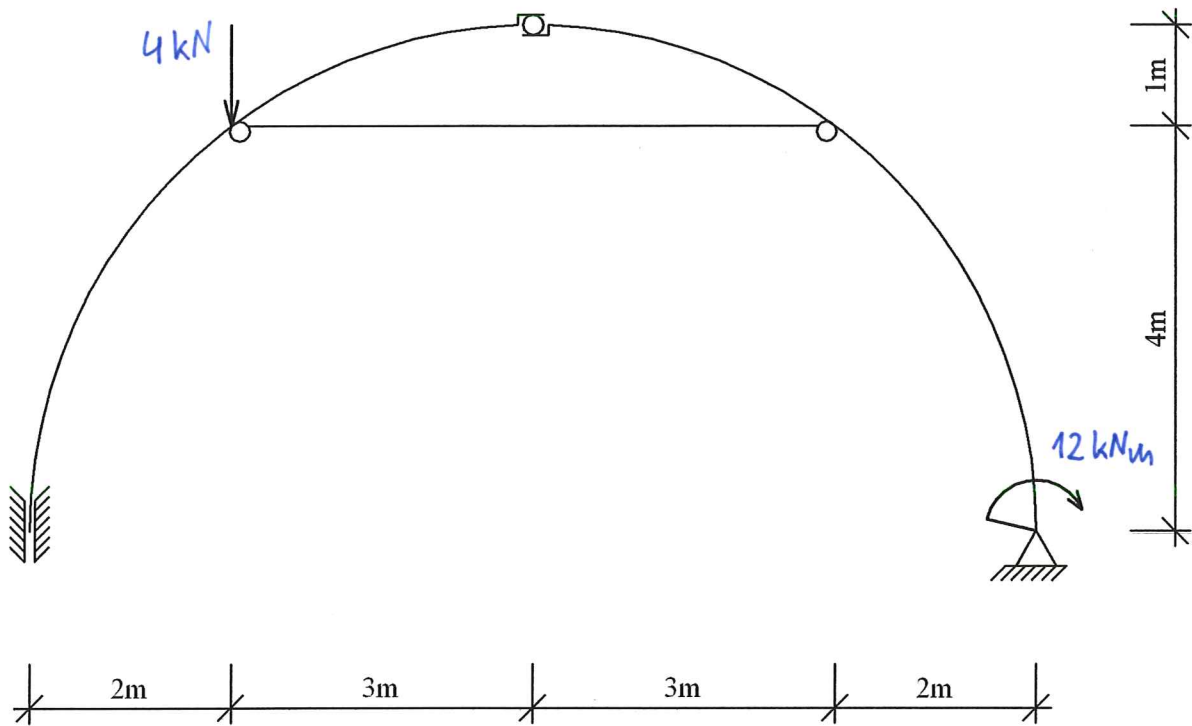
PPR



Warunki zgodności przemieszczeń: $\psi_1 = \psi_2$, $3\psi_2 = 6\psi_3$

$$L = M_A \cdot \psi_1 - 72 \cdot \psi_1 - 3\psi_1 \cdot 24 + \frac{1}{2} \cdot 3 \cdot 6 \cdot 2\psi_2 + 6 \cdot 6 \cdot 3\psi_3 + \frac{1}{2} \cdot 6 \cdot 12 \cdot 2\psi_3 - 36 \cdot \psi_3 = 0$$

$$M_A = 72 + 72 - 18 - 54 - 36 + 18 \rightarrow \boxed{M_A = 54 \text{ kNm}}$$



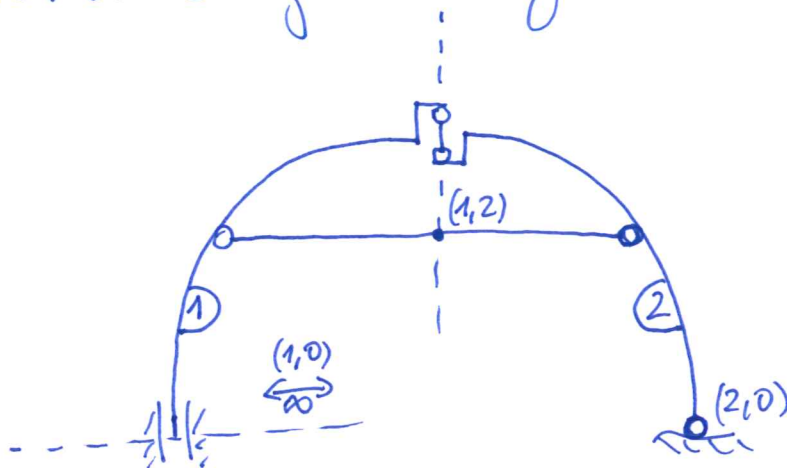
1 Sprawdzenie GN i SW

* warunki ilościowy

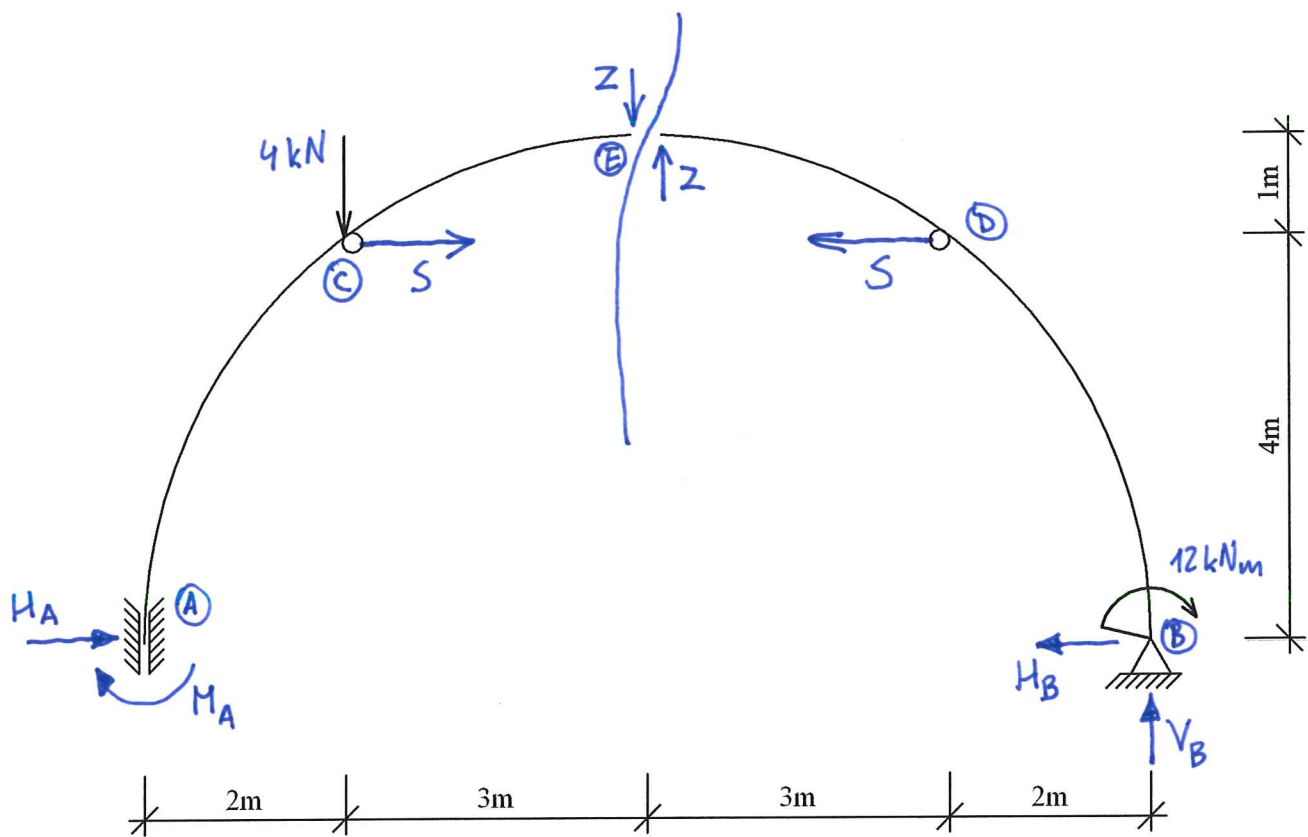
$$n = e - 3t \quad , \quad e = 9 \quad , \quad t = 3$$

$$n = 9 - 3 \cdot 3 = 0$$

* warunki jakościowy



Na mocy twierdzenia o 3-ech warunkach wystarczy GN, a przy tym SW ($n=0$).



2. Wymaganie reakcji oraz sił w zwojniki i ściągę

Warunki globalny $\sum M_B = 0$

$$M_A - 8 \cdot 4 + 12 = 0$$

$$\underline{M_A = 20 \text{ kNm}}$$

$$\sum M_A = 0 \rightarrow M_A + 2 \cdot 4 + 12 - 10 V_B = 0$$

$$10 V_B = 20 + 8 + 12$$

$$\underline{V_B = 4 \text{ kN}}$$

Warunki konstrukcyjne

$$\sum y_P = 0 \rightarrow -Z - V_B = 0$$

$$Z = -4 \text{ kN}$$

$$\sum M_B^P = 0 \rightarrow 5 \cdot 2 - 4S + 12 = 0$$

$$5 \cdot (-4) - 4S + 12 = 0$$

$$\underline{S = -2 \text{ kN}}$$

$$\sum X_P = 0 \rightarrow -S - M_B = 0$$

$$\underline{M_B = 2 \text{ kN}}$$

Wannsch globalung

$$\sum X = 0 \rightarrow H_A - M_B = 0$$

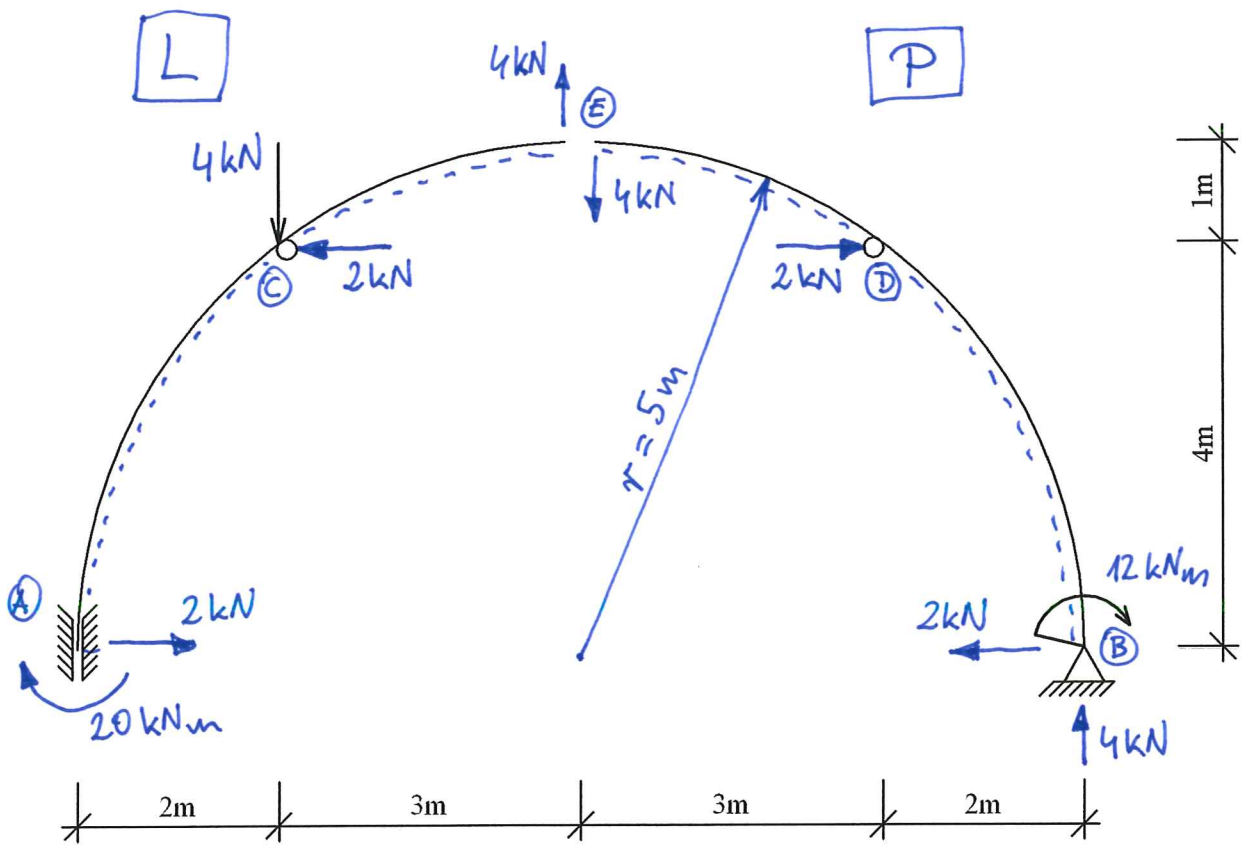
$$\underline{H_A = 2 \text{ kN}}$$

Sprennenie

$$\sum M_c = 0$$

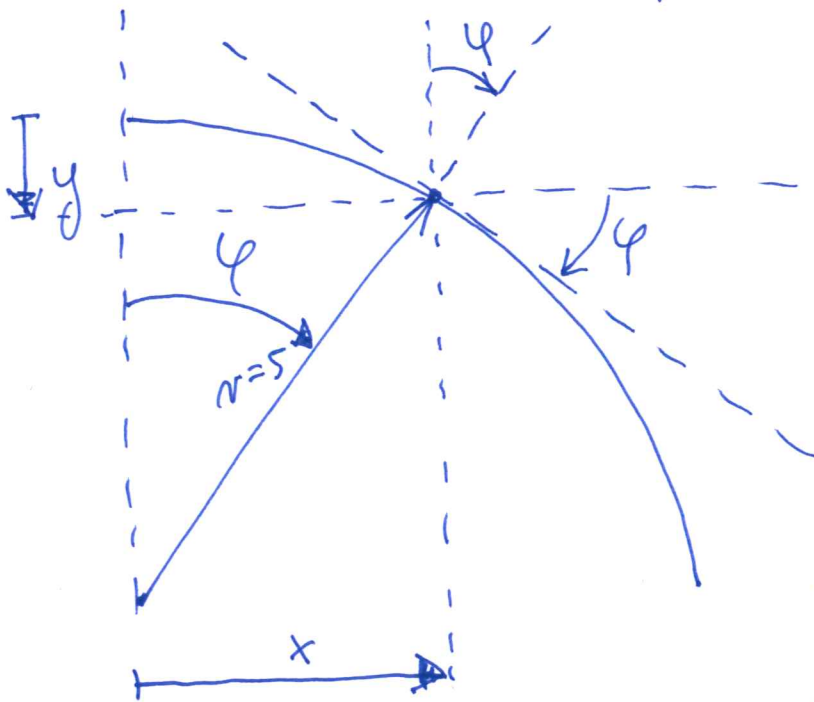
$$M_A - 4H_A + 4M_B - 8V_B + 12 =$$

$$= 20 - 4 \cdot 2 + 4 \cdot 2 - 8 \cdot 4 + 12 = 0$$



3. Wyznaczenie sił przekrojowych

3.1. Łuk z prawej **P** $0^\circ \leq \varphi \leq 36,87^\circ$



$$y = r - r \cos \varphi$$

$$x = r \sin \varphi$$

$$\begin{aligned} M_{ED}(\varphi) &= -4 \cdot x = \\ &= -4 \cdot 5 \cdot \sin \varphi = \\ &= -20 \sin \varphi \end{aligned}$$

$$T_{ED}(\varphi) = -4 \cdot \cos \varphi$$

$$N_{ED}(\varphi) = -4 \sin \varphi$$

φ [deg]	$M_{ED}(\varphi)$ kNm	$T_{ED}(\varphi)$ kN	$N_{ED}(\varphi)$ kN
0	0,000	-4,000	0,000
5	-1,743	-3,985	-0,349
10	-3,473	-3,939	-0,695
15	-5,176	-3,864	-1,035
20	-6,840	-3,759	-1,368
25	-8,452	-3,625	-1,690
30	-10,000	-3,464	-2,000
36,87	-12,000	-3,200	-2,400

3.2. Łuk z pręcej \boxed{P} $36,87^\circ \leq \varphi \leq 90^\circ$

$$\begin{aligned}
 M_{DB}(\varphi) &= -4 \cdot x + 2 \cdot (y-1) = \\
 &= -4 \cdot 5 \cdot \sin \varphi + 2 \cdot (5 - 5 \cos \varphi - 1) = \\
 &= -20 \sin \varphi + 8 - 10 \cos \varphi
 \end{aligned}$$

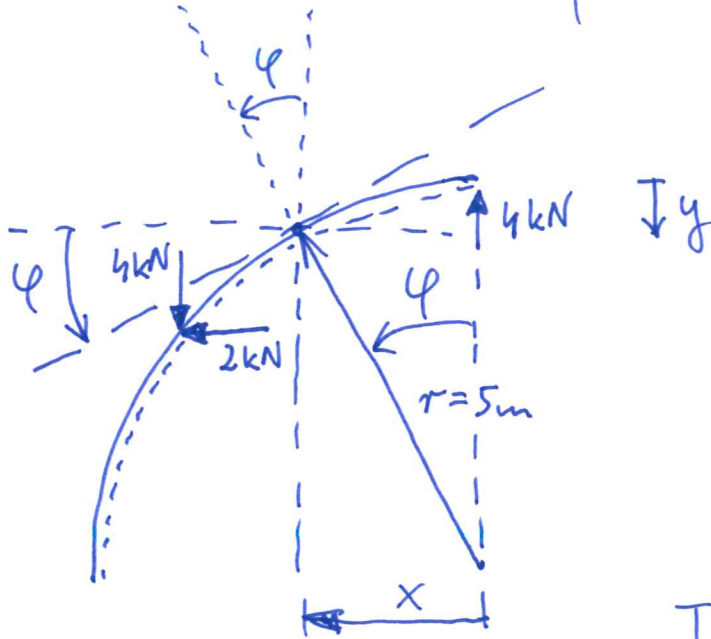
$$T_{DB}(\varphi) = -4 \cdot \cos \varphi + 2 \cdot \sin \varphi$$

$$N_{DB}(\varphi) = -4 \cdot \sin \varphi - 2 \cdot \cos \varphi$$

φ [deg]	$M_{DB}(\varphi)$ kNm	$T_{DB}(\varphi)$ kN	$N_{DB}(\varphi)$ kN
36,87	-12,000	-2,000	-4,000
40	-12,516	-1,779	-4,103
45	-13,213	-1,414	-4,243
50	-13,749	-1,039	-4,350
55	-14,119	-0,656	-4,424
60	-14,321	-0,268	-4,464
65	-14,352	0,122	-4,470
70	-14,214	0,511	-4,443
75	-13,907	0,897	-4,381
80	-13,433	1,275	-4,287
85	-12,795	1,644	-4,159
90	-12,000	2,000	-4,000

3.3. Lük 2 lewyj L

$$0^\circ \leq \varphi \leq 36,87^\circ$$



$$y = r - r \cos \varphi$$

$$x = r \sin \varphi$$

$$M_{EC}(\varphi) = 4 \cdot x =$$

$$= 4 \cdot 5 \cdot \sin \varphi = 20 \sin \varphi$$

$$T_{EC}(\varphi) = -4 \cdot \cos \varphi$$

$$N_{EC}(\varphi) = 4 \cdot \sin \varphi$$

φ [deg]	$M_{EC}(\varphi)$ kNm	$T_{EC}(\varphi)$ kN	$N_{EC}(\varphi)$ kN
0	0,000	-4,000	0,000
5	1,743	-3,985	0,349
10	3,473	-3,939	0,695
15	5,176	-3,864	1,035
20	6,840	-3,759	1,368
25	8,452	-3,625	1,690
30	10,000	-3,464	2,000
36,87	12,000	-3,200	2,400

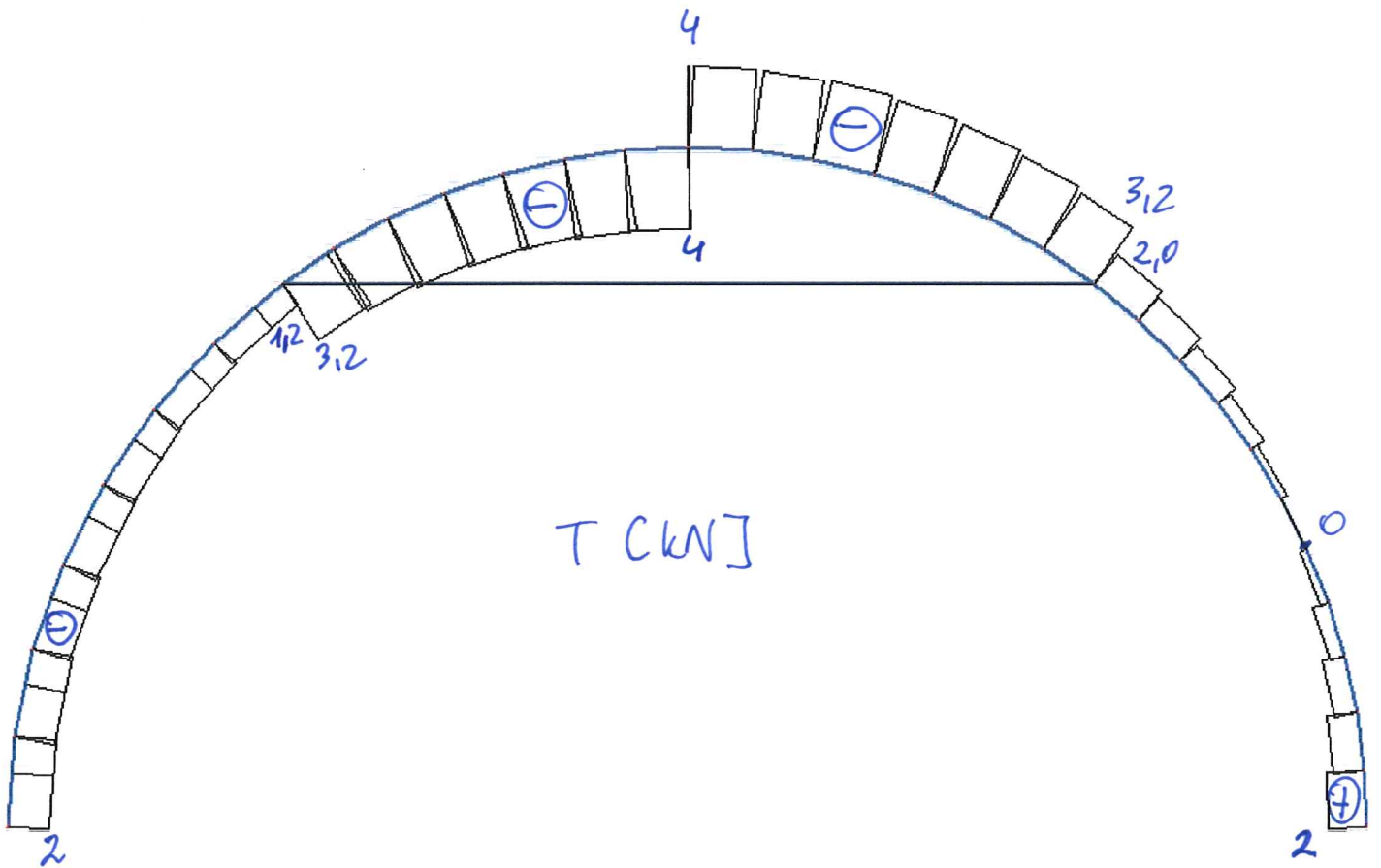
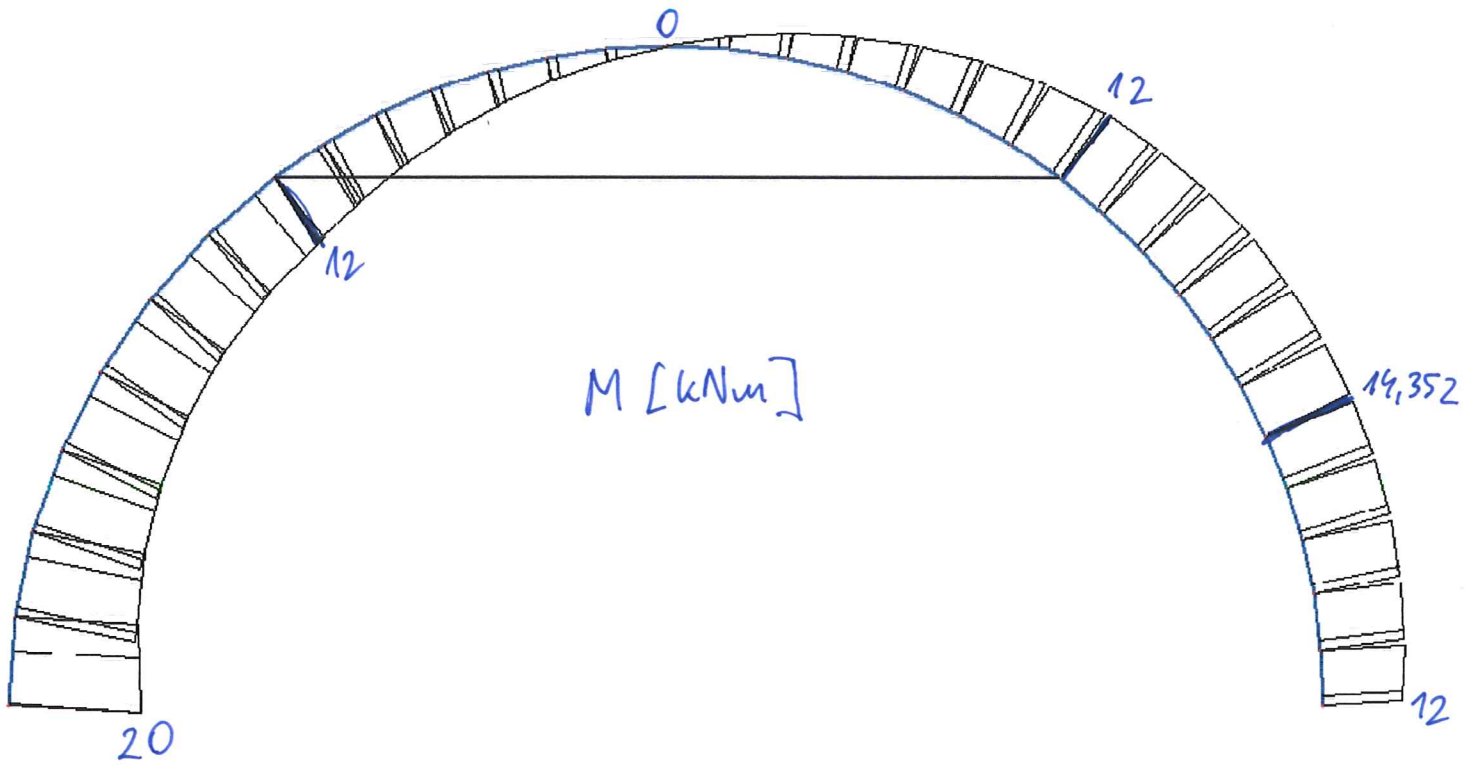
3.4. zůvek 2 lewuj \square $36,87^\circ \leq \varphi \leq 90^\circ$

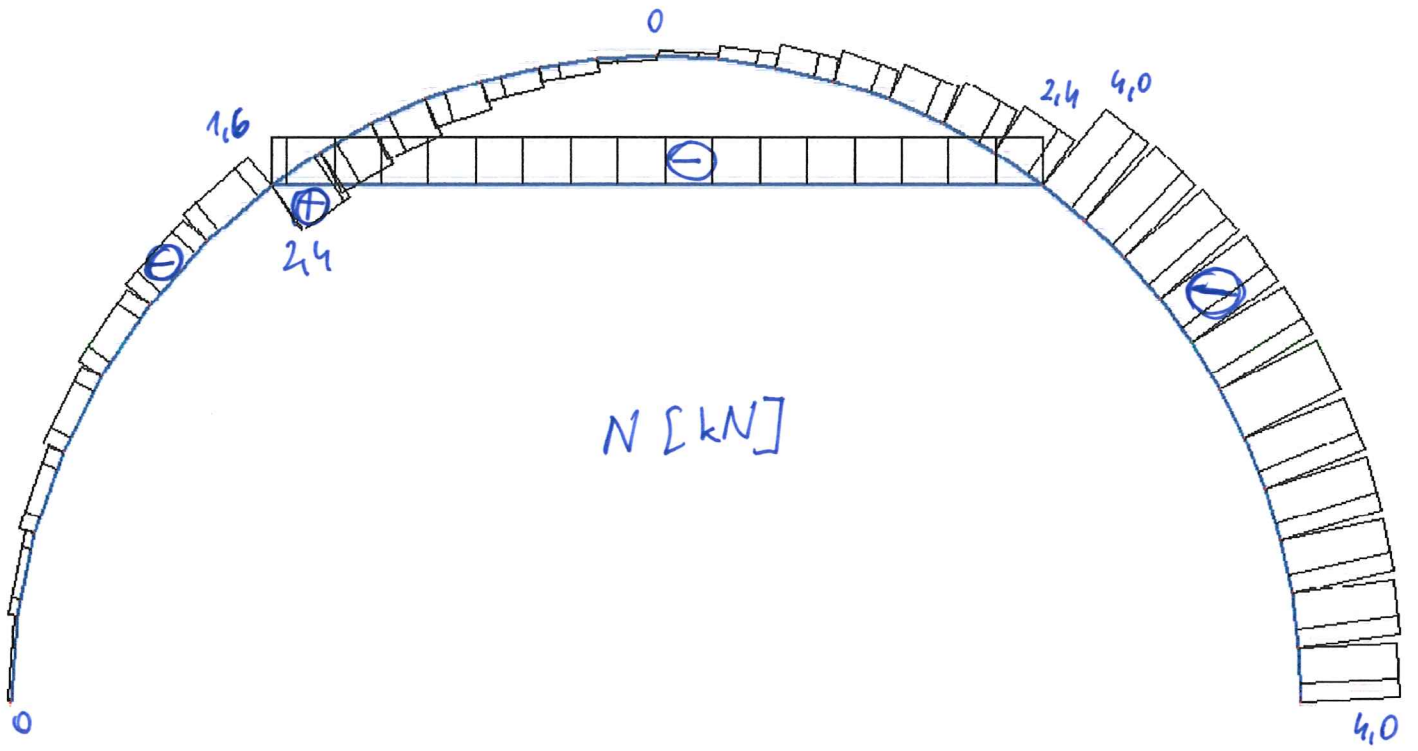
$$\begin{aligned}M_{CA}(\varphi) &= 4x - 4(x-3) + 2(y-1) = \\&= 4 \cdot 5 \sin \varphi - 4(5 \sin \varphi - 3) + 2(5 - 5 \cos \varphi - 1) = \\&= 12 + 8 - 10 \cos \varphi = 20 - 10 \cos \varphi\end{aligned}$$

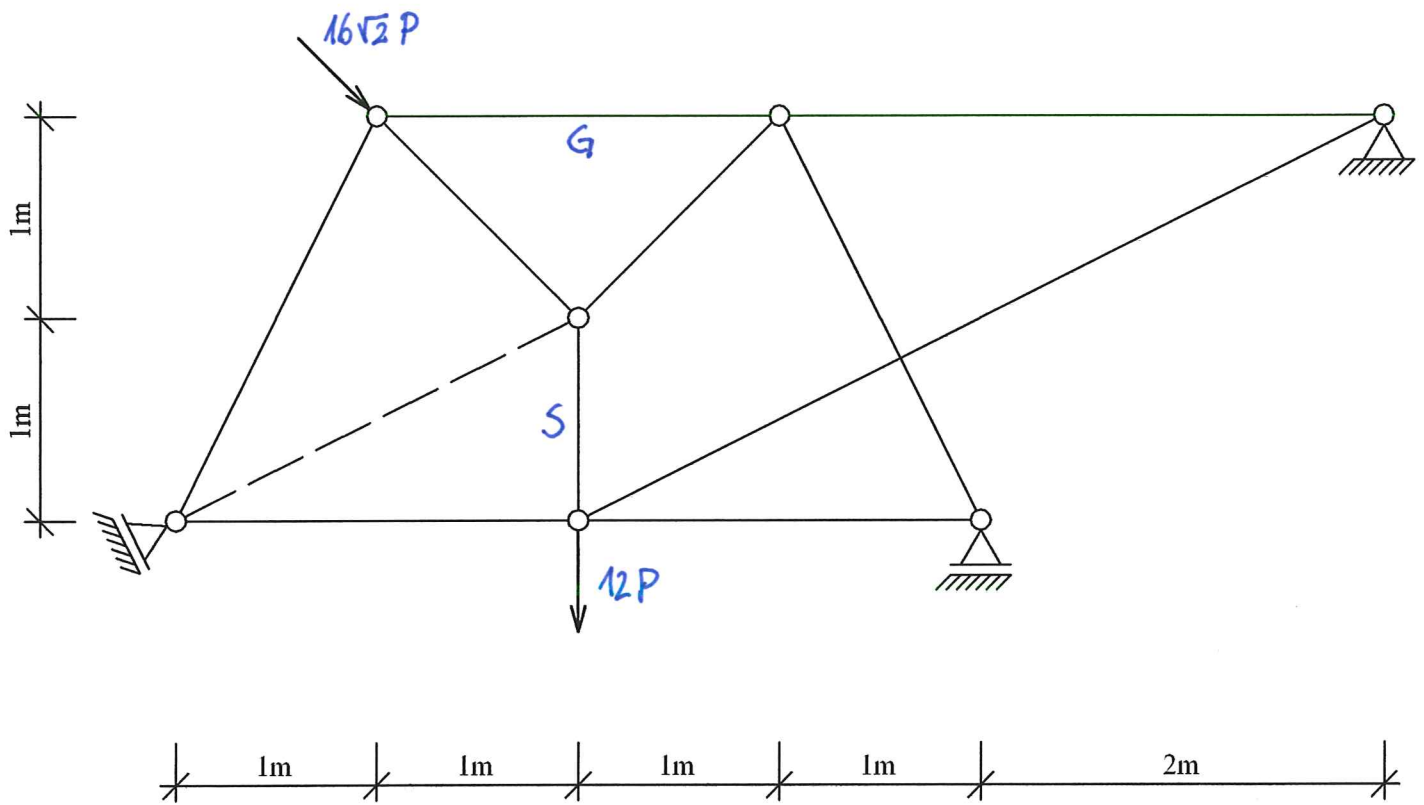
$$T_{CA}(\varphi) = -4 \cos \varphi + 4 \cos \varphi - 2 \sin \varphi = -2 \sin \varphi$$

$$N_{CA}(\varphi) = 4 \sin \varphi - 4 \sin \varphi - 2 \cos \varphi = -2 \cos \varphi$$

φ [deg]	$M_{CA}(\varphi)$ kNm	$T_{CA}(\varphi)$ kN	$N_{CA}(\varphi)$ kN
36,87	12,000	-1,200	-1,600
40	12,340	-1,286	-1,532
45	12,929	-1,414	-1,414
50	13,572	-1,532	-1,286
55	14,264	-1,638	-1,147
60	15,000	-1,732	-1,000
65	15,774	-1,813	-0,845
70	16,580	-1,879	-0,684
75	17,412	-1,932	-0,518
80	18,264	-1,970	-0,347
85	19,128	-1,992	-0,174
90	20,000	-2,000	0,000







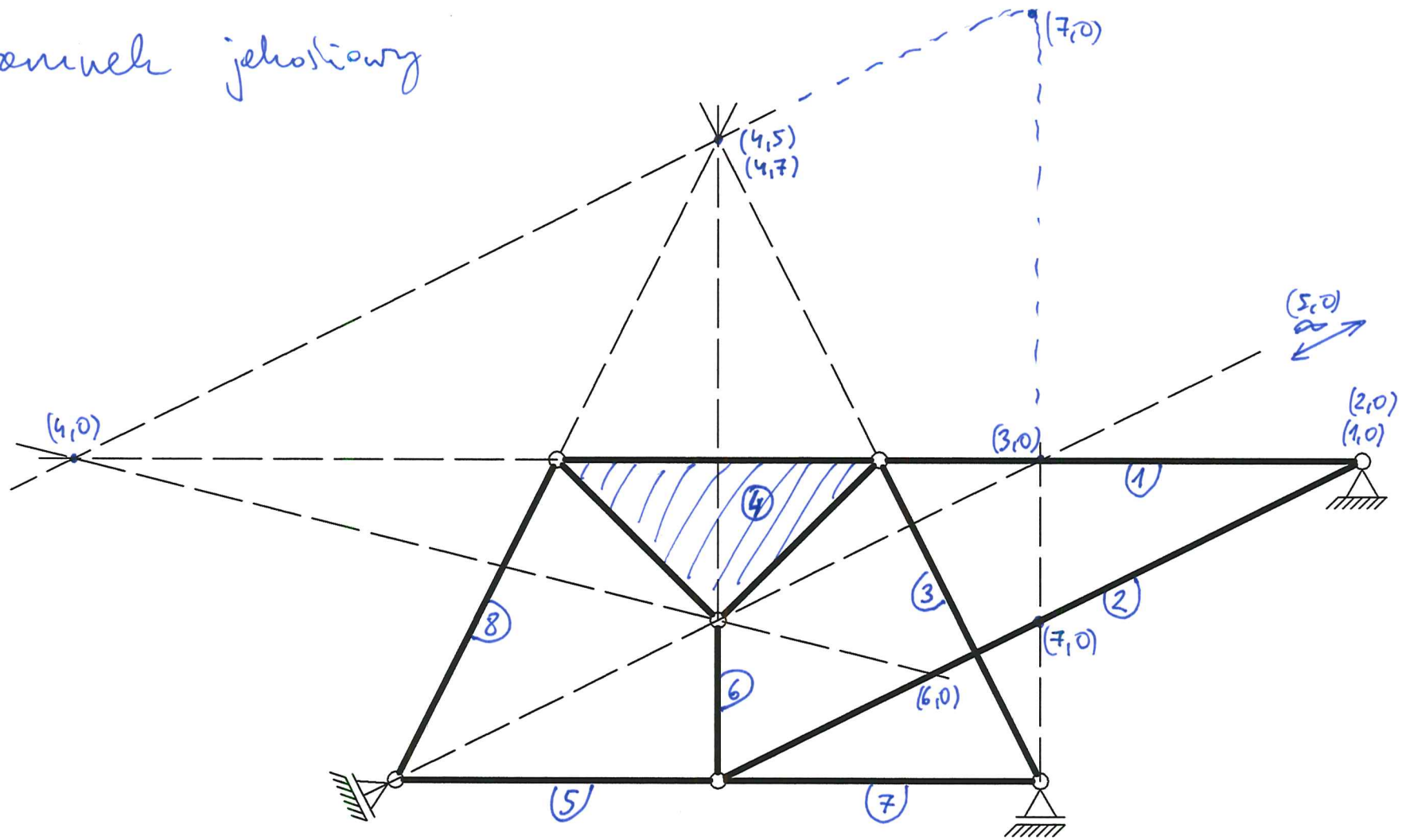
1. Sprawdzenie GN i SW

* warunki ilościowy

$$n = 2W - p - r$$

$$n = 2 \cdot 7 - 10 - 4 = 0$$

* warunki jęhoskowy



a) (1,0), (2,0) - w przębie

b) (3,0) - $3 \begin{matrix} e \\ 1 \\ 0 \end{matrix}$

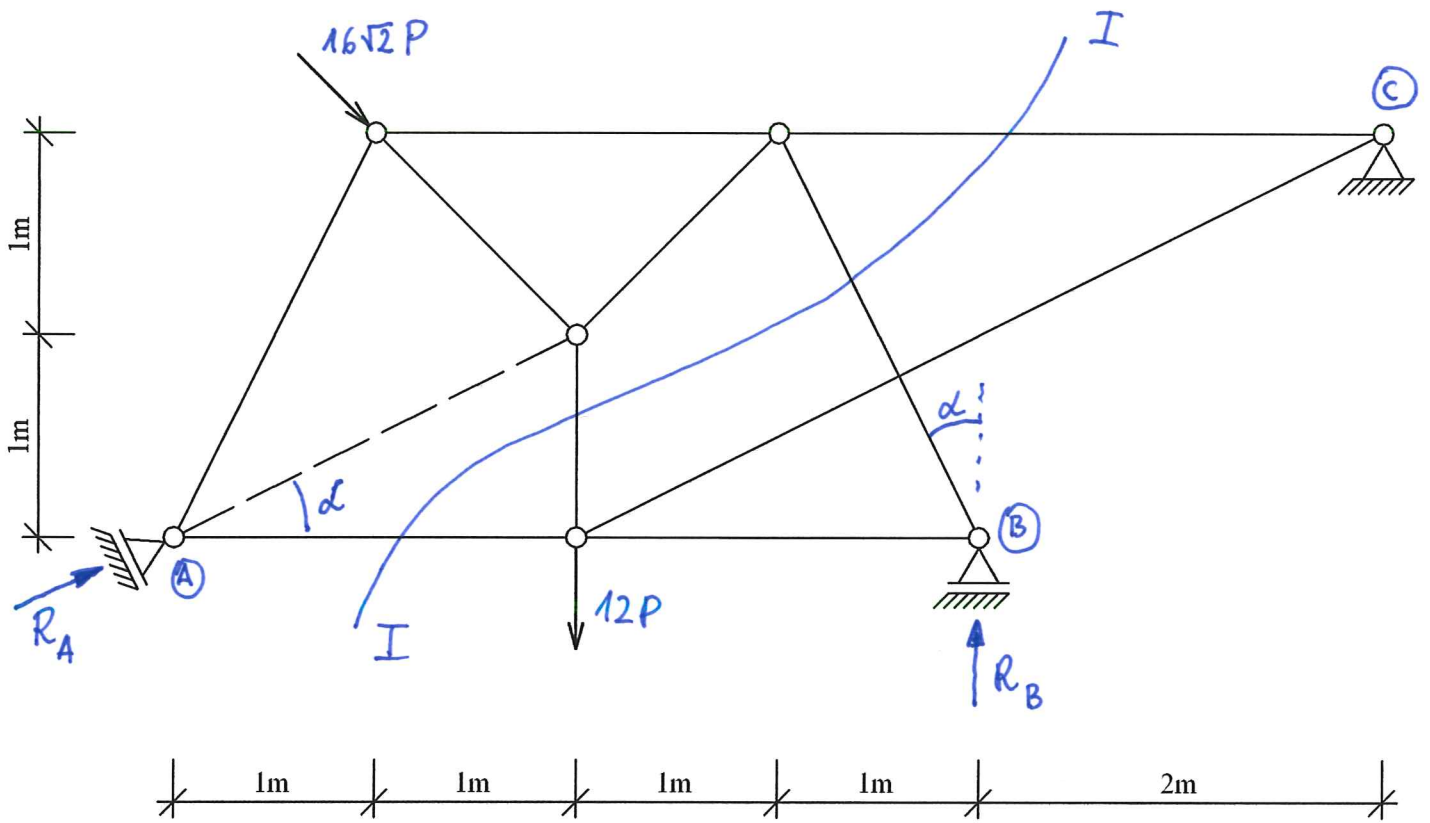
c) (5,0) - $5 \begin{matrix} e \\ 2 \\ 0 \end{matrix}$

d) (4,0) - $4 \begin{matrix} 1 \\ 5 \\ 0 \end{matrix}$, (4,5) - $4 \begin{matrix} 6 \\ 8 \\ 5 \end{matrix}$

e) (6,0) - $6 \begin{matrix} 4 \\ 2 \\ 0 \end{matrix}$

f) (7,0) - $7 \begin{matrix} 4 \\ e \\ 0 \end{matrix}$ lub $7 \begin{matrix} 2 \\ e \\ 0 \end{matrix}$ -

↓
↓
symmetry w położeniu
środków ciężkości → udział GN



2. Wyznaczenie sił S oraz G bez użycia reakcji

$$\sum M_C = 0$$

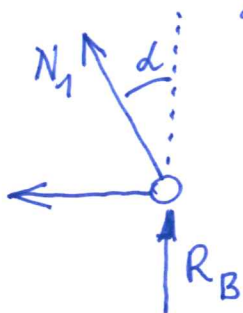
$$\sin \alpha = \frac{1}{\sqrt{5}}, \quad \cos \alpha = \frac{2}{\sqrt{5}}$$

$$-2 \cdot \frac{2}{\sqrt{5}} R_A + 6 \cdot \frac{1}{\sqrt{5}} R_A - 5 \cdot 16P - 4 \cdot 12P + 2 \cdot R_B = 0$$

$$2R_B = -\frac{2}{\sqrt{5}} R_A + 128P$$

$$R_B = 64P - \frac{1}{\sqrt{5}} R_A$$

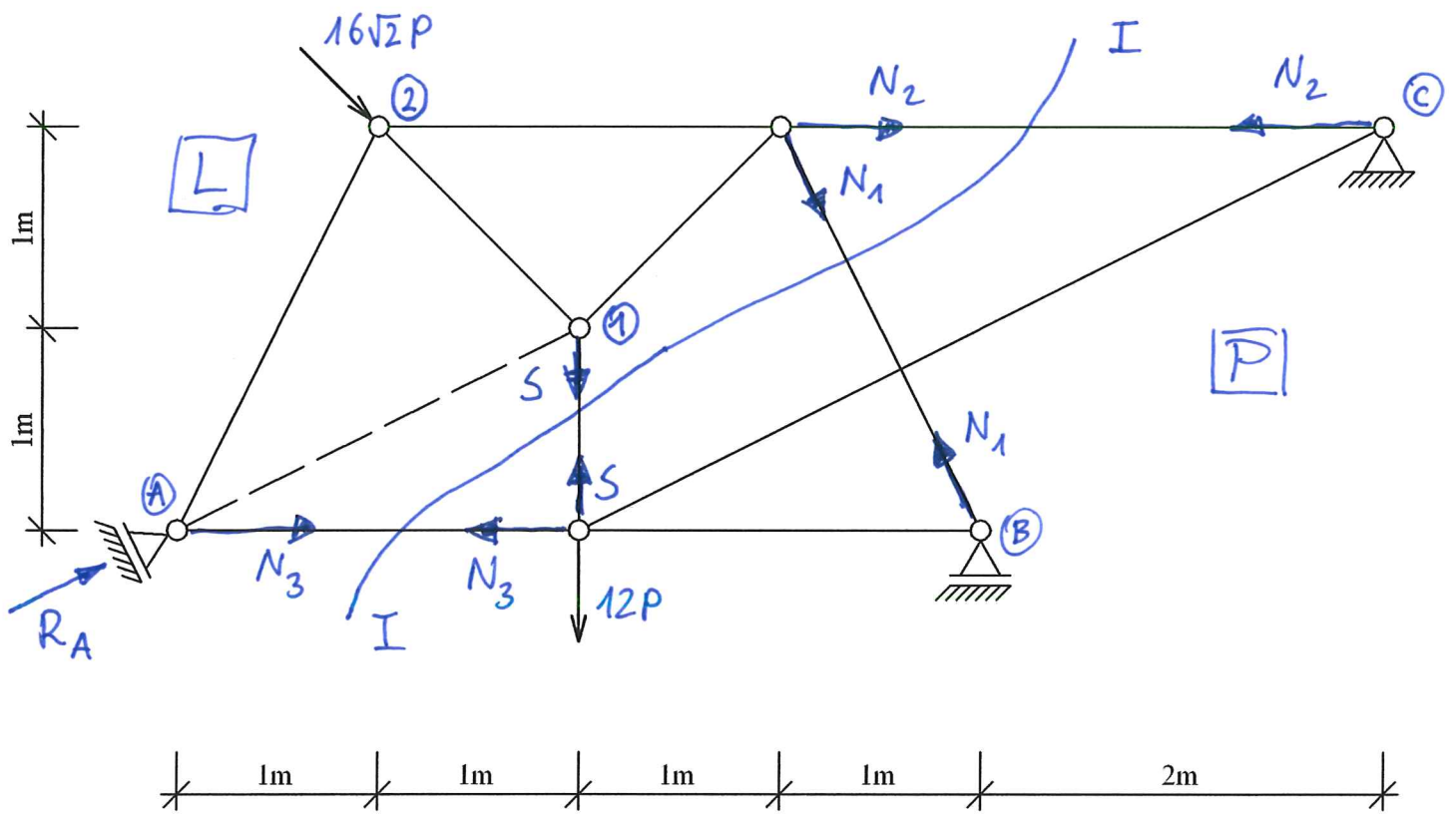
* równowaga węzła B



$$\sum y = 0$$

$$-\frac{2}{\sqrt{5}} N_1 - R_B = 0$$

$$N_1 = \frac{1}{2} R_A - 32\sqrt{5}P$$



* präzise Ritterer I-I

$$\sum y_L = 0$$

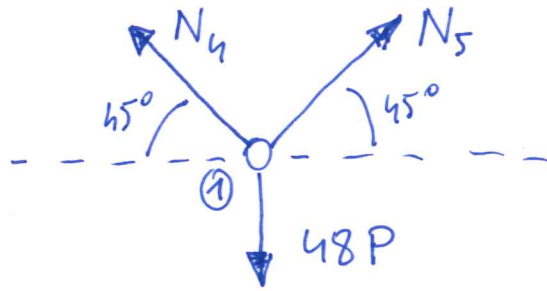
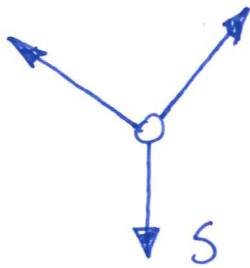
$$S + \frac{2}{\sqrt{5}} N_1 + 16P - \frac{1}{\sqrt{5}} R_A = 0$$

$$S = -\frac{2}{\sqrt{5}} \left(\frac{1}{2} R_A - 32\sqrt{5}P \right) - 16P + \frac{1}{\sqrt{5}} R_A$$

$$S = -\frac{1}{\sqrt{5}} R_A + \frac{1}{\sqrt{5}} R_A + 64P - 16P$$

$$\underline{S = 48P}$$

* Równowaga węzła (1)



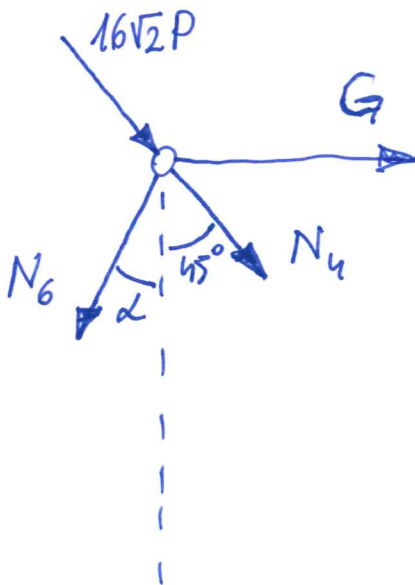
$$\sum x = 0 \rightarrow N_4 = N_5$$

$$\sum y = 0 \rightarrow -\frac{\sqrt{2}}{2}N_4 - \frac{\sqrt{2}}{2}N_5 + 48P = 0$$

$$\sqrt{2}N_4 = 48P$$

$$N_4 = 24\sqrt{2}P$$

* Równowaga węzła (2)



$$\sum y = 0$$

$$16P + \frac{\sqrt{2}}{2}N_4 + \frac{2}{\sqrt{5}}N_6 = 0$$

$$16P + 24P + \frac{2}{\sqrt{5}}N_6 = 0$$

$$N_6 = -20\sqrt{5}P$$

$$\sum x = 0$$

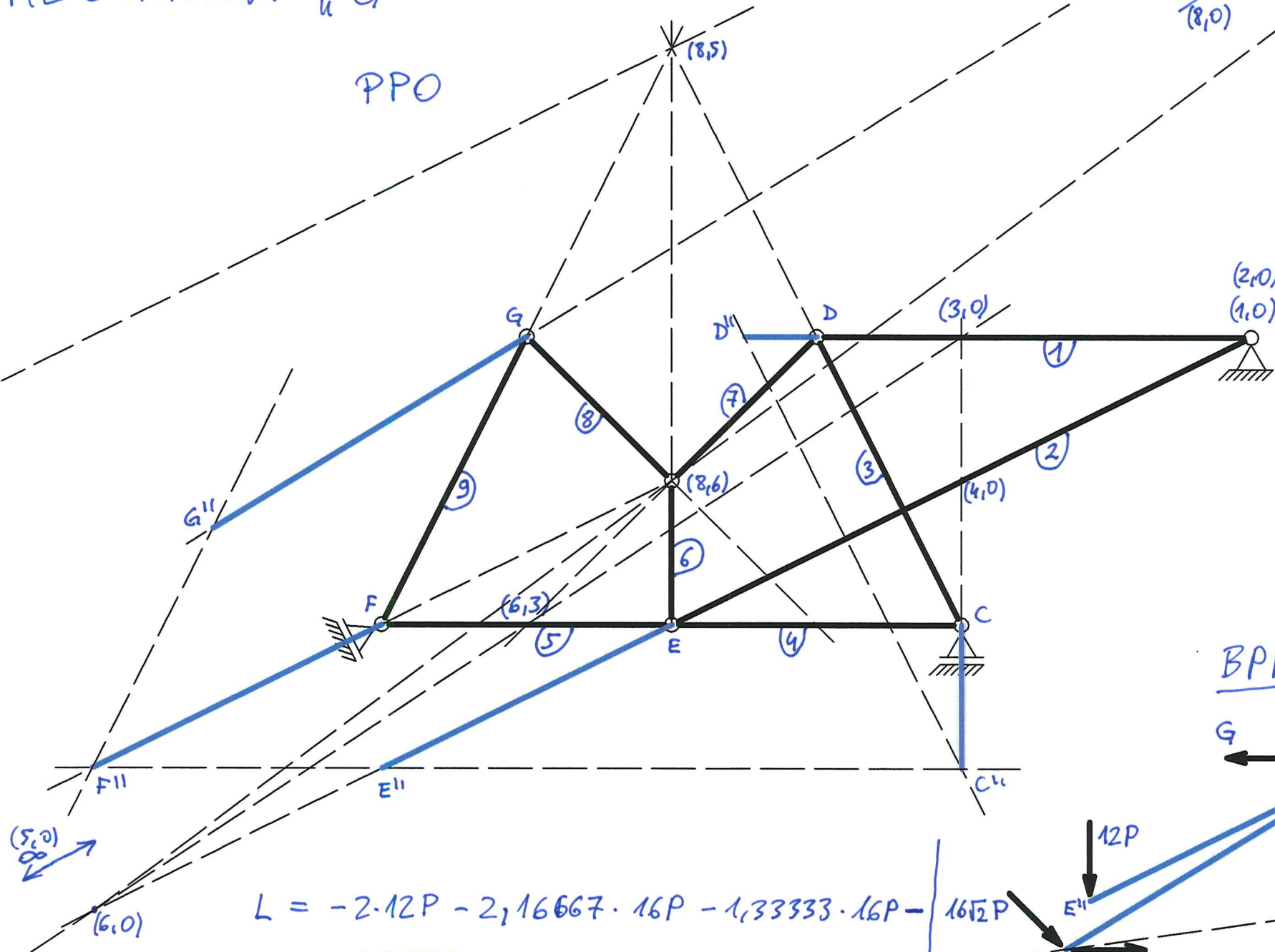
$$16P + G + \frac{\sqrt{2}}{2}N_4 - \frac{1}{\sqrt{5}}N_6 = 0$$

$$16P + G + 24P + 20P = 0$$

$$\underline{\underline{G = 60P}}$$

MECHANIZM u G''

PPO



$(1,0)$
 $(2,0)$ w
pnequbic

$$(3,0) - 3^1 e_0$$

$$(4,0) - 4^2 e_0$$

$$(5,0) - 5^2 e_0$$

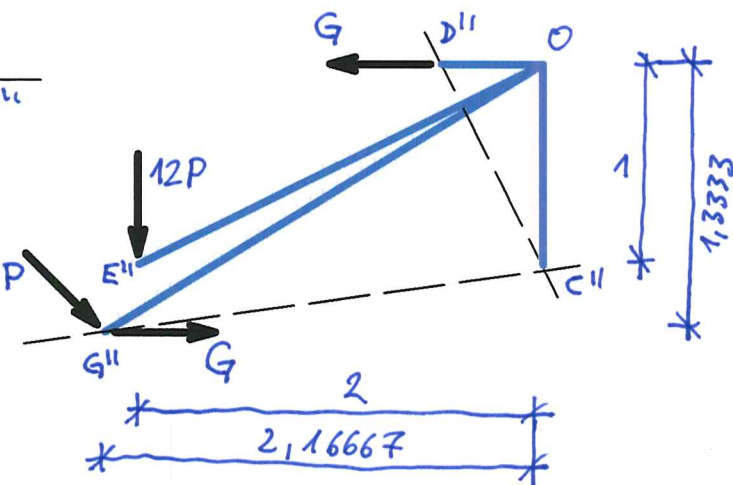
$$(6,0) - 6^3 e_0$$

$$(6,3) - 6^4 e_3$$

$$(8,0) - 8^5 e_0$$

$$(8,5) - 8^6 e_5$$

BPPO



$$L = -2 \cdot 12P - 2,16667 \cdot 16P - 1,33333 \cdot 16P - 16\sqrt{2}P - 1,33333 \cdot G = 0$$

$$G = -60P$$

